Real-Option Valuation of Build-Operate-Transfer Infrastructure Projects under Performance Bonding

Yu-Lin Huang[1] and Chia-Chi Pi[2]

Abstract: This study assesses the effect of performance bonding on the valuation of a build-operate-transfer (BOT) project by extending the classical Black-Scholes-Merton (BSM) call option model. As common features in BOT contracts, a performance bond is a penalty imposed on concessionaires who exercise contractual rights to terminate participation in a project. In the real-option context, termination rights grant concessionaires the flexibility in managing market uncertainties that can increase the valuation of an infrastructure project, but the penalty impairs this flexibility and reduces valuations. A case study numerically illustrates the BSM model and indicates that performance bonding can destroy the flexibility and project valuations inherent in termination rights even when the penalty is moderate. Balancing performance bonds and termination rights is necessary because both are important in establishing and maintaining long-term contractual relationships in privatized BOT infrastructure projects. **DOI: 10.1061/(ASCE)CO.1943-7862.0000821.**

Author keywords: Build-operate-transfer; Performance bond; Real option; Infrastructure privatization; Valuation; Concession contract.

Background and Introduction

For nearly three decades, privatization has become an established approach for providing infrastructure services. Huang (1995) documented more than 80 privatized infrastructure projects worldwide, focusing on their institutional and regulatory designs. Tam (1999) and Kumaraswamy and Morris (2002) investigated build-operate-transfer (BOT) infrastructure projects in Asia. Chen and Messner (2005) investigated BOT water-supply projects in China. Kleiss and Imura (2006) investigated private finance initiatives (PFI) in Japan. Winch (2000) investigated PFI public work projects in the United Kingdom. Koch and Buser (2006) investigated governance of public-private partnerships (PPP) in Denmark. Fischer et al. (2006) investigated PPP infrastructure developments in Germany. Vazquez and Allen (2004) investigated BOT highway projects in Central America and Mexico. Algarni et al. (2007) investigated BOT infrastructure projects in the United States.

Among PFI and other privatization approaches, the BOT model is favored by governments for large-scale infrastructure developments, which usually require long-term sunk investments in facilities exposed to substantial uncertainties (Grimsey and Lewis 2002). To attract private sector investment in BOT projects, host governments offer risk-management concessions, including loan repayment and minimum revenue guarantees, rights to incremental expansions, and rights to premature termination (Huang 1995; Wibowo 2004). Availability of these arrangements means that traditional methods, such as discounted cash flow models, are no longer satisfactory for valuing BOT projects.

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Researchers have developed numerous real-option models to treat complex BOT valuation issues. For example, Rose (1998) evaluated interacting toll road investment options. Smit (2003) provided a real-option-based game theory model to evaluate airport expansions. Garvin and Cheah (2004) proposed a real-option pricing model for analyzing toll road investments. Wand and Min (2006) evaluated interrelationships among power generation projects. Huang and Chou (2006) evaluated minimum revenue guarantees. Mayer and Kazakidis (2007) provided a strategic decision-making tool based on real-option valuation techniques for mine planners to determine the value of flexibility-enhancing alternatives incorporated into the mine plan. Damnjanovic et al. (2008) evaluated the interconnectivity and flexibility of toll road expansions. Menassa et al. (2009) evaluated alternative dispute resolution investments. Huang and Pi (2009) developed a European-style sequential compound call option (SCCO) model to evaluate multistage BOT projects involving dedicated asset investments. Huang and Pi (2011) extended the SCCO model to assess the effects of competition and technological obsolescence on project valuation in privatized infrastructure markets.

Real-option models have proven to be powerful valuation tools for complex BOT concession arrangements and for information technology and other types of investment projects. Panayi and Trigeorgis (1998) developed a real-option model to value multistage information technology projects. Yeo and Qiu (2003) discussed the valuation of investment flexibility in technology investment projects using the real-option approach. Chen et al. (2009) developed an integrated real-option approach to value information technology projects. Eckhause et al. (2009) developed a real-option approach for vendor selection in multistage R&D acquisitions.

