

# Bid Compensation Theory and Strategies for Projects with Heterogeneous Bidders: A Game Theoretic Analysis

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**Abstract:** Practitioners involved in large-scale, complex projects have long been searching for strategies that encourage bidders to invest more effort in project planning and schematic design during the bid preparation process. According to conventional wisdom, one of the strategic alternatives for encouraging bidders to make extra efforts in the early stage is to offer bid compensation to unsuccessful bidders. However, although the argument for using bid compensation is intuitively sound, rigorous investigations on the effectiveness of bid compensation are scant. In this study, game theoretic analysis is applied to analyze strategic interactions among project bidders. In large-scale, complex projects, it is not unusual to have one or two bidders who have stronger competitive power than other bidders do, such as reputation, specialties, and financing capability. Thus, a more general scenario is examined where the bidders are heterogeneous in terms of their competitive advantages. Through the analysis, the authors show that, under certain conditions, the offering of bid compensation can effectively encourage the stronger bidders to make extra efforts in the early stage. Conditions for the effective use of bid compensation are identified. Strategic implications for practitioners are derived and suggested accordingly. DOI: 10.1061/(ASCE)ME.1943-5479.0000212. © 2014 American Society of Civil Engineers.

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## Introduction

Planning and design efforts in the early stage of a project life cycle have been widely considered to have crucial impacts on project performance. Based on the study of nine construction projects, Burati et al. (1992) estimate that design flaws alone contribute to 79% of the total project cost deviations and that the estimated loss amounts to at least 9.5% of total construction costs. Therefore, for projects with costly bid preparation, practitioners have been searching for strategies that may encourage the bidders to invest more effort in preplanning during the tendering process. For example, in public-private partnership (PPP) projects, because of the characteristics of the delivery process and the emphases on creativity and business models, project planning and schematic design at the early stage have a great impact on project implementation and performance. The desired extra efforts from bidders may include employing top-tier consultants and professionals, conducting more thorough investigations beyond preliminary design and analysis, and exploring other alternatives that create values.

Based on conventional wisdom, one of the strategic alternatives for encouraging bidders to make more efforts in the early stage is to offer bid compensation to unsuccessful bidders (typically the second-ranked bidder). The Design-Build Institute of America's (1996) *Design-Build Manual of Practice* suggests that "the provision of reasonable compensation will encourage the more sought-after design-build teams to apply and, if short listed, to make an

extra effort in the preparation of their proposal." However, although intuitively using bid compensation may seem apparent, rigorous investigations on the effectiveness of bid compensation are scarce, and no theoretical frameworks have been proposed to support the use of bid compensation. Although auction or bidding problems have been extensively studied by economists, the focuses of these studies are mainly on pricing and mechanism design issues, distinctly different from the nature of the bid compensation problems analyzed in this paper. An attempt of such investigations is by Ho (2005), who studied the effectiveness of bid compensation through game theoretic analysis. The authors know of no other theoretical studies on bid compensation problems in the literature. Surprisingly, Ho concludes that bid compensation is generally not necessary for the desired extra efforts from bidders. However, this result is based on a restrictive assumption: all bidders are homogeneous; i.e., they have similar capabilities in terms of winning and completing the project.

The point of departure is that Ho's (2005) conclusion might not hold if the homogeneous bidder assumption is relaxed by considering *heterogeneous bidders*. In practice, it is not unusual to have one or two bidders who have stronger competitive power such as reputation, specialties, and financing ability. Therefore, it is crucial to study the effectiveness of bid compensation under a more realistic and general setting. As such, in this study, Ho's homogeneous or average bidder assumptions are relaxed by differentiating the bidders into *strong* and *regular* ones and by investigating the effectiveness of bid compensation with heterogeneous bidders. Game theory is also applied in this study to analyze the interaction dynamics among heterogeneous bidders. Specifically, the authors develop several bid compensation game models to analyze different *heterogeneous* scenarios, which include bidding involving two bidders, three bidders, and four bidders, each with various numbers of strong bidders. Based on the game equilibria obtained, conditions for the effective use of bid compensation are identified. Strategic implications for practitioners are derived and suggested accordingly.

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## Research Methodology: Game Theoretic Modeling and Analysis

Game theory can be defined as “the study of mathematical models of conflict and cooperation between intelligent rational decision-makers” (Myerson 1991). Game theory has been applied to important topics in economics (see Mas-Colell et al. 1995) and in many other disciplines. In construction management, game theory modeling has been applied to analyze competitive bidding schemes (Drew and Skitmore 2006), strategies for subcontractor selection (Unsal and Taylor 2011), PPP renegotiation and governance (Ho 2006), bid compensation (Ho 2005), and knowledge sharing (Levitt et al. 2013; Ho et al. 2011). *Games* can be classified according to the completeness of information and the way in which games are played. There are two basic types of games: static games and dynamic games. In a static game, the players act simultaneously, meaning that each player chooses an action without knowing the decisions of others. In contrast, in a dynamic game, players act sequentially and observe other players’ actions in previous moves. In this study, static games are used to model the bidders’ strategic interactions.

To solve a game model is to solve for the *Nash equilibrium* (NE) of the game. Nash equilibrium is a set of strategies, each of which determines the optimal response by each player, given other players’ strategies in the equilibrium. Because the strategies in the NE are the best possible responses of each player based on the decisions of others, the set of strategies can be viewed as a stable equilibrium. In other words, in a NE, no player can increase its payoff by unilaterally deviating from the equilibrium solution. Thus, the equilibrium is “strategically stable” and “self-enforcing” (Gibbons 1992). The Nash equilibria can be pure strategy NE (PSNE) and/or mixed strategy NE (MSNE). In a PSNE, each player’s equilibrium strategy is to play a particular strategy, whereas in a MSNE each player randomizes their strategy according to the equilibrium probability distribution. In fact, a PSNE can be considered a special case of MSNE, with 100% probability of playing the pure strategy. The major concern in an MSNE is the probability of playing each strategy.

Ho et al. (2011) proposed a game theoretic modeling approach to studying strategic management issues in engineering. The game theoretic modeling generally consists of three steps. The first step is to abstract the problem under study and develop a game model for the problem. The second step is to solve for the equilibrium solutions of the game model. The third step is to identify possible contextual variables and then to derive strategy implications or to build a new theory by differentiating the solutions so as to reflect different strategic or problem contexts. In this research, the three steps of game theoretic modeling will be followed to study the bid compensation strategy and derive strategic implications.

### Model Assumptions and Setup

In game theoretic modeling, it is critical to abstract the problem and make some simplifying assumptions to focus on the issues of concern and obtain insightful results. The assumptions and model setup are summarized and discussed as follows.

#### **Players and Information: Heterogeneous Bidders with Complete Information**

For companies that enter the bidding, it is common to find that bidders with varying strengths such as company size, technical capability, and reputation, among which some firm traits may stand

out and be preferred by the owner. For example, possessing key knowledge relevant to the project or having experience or a track record in similar projects can be the key factor in companies having a competitive advantage over competitors. In general, companies that own a larger amount of critical resources, exhibit stronger technical capabilities, or have a better reputation than their competitors are considered superior. These companies may be preferred by owners and are more likely to win the bid. In this study, companies with preferred competitive advantages are regarded as *strong bidders*. Specifically, the bidders are grouped into two categories: strong bidders and regular bidders. Regular bidders, thus, refer to bidders with less competitive power than their competitors. Ho’s (2005) *average bidders* can be considered as *regular bidders*, who are equal in terms of their overall capabilities and competitive power. Since the prequalification process imposed during procurement reduces the variation of the quality of bidders, it is reasonable to assume in the model that those who are not strong bidders are equally good.