Although the real-option approach is popular and powerful, the existing BOT real-option valuation models have not incorporated the influence of performance bonds on project valuation. At the time when contracts are signed, the value of BOT projects is uncertain and must be reevaluated as information arises during construction. This presents the possibility of hold-up. Host governments may seek court enforcement of strict contract terms, requiring concessionaires to continue construction even when re-evaluation deems it infeasible. BOT contracts often grant concessionaires termination rights that potentially forestall such hold-up events. According to Klein (1996), voluntary termination can prevent hold-up events. In turn, however, host governments may require concessionaires to accept a performance bond that, as Vandegrift (1998) suggested, assures concessionaires do not exercise termination rights arbitrarily.

In real-option theories, termination rights give concessionaires flexibility to minimize irreversible sunk investments under adverse market conditions (Dixit and Pindyck 1994). This flexibility is potentially valuable and can encourage private sector investments in BOT projects (Huang and Pi 2009, 2011). Nevertheless, a bonding requirement imposes a penalty that reduces the value of termination rights although their arbitrary exercise can be avoided. If the effect of performance bonds on a BOT project's valuation is not assessed using real-option valuation, its valuation would tend to be overstated, perhaps misleading investment decisions.

This study extends the classical Black-Scholes-Merton (BSM) model to a situational European-style call option to assess the impact of performance bonding. Risk-neutral pricing theory is applied to derive a closed-form pricing solution for the extended model. Sensitivity analysis shows that the value of the extended European-style call option is decreasing in the bond value. Numerical analysis of a case study shows that performance bonds can destroy the value of flexibility granted by termination rights even when the bond penalty is moderate. Final analysis suggests that performance bonds can increase what Klein (1995) called the self-enforcing range of contracts with incomplete performance specifications. Balancing performance bonds and termination rights is necessary to assure concessionaires' investment and performance.

This paper proceeds as follows: First, the valuation model is presented, and sensitivity analysis is provided to examine the effect of the performance bond; next, the Taiwan high-speed rail project case study is presented; the need to balance performance bonds and termination rights is discussed; and finally, results are summarized and future research directions are suggested.

The Valuation Model

As Fig. 1 shows, BOT projects usually involve three major stages: preconstruction (Stage I), construction (Stage II), and operation (Stage III). The concessionaire signs its contract with the host government at time, t_0, and is obligated to start construction at t_1. The parameter K denotes the project's total construction cost. If the concessionaire decides to invest K at t_1 and completes construction on time at t_2, the project would be operated until the concession period expires at t_n. If the concessionaire decides not to invest at t_1, then a performance bond, whose value is denoted by B, would be executed by the host government as compensation.

As described, this contractual setting presents the situational equivalent of a European-style call option for the concessionaire to decide whether to invest at t_1. When the concessionaire signs the contract at t_0, the project has an expected value of S_0. However, this valuation is uncertain because complete information is unavailable during preconstruction. The concessionaire reassesses the project's value between t_0 and t_1 when new information appears. At t_1, a rational concessionaire would invest if the reassessed valuation S_{t_1} equals or exceeds K, which by definition is the exercise price of the call option.

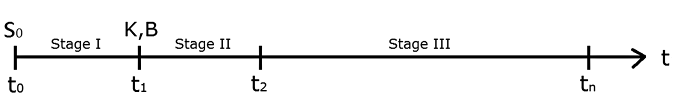

Fig. 1. Real-option valuation model settings in a typical BOT project lifecycle

This situational equivalent of a European-style call option offers the opportunity to apply the classical BSM model because the concessionaire faces a penalty of B by deciding not to invest at t_1. The BSM model has no penalty term when the call option is unexercised (Black and Scholes 1973; Merton 1973). Suppose all investors are risk-neutral and there is no market arbitrage (Harrison and Kreps 1979; Harrison and Pliska 1981). Then the time-t payoff of the call option under performance bonding is provided by

$$C_t = E^Q[\max(S_t - K, -B)] = E^Q\{\max[S_t - (K - B), 0]\} - B \quad (1)$$

where E^Q denotes a conditional expectation operator under the risk-neutral probability measure Q; and S_t denotes the reassessed project valuation at time-t. To determine the value of this option, assume that uncertainty in project valuation exhibits the following stochastic behavior (Black and Scholes 1973; Merton 1973):

$$\frac{dS_t}{S_t} = (r - q)dt + \sigma dz_t^Q \quad (2)$$

where r = a risk-free rate of return; q = the project's dividend payout rate; and z_t^Q = standard Brownian motion in a risk-neutral environment. Parameter σ denotes volatility in asset values and is assumed to have a deterministic value. On the basis of the foregoing model settings, an analytical solution of the real-option value can be derived by the risk-neutral pricing approach.

Theorem: Pricing Formula for a European-style Call Option with Performance Bonding

Assuming the aforementioned conditions, the value of the situational European-style call option at t_0 is provided by

$$\begin{aligned} C_0 &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} N(d_1) - (K - B) e^{-\int_{t_0}^{t_1} r(u)du} N(d_2) \\ &\quad - B e^{-\int_{t_0}^{t_1} r(u)du} \\ &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} N(d_1) - K e^{-\int_{t_0}^{t_1} r(u)du} N(d_2) \\ &\quad - B e^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) \end{aligned} \quad (3)$$

where

$$d_1 = \frac{\ln(\frac{S_0}{K-B}) + \int_{t_0}^{t_1}[r(u) - q(u) + \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} = d_2 + \sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}$$

$$d_2 = \frac{\ln(\frac{S_0}{K-B}) + \int_{t_0}^{t_1}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$$

In this valuation model, imposing a performance bond detracts from project valuation—the third term on the right-hand side of Eq. (3). Bond also affects the expected project valuation (the first term on the right) and the expected exercise price (the second term on the right) through the cumulative normal distributions $N(d_1)$ and $N(d_2)$, respectively (see Appendix I for proof).

Sensitivity Analysis

From the pricing formula [Eq. (3)], the value of the situational European-style call option C_0 at time-t_0 is proposed to be monotonic and strictly decreasing in B

$$\frac{\partial C}{\partial B} = -e^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) < 0 \tag{4}$$

This proposition reflects that a performance bond creates a penalty that must be subtracted from the estimated project valuation. The higher the required bond value, the higher is its penalty (see Appendix II for proof).

Numerical Case Study

This section discusses the Taiwan high-speed rail project as a numerical application of the valuation model. A *MATLAB*-based computer program supports the implementation.

Project Profile

The undertaking is a large-scale BOT project with a total construction cost of 446.58 billion New Taiwan dollars. Taiwan High-Speed Railroad Corporation (THSRC) is the concessionaire. Its July 1998 contract with the Taiwanese government grants it exclusive rights to build and operate the project until July 2033, at which time the project reverts to the government. According to the contract, construction began in March 2000, and so the preconstruction stage was less than 2 years. THSRC could terminate the project prematurely, but a performance bond was required to guarantee that it adhered to the contract. The initial bond was 15 billion New Taiwan dollars, approximately 4% of the total construction cost. The required bond declines as construction progresses.

Valuation Parameters

According to government data, the project's initial valuation, S_0, was 385.27 billion New Taiwan dollars at t_0, a figure calculated from the discounted value of the project's earnings before interest, tax, and depreciation. Taiwan's government set the service rate to achieve a 10% return on investment; therefore, this study sets the project's discount rate at 10%. Accordingly, at time-t_1, the discounted value of the construction cost was 400.47 billion New Taiwan dollars.

The risk-free interest rate of the project is 6.35%, estimated from the monthly spot rates of the 10-year treasury bonds reported by the Taiwan Central Bank between January 1993 and July 1998. Historical asset volatility is 0.3687, estimated from the series of 1-year stock prices of eight transportation companies reported in the 1998 Taiwan Economic Journal database. This case study disregards dividends and assumes $q = 0$. Table 1 summarizes the base-case valuation parameters.