Concerning the information, the authors argue that it is reasonable to impose the complete information assumption for the purpose of game modeling. With current advancements in information technology, information and knowledge about project bidders can be easily obtained. In addition, since the number of potential bidders for large-scale projects is often small, owners and bidders tend to be familiar with other bidders in terms of their competitive power, weaknesses, and the potential costs and profits for a project. Therefore, it is not unrealistic to assume the completeness of information in modeling. Under the complete information assumption, all players know which competitors are strong or regular bidders, and they have the ability to reasonably assess each player’s project payoffs, including profit margins and costs.

#### **Bidding Strategies and Project Awarding: Two Levels of Effort under Best Value Competition**

For modeling tractability, it is assumed that there are two discrete levels of effort, *High* and *Average*, in preparing a proposal and planning for the project, denoted by “*H*” and “*A*,” respectively. Effort “*A*” is defined as the level of effort that is typically observed in practice and exercised by bidders. Effort “*H*” is defined as employing extra efforts (which incur extra costs, denoted by *E*) in order to improve the quality of a proposal or project planning. The extra efforts may include, for example, employing top-tier consultants and professionals, conducting more thorough investigations beyond preliminary design and analysis, exploring more alternatives that create value, and being more engaged in tendering negotiation or discussion. In terms of project awarding, following the common practice in many design-build or PPP projects, it is assumed that the bid evaluation is based on *best value*, instead of best (lowest) price, and the value is reflected in the bid proposal and the interaction between the project owner and bidders. Accordingly, for the same type of bidders (strong or regular), the value difference among the same type of bidders who use the same level of effort is considered insignificant. Moreover, by the definitions of the bidder types, the authors infer and assume that if the same level of effort is adopted by the bidders, the strong bidders will outperform the regular bidders. In abstracting the problem, it is further assumed that the overall quality of bid preparation by a regular bidder with effort *H* will roughly have the same quality of a strong bidder with effort *A* and that if there are multiple bidders who simultaneously have the best value proposal for winning the project, each of these bidders is assumed to have the same probability of winning.

## Number of Bidders

Since the complexity of the bidding model with heterogeneous bidders grows exponentially with the number of bidders, in this study, the number of bidders is limited to four so that the model is analytically tractable. Yet, the applicability of the results is not as limited as it may seem, because, in those projects with costly bid preparation, the number of bidders for a project tends to be small. This is mainly due to the relatively large scale of projects and the common *short-listing* practice. Given such, several bid compensation game models have been developed to analyze different *heterogeneous* scenarios, which include bidding with two bidders, three bidders, and four bidders, each with various numbers of strong bidders.

Since this bid compensation model assumes fixed numbers of strong and regular bidders, the model is limited by not being able to predict or answer whether more potential bidders will be attracted to use bid compensation. Therefore, the effectiveness of bid compensation in this study is not defined in terms of attracting additional bidders. The effectiveness of bid compensation will formally be defined later in the paper.

## Payoffs of Bidders

The payoffs of bidders are modeled by three related components or model variables: (1)  $P$ , the profit margin from winning and performing the project; (2)  $S$ , the bid compensation offered; and (3)  $E$ , the extra costs (i.e., marginal costs) for playing  $H$ . In this study  $E$  will be specified as a proportion of  $P$ , e.g.,  $E = 0.1P$ .  $S$  will be expressed as a proportion or multiple of  $E$ , e.g.,  $S = 0.8E$ , which can be further expressed in terms of a proportion of  $P$ . e.g.,  $S = 0.8E = 0.8(0.1P) = 0.08P$ . Since  $S$  and  $E$  can both be expressed in terms of  $P$ , as shall be shown later in model solutions, the variable  $P$  will be canceled out eventually, and thus the numerical values of model solutions can be obtained once the relationship between  $E$ ,  $S$ , and  $P$  is specified. As for the bid compensation, based on the common practice, it shall be assumed that a fixed amount of bid compensation announced before bid submission is offered to the second best bidder, i.e., the highest-ranked unsuccessful bidder. For convenience, it is assumed that  $E$ , the extra cost for effort “ $H$ ,” will not be added to the project cost or bid price, but will be reflected as a negative payoff,  $-E$ , in a bidder’s payoff profile. This assumption is to simplify the trade-off between quality improvement and payoff reduction by fixing the bid price and letting the high effort proposal be preferred or chosen in the best-value competition.