Valuation Outcomes

Ignoring the effect of the performance bond (that is, $B = 0$), the base-case parameters yield a project valuation of 83.32 billion New Taiwan dollars at t_0. This valuation exceeds the project's net present value (NPV) at t_0, i.e., 65.71 billion New Taiwan dollars calculated using the 10% discount factor. This result indicates that the European-style situational option to terminate investment at t_1 has monetary value.

Table 1. Base-Case Project Valuation Parameters

Variable	Value
Time-t_0 discounted value of the underlying project asset (S_0)	385.273 billion New Taiwan dollars
Time-t_1 discounted value of the construction cost (K)	400.469 billion New Taiwan dollars
Performance bond value (B)	15 billion New Taiwan dollars
Risk-free interest rate (r)	6.35%
Asset return volatility (σ)	0.3687
Dividend payout rate (q)	Not applicable

However, when the effect of the performance bond is considered (that is$B = 15$), the project valuation declines from 83.32 billion New Taiwan dollars at t_0 to 76.26 billion New Taiwan dollars at t_1. Although this reduced valuation still exceeds the project's NPV, the reduction is substantial, approximately 8.5% below the base case. This result indicates that the effect of performance bonding on project valuation must be carefully assessed; otherwise, valuations will be overstated, perhaps misleading investment decisions.

Futher sensitivity analysis in Fig. 2 shows that the right to terminate investment becomes more valuable when the project's asset return volatility increases. This result is reasonable because volatility in returns is an effective proxy for project risk and uncertainty. When the volatility falls below 0.3, the project value will be less than the NPV if the performance bond is required. However, at the same level of volatility, the base-case project valuation remains above NPV. In other words, at volatility of 0.3, the prospect of paying the performance bond destroys all value inherent in the right to terminate. This shows how ignoring performance bonding may generate flawed decisions.

Figs. 3 and 4 show the sensitivity of the project's valuation to changes in performance bond requirements. As shown, the value of the termination right vanishes when the required bond approaches 10% of the total construction cost, a percentage considered modest for smaller BOT projects. Therefore, assessing the impact of performance bonding on project valuation is sensible on a case-by-case basis.