## Model Solutions and Effectiveness of Bid Compensation

The bidder’s payoffs for each possible equilibrium in a certain game scenario will be modeled in this section. The possible equilibria include pure strategy NE and mixed strategy NE. In a mixed strategy NE, the probability of playing effort  $H$  and that of playing  $A$  are modeled. The modeling begins with two-bidder games with one strong bidder, continues to three-bidder and four-bidder games with one strong bidder, and finishes at games with two strong bidders. A particular interest is on the strong bidder’s probability of playing  $H$ . In terms of the effectiveness of bid compensation, the effectiveness is defined by the amount of bid compensation required to meet the strong bidder’s probability of playing  $H$  desired by the owner. The lower the amount of required bid compensation is, the higher the effectiveness is. There may be some unrealistic values for the model variables in certain equilibria; however, these

theoretical values or equilibrium conditions are merely for analytical purpose and do not suggest that these situations may be observed in practice. In terms of notation, because many variables/symbols were used in this paper, a notation list is provided at the end of the paper for quick reference.

## Two-Bidder Games with One Strong Bidder

In a two-bidder game with one strong bidder, there is one strong bidder and one regular bidder. The possible payoffs for each bidder in the games can be expressed by a payoff matrix as shown in Table 1, where bidder 1 is the strong bidder and bidder 2 is the regular one. Throughout this paper bidder 1 *always* represents the strong bidder. The payoffs in each cell of Table 1 are derived by following the aforementioned assumptions and setup concerning the relative competitive strength determined by the types of the bidders (strong or regular) and levels of effort ( $H$  or  $A$ ). Accordingly, if both bidders choose “ $H$ ,” denoted by  $(H, H)$ , bidder 1 as the strong bidder will win the bid and bidder 2 will receive the bid compensation. As a result, the payoffs for the bidders in  $(H, H)$  equilibrium are  $(P - E, S - E)$ . For a more detailed description of an equilibrium, in this paper, brackets are sometimes used to combine the equilibrium strategies and the equilibrium payoffs. For example, a full expression of equilibrium  $(H, H)$  can also be denoted as  $[(H, H); (P - E, S - E)]$ . Next, if both bidders choose  $(A, A)$ , bidder 1 will still win and the payoffs for both bidders are  $(P, S)$ . Again, equilibrium  $(A, A)$  can be expressed as  $[(A, A); (P, S)]$ . Following the same reasoning, the payoffs of  $(H, A)$  equilibrium are  $(P - E, S)$ . In  $(A, H)$ , according to the model setup, since a strong bidder with effort  $A$  and a regular bidder with effort  $H$  perform equally, both bidders have a 50% chance of winning the bid, and the expected payoffs are  $(S/2 + P/2, S/2 + P/2 - E)$ . The same principles for deriving the payoff matrix are applied throughout the analyses in this study.

## Pure Strategy Nash Equilibrium

Having seen that the payoffs in each equilibrium are expressed as functions of  $S$ ,  $P$ , and  $E$  rather than particular numerical values, the analysis is now focused on solving for the conditions of each possible Nash equilibrium of the game. The approach to solving for the equilibrium conditions is to find the conditions that ensure the self-enforcing requirements of Nash equilibrium. This technique will be applied throughout the paper.

There are four possible Nash equilibria in this game:  $(H, H)$ ,  $(A, A)$ ,  $(H, A)$ , and  $(A, H)$ . First, check equilibrium  $[(H, H); (P - E, S - E)]$ . For the strong bidder (bidder 1) not to deviate from  $[(H, H); (P - E, S - E)]$  to  $[(A, H); (P/2 + S/2, P/2 + S/2 - E)]$ ,

$$P - E > \frac{S}{2} + \frac{P}{2} \rightarrow S < P - 2E \text{ (for strong bidder)} \quad (1)$$

is needed.

On the other hand, for the regular bidder (bidder 2) not to deviate from  $[(H, H), (P - E, S - E)]$  to  $[(H, A), (P, S)]$ ,

**Table 1.** Payoffs for Two-Bidder Games with One Strong Bidder

Bidder 1 (strong)	Bidder 2	
	$H$	$A$
$H$	$(P - E, S - E)$	$(P - E, S)$
$A$	$(S/2 + P/2, S/2 + P/2 - E)$	$(P, S)$



$$S - E > S \rightarrow E < 0 \rightarrow \text{not possible (for regular bidder)} \quad (2)$$

is needed.

Since extra costs ( $E$ ) can never be negative, Eq. (2) cannot be satisfied, and therefore  $(H, H)$  cannot be the equilibrium for this game.

Second, consider  $[(A, A); (P, S)]$ . For the strong bidder not to deviate from  $(A, A)$  to  $[(H, A); (P - E, S)]$ ,

$$P > P - E \rightarrow E > 0 \text{ (for strong bidder)} \quad (3)$$

is needed.

For the regular bidder not to deviate from  $(A, A)$  to  $[(A, H); (P/2 + S/2, P/2 + S/2 - E)]$ ,

$$S > S/2 + P/2 - E \rightarrow S > P - 2E \text{ (for regular bidder)} \quad (4)$$

is needed.

Thus, condition (4) alone will guarantee  $(A, A)$  to be a Nash equilibrium. Condition (4) has two implications. On the one hand, when  $S$  is large, both players will be motivated to play  $A$ . This is because that, when  $S$  is large, the regular bidder will be better off by receiving  $S$  for sure by playing  $A$  and, as a result, the strong bidder can win the project without playing  $H$ . On the other hand, condition (4) also implies that, when  $P - 2E < 0$ , i.e.,  $E > P/2$ , condition (4) will always be satisfied. Therefore,  $(A, A)$  will be the equilibrium solution when  $S$  is large or  $E > P/2$ .

Third, check equilibrium  $[(H, A); (P - E, S)]$ . For the strong bidder not to deviate from  $(H, A)$  to  $[(A, A); (P, S)]$ , the model must have

$$P - E > P \rightarrow E < 0 \rightarrow \text{not possible (for strong bidder)} \quad (5)$$

For the regular bidder not to deviate from  $(H, A)$  to  $[(H, H); (P - E, S - E)]$ , the model must have

$$S > S - E \rightarrow E > 0 \text{ (for regular bidder)} \quad (6)$$

Since  $E$  cannot be negative, as required by Eq. (5),  $(H, A)$  equilibrium cannot exist.

Fourth, check the  $(A, H)$  equilibrium. Following the same logic, for  $(A, H)$  to be the solution, the model must have condition  $S/2 + P/2 > P - E \rightarrow S > P - 2E$  for the strong bidder and condition  $S/2 + P/2 - E > S \rightarrow S < P - 2E$  for the regular bidder. Since it is not possible to satisfy both  $S > P - 2E$  and  $S < P - 2E$  at the same time, the  $(A, H)$  equilibrium cannot exist.

To summarize,  $(A, A)$  is the only possible *pure strategy* NE for two-bidder games with one strong bidder. The fact that  $(A, A)$  is the only possible pure strategy NE does not mean that  $(A, A)$  is the only possible solution. In fact, when Eq. (4) is not satisfied (i.e.,  $S < P - 2E$ ), the NE will be a mixed strategy NE, which is discussed next.