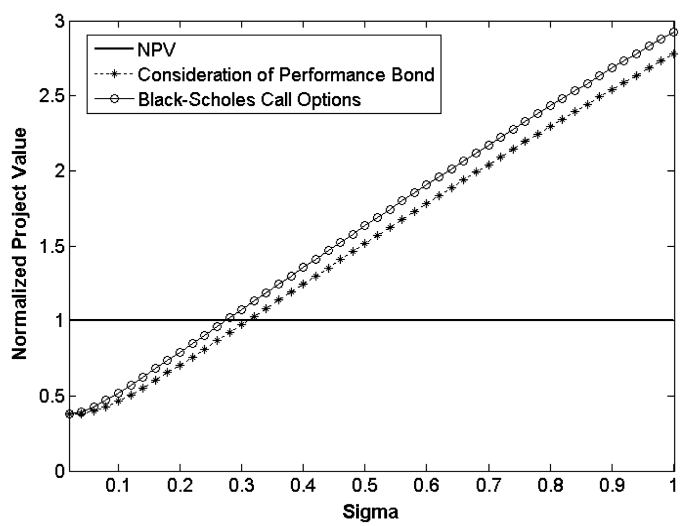

Fig. 2. Sensitivity analysis with respect to asset return volatility

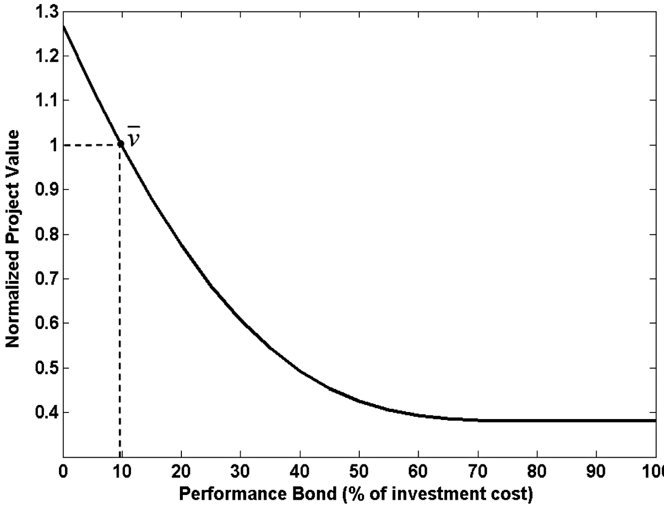

Fig. 3. Sensitivity analysis with respect to performance bond value

Reconciliation between Performance Bonds and Termination Rights

The THSRC case shows that termination rights can increase project valuation, and performance bonds can destroy it. Then, why does performance bonding coexist with termination rights? Is it as Williamson (1983) said, "an arbitrary exercise of power" or that "the stronger 'demands' a hostage from the weaker, who accedes it because it has no other choice"?

According to Williamson (1983), when contracts involve long-term sunk investments, bonds are ideal hostages for the buyer because they mitigate the hazard of the concessionaire cancelling delivery under market uncertainty. However, BOT performance bonds are not hostages in that sense because the concessionaire holds the termination rights, not the government. As the seller of services, the concessionaire probably accepts performance bonding as a reciprocity for the government's granting it termination and other rights. As Huang (1995, p. 103) noted, BOT concession contracts involve concerted reciprocity to protect the concessionaire's rights to serve, manage market uncertainty, and assure full recovery of capital. Contractual termination clauses can avoid the hold-up events mentoined in Klein (1996) by granting the concessionaire an exit if the project becomes infeasible after construction begins. Performance bonding alongside a termination clause can strengthen trust between concessionaire and government, and potential abuses of the termination clause and other rights can be prevented (Vandegrift 1998).

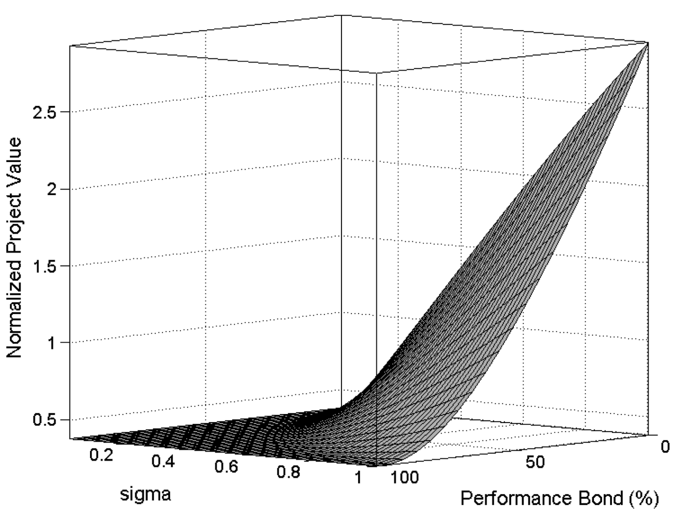

Fig. 4. Sensitivity analysis with respect to asset return volatility and performance bond value

In addition, contracts with imprecise performance specifications are difficult to enforce and need self-enforcement mechanisms to assure performance (Klein 1995). Performance specifications in BOT concession contracts are necessarily imprecise because during negotiations, the underlying projects are in their infancy, when information is insufficient for contracting parties to detail all performance elements and contingencies. Although incomplete performance specifications post a challenge to court enforcement, performance bonding increases what Klein (1996) called the contract's self-enforcing range, and this increases the probability of concessionaires' performance even without close government monitoring.

To see how performance bonding influences concessionaires' performance, suppose (1) the amount of the performance bond B is fixed during contract execution; (2) construction cost K is completely sunk once invested and unrecoverable upon voluntary termination; and (3) the concessionaire incurs no loss of reputational capital (Coase 2006) and other costs by terminating. The concessionaire would then perform after signing the contract at t_0 if and only if the expected gain of performing exceeds the expected loss of not performing. That is, for $t_1 \leq t \leq t_n$, the concessionaire performs when

$$S_t' \geq -B \tag{5}$$

where S_t' denotes the time-t reassessed net asset value of the project. Eq. (5) is the self-enforcing range of the BOT concession contract. According to Klein (1996), this range covers all the likely expost conditions in which gains from performing exceed gains or savings from not performing. Clearly, performance bonding broadens the self-enforcing range because it lowers the down-side risk the concessionaire can bear, thereby increasing its probability of performing. For this, the time-t probability of concessionaires' performance under a specific S_t' valuation is provided by

$$p_t = \Pr(S_t' \geq -B | S_0 \geq 0) \tag{6}$$

where the initial condition $S_t' \geq 0$ is necessary to induce the concessionaire to sign the contract at time-t_0. Given the uncertainty inherent in determining the project's valuation, the cumulative probability of concessionaires' performance is further provided by

$$P_t = \int_{-B}^{\infty} f(S_t') dS_t' \tag{7}$$

where $f(S_t')$ = probability density function of S_t'. Clearly, the higher the amount of the bond, the higher the cumulative probability.

In brief, BOT performance bonding is not an arbitrary exercise of power by government, but a contractual mechanism to avoid concessionaires' arbitrarily terminating projects and to increase the probability of self-enforcement by the concessionaire. A tradeoff between performance bonds and termination rights must be established, because the latter can avoid hold-ups and enhance project valuation to concessionaires.

More specifically, to maximize the expost probability of concessionaires' performance, government can further increase the amount of the bond, but doing so reduces the project valuation to the concessionaire. Government must assess whether the bond penalty reduces project valuation to the degree that the concessionaire does not sign the contract. If the penalty is unacceptable, government could reduce it and reassess whether the resulting probability of performance is acceptable. Alternatively, the government could offer inducements that increase the probability of performance. This adjustment process is necessarily iterative if it is to achieve desirable outcomes.

Summary and Future Research Directions

Real-option theory is popular in managing complex valuation problems in BOT undertakings. However, previous BOT real-option models have not incorporated performance bonds in their assessments of valuation, thereby potentially overstating the valuation of BOT projects that have termination rights. The valuation model discussed in this paper overcomes this issue. Sensitivity analysis shows that termination rights lose value as the amount of the performance bond is increased. The case study of the Taiwan high-speed rail project indicated that the value can be lost even if the amount of the bond is modest.

Finally, this study argued that performance bonds are a type of reciprocity that enhances a concessionaire's self-enforcement of performance specifications. This study has demonstrated that an iterative process is important in establishing a tradeoff in the amount of performance bonds, the value of termination rights, and the probability of performance.

This study's contribution to the BOT literature is twofold. First, it used the real-options approach to analytically assess how performance bonding influences the value inherent in the flexibility provided by termination rights. However, the present model is insufficient to assess the effect of performance bonding on complex BOT projects, such as those involving multistage expansions. Future research is needed in this area. Second, this study has shown how to assess the tradeoff between performance bonding and termination rights using Klein's (1996) concept of a self-enforcing range in contracts. Still, the present analysis is conceptual. To balance the tradeoff in designing better BOT concession, future research should develop analytical models that directly estimate the probability of concessionaires' performance using Klein's (1996) contract theory.

Appendix I. Proof of Theorem

According to Cox and Ross (1976), Harrison and Kreps (1979), and Harrison and Pliska (1981), the fair price of the European call option is provided by

$$C_0 = e^{-\int_{t_0}^{t_1} r(u)du}(E^Q\{\max[S_t-(K-B),0]\}-B)$$
$$= e^{-\int_{t_0}^{t_1} r(u)du}E^Q\{[S_t-(K-B)]\cdot 1_{[S_t>(K-B)]}\} - Be^{-\int_{t_0}^{t_1} r(u)du}$$

$$\begin{aligned} C_0 &= e^{-\int_{t_0}^{t_1} r(u)du}(E^Q\{\max[S_t-(K-B),0]\}-B) \\ &= e^{-\int_{t_0}^{t_1} r(u)du}E^Q\{[S_t-(K-B)]\cdot 1_{[S_t>(K-B)]}\} - Be^{-\int_{t_0}^{t_1} r(u)du} \\ &= e^{-\int_{t_0}^{t_1} r(u)du}E^Q\{S_t\cdot 1_{[S_t>(K-B)]}\} - (K-B)e^{-\int_{t_0}^{t_1} r(u)du} \\ &\quad \times E^Q\{1_{[S_t>(K-B)]}\} - Be^{-\int_{t_0}^{t_1} r(u)du} \end{aligned} \tag{8}$$

According to Shreve (2004), the solution of the stochastic differential equation in Eq. (2) is provided by

$$S_t = S_0 e^{\int_{t_0}^{t_1}[r(u)-q(u)-\frac{1}{2}\sigma^2(u)]du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$$

Substituting this solution into Eq. (8) yields

$$\begin{aligned} C_0 &= S_0 e^{-\int_{t_0}^{t_1} q(u)du}E^Q\left\{e^{-\int_{t_0}^{t_1}\frac{1}{2}\sigma^2(u)du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}\cdot 1_{[S_t>(K-B)]}\right\} \\ &\quad -(K-B)e^{-\int_{t_0}^{t_1} r(u)du}E^Q\{1_{[S_t>(K-B)]}\} - Be^{-\int_{t_0}^{t_1} r(u)du} \end{aligned} \tag{9}$$

The Girsanov theory can be used to eliminate the uncertain term in the expectation operatorE^Q. First, define the Radon-Nikodym derivative as

$$\frac{dR}{dQ} = -\int_{t_0}^{t_1}\frac{1}{2}\sigma^2(u)du + z^Q\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du} \tag{10}$$

Then, the Brownian motion term in Eq. (2) can be rewritten as $dz_t^Q = dz_t^R + \sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}$, and the original stochastic asset process becomes

$$\frac{dS_t}{S_t} = (r-q+\sigma^2)dt + \sigma dz_t^R$$

where dz_t^R represents a standard Brownian motion under the measure R, using the underlying asset valuation as a numéraire (for example, see Shreve 2004). By Ito's lemma, the solution of this stochastic asset valuation process is

$$S_t = S_0 e^{\int_{t_0}^{t_1}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du + z^R\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$$

Substituting this solution into Eq. (9) yields

$$\begin{aligned} C_0 &= S_0 e^{-\int_{t_0}^{t_1} q(u)du}E^Q\left\{\frac{dR}{dQ}\cdot 1_{[S_t>(K-B)]}\right\} \\ &\quad -(K-B)e^{-\int_{t_0}^{t_1} r(u)du}E^Q\{1_{[S_t>(K-B)]}\} - Be^{-\int_{t_0}^{t_1} r(u)du} \\ &= S_0 e^{-\int_{t_0}^{t_1} q(u)du}E^R\{1_{[S_t>(K-B)]}\} \\ &\quad -(K-B)e^{-\int_{t_0}^{t_1} r(u)du}E^Q\{1_{[S_t>(K-B)]}\} - Be^{-\int_{t_0}^{t_1} r(u)du} \\ &= S_0 e^{-\int_{t_0}^{t_1} q(u)du}P^R[\ln S_t > \ln(K-B)] \\ &\quad -(K-B)e^{-\int_{t_0}^{t_1} r(u)du}P^Q[\ln S_t > \ln(K-B)] - Be^{-\int_{t_0}^{t_1} r(u)du} \end{aligned} \tag{11}$$

where

$$\begin{aligned} &P^R[\ln S_t > \ln(K-B)] \\ &\quad = P^R\left\{\ln S_0 + \int_{t_0}^{t_1}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du \right. \\ &\qquad \left. + z^R\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du} > \ln(K-B)\right\} \\ &\quad = P^R\left\{-z^R < \frac{\ln(\frac{S_0}{K-B}) + \int_{t_0}^{t_1}[r(u)-q(u)+\frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}\right\} \\ &\quad = P^R(-z^R < d_1) \end{aligned}$$

Similar derivation yields

$$P^Q[\ln S_t > \ln(K-B)] = P^Q(-z^Q < d_2) \tag{12}$$

where

$$d_2 = \frac{\ln(\frac{S_0}{K-B}) + \int_{t_0}^{t_1}[r(u) - q(u) - \frac{1}{2}\sigma^2(u)]du}{\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}}$$

Accordingly, the time-t_0 value of the call option is provided by

$$\begin{aligned} C_0 &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} P^R(-z^R < d_1) \\ &\quad - (K-B)e^{-\int_{t_0}^{t_1} r(u)du} P^Q(-z < d_2) - Be^{-\int_{t_0}^{t1} r(u)du} \end{aligned} \tag{13}$$

where both z^R and z^Q = standard Brownian motions. Finally, the pricing formula in Eq. (3) is proven by the fact that $z \sim N(0,1)$ and $1 - N(d_2) = N(-d_2)$ in Eq. (13).

Appendix II. Proof of Proposition

According to Eq. (3)

$$\begin{aligned} \frac{\partial C}{\partial B} &= \frac{\partial}{\partial B}[S_0 e^{-\int_{t_0}^{t_1} q(u)du} N(d_1) - Ke^{-\int_{t_0}^{t_1} r(u)du} N(d_2) \\ &\quad - Be^{-\int_{t_0}^{t_1} r(u)du} N(-d_2)] \\ &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} \frac{\partial}{\partial B} N(d_1) - Ke^{-\int_{t_0}^{t_1} r(u)du} \frac{\partial}{\partial B} N(d_2) \\ &\quad - e^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) - Be^{-\int_{t_0}^{t_1} r(u)du} \frac{\partial}{\partial B} N(-d_2) \end{aligned} \tag{14}$$

By the chain rule

$$\begin{aligned} \frac{\partial}{\partial B} N(d_i) &= \frac{\partial N(d_i)}{\partial d_i} \cdot \frac{\partial d_i}{\partial B} \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_i^2} \frac{1}{(K-B)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \quad \text{for } i = 1, 2 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial}{\partial B} N(-d_2) &= \frac{\partial N(-d_2)}{\partial(-d_2)} \cdot \frac{\partial(-d_2)}{\partial B} \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(-d_2)^2} \frac{-1}{(K-B)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{\partial C}{\partial B} &= S_0 e^{-\int_{t_0}^{t_1} q(u)du} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{1}{(K-B)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \\ &\quad - e^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) - Ke^{-\int_{t_0}^{t_1} r(u)du} \\ &\quad \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \frac{1}{(K-B)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} - Be^{-\int_{t_0}^{t_1} r(u)du} \\ &\quad \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(-d_2)^2} \frac{-1}{(K-B)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} = S_0 e^{-\int_{t_0}^{t_1} q(u)du} \\ &\quad \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_1^2} \frac{1}{(K-B)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} - e^{-\int_{t_0}^{t_1} r(u)du} \\ &\quad \times N(-d_2) - (K-B)e^{-\int_{t_0}^{t_1} r(u)du} \\ &\quad \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}d_2^2} \frac{1}{(K-B)\sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}} \end{aligned} \tag{15}$$

Replacing d_1 by $d_2 + \sqrt{\int_{t_0}^{t_1}\sigma^2(u)du}$ and by some calculation yields

$$\frac{\partial C}{\partial B} = -e^{-\int_{t_0}^{t_1} r(u)du} N(-d_2) < 0 \tag{16}$$

This proves the proposition.

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