### Mixed Strategy Nash Equilibrium

The concept of mixed strategy NE is adopted to solve for the equilibrium strategies when  $S$  is smaller than  $P - 2E$ . In a mixed strategy NE, bidders randomize actions  $H$  and  $A$  with a certain probability that the other players are indifferent to playing any pure strategies. From a more dynamic perspective, a mixed strategy can be taken to be that every player observes which pure strategy works better and the player would change their strategy if the one used did not perform as well as other strategies. A mixed strategy can occur when there are multiple pure strategy NE or when there is no pure strategy NE. Therefore, the major concern in a mixed strategy equilibrium is the players' probability of playing each strategy. Following the concept of equilibrium, the mathematical requirement for the equilibrium probabilities is that each player's mix strategy

equilibrium probabilities will make the other player indifferent to potential strategies (Gibbons 1992).

Let  $u$  be the strong bidders' probability of choosing  $H$  and  $v$  be the regular bidder's probability of choosing  $H$ . The conditions for the equilibrium probabilities are that equilibrium probability  $u^*$  has to balance the regular bidder's payoffs and make the regular bidder indifferent to choosing  $H$  and  $A$  and that equilibrium probability  $v^*$  has to make the strong bidder indifferent to choosing  $H$  and  $A$ . Therefore, the equilibrium probabilities must satisfy the following two conditions: First, for the strong bidder to use  $u^*$  for balancing the regular bidder's payoffs

$$(S - E)u^* + (S/2 + P/2 - E)(1 - u^*) = Su^* + S(1 - u^*) \text{ (balancing regular bidder's payoffs)} \quad (7)$$

is needed.

Second, for the regular bidder to use  $v^*$  for balancing the strong bidder's payoffs

$$(P - E)v^* + (P - E)(1 - v^*) = (S/2 + P/2)v^* + P(1 - v^*) \text{ (balancing strong bidder's payoffs)} \quad (8)$$

is needed.

Solving Eqs. (7) and (8)

$$u^* = 1 - [2E/(P - S)] \quad (9)$$

$$v^* = 2E/(P - S) \quad (10)$$

is obtained.

In addition, since the restriction for the mixed strategy probability is that  $0 < u^* < 1$ ,  $0 < 1 - [2E/(P - S)] < 1 \rightarrow S < P - 2E$ .  $S < P - 2E$  further implies that  $P - 2E > 0 \rightarrow E < P/2$ . To summarize, when  $E > P/2$ , the game equilibrium is  $(A, A)$ ; when  $E < P/2$ , the game equilibrium is a mixed strategy NE.

### Impact of Bid Compensation

In practice,  $S$  and  $E$  are usually relatively small compared with  $P$ ; therefore, the condition for the mixed strategy NE,  $S < P - 2E$ , can be easily satisfied. In other words, the most likely equilibrium in this game is a mixed strategy NE. Furthermore, according to Eqs. (9) and (10), when  $E$  is small,  $u^*$  tends to be large and  $v^*$  tends to be small, meaning that the strong bidder is inclined to choose  $H$  and vice versa. Surprisingly, Eqs. (9) and (10) also indicate that the offering of bid compensation will *discourage* the strong bidder from choosing  $H$  but encourage the regular bidder to choose  $H$ . Therefore, because project owner's concern is to facilitate high efforts from the strong bidder, bid compensation should not be used in most cases. This counterintuitive result shall be explained later in greater detail. Furthermore, the probability of the strong bidder playing  $H$  may be estimated numerically. For example, when  $E = 0.1P$  and  $S = 0$ ,  $u^* = 1 - (2E/P) = 1 - 0.2 = 0.8$ , according to Eq. (9). Practically, this model can help the project owner assess the strong bidder's probability of playing  $H$ , as a function of  $E$ . As discussed earlier, once the relationship between  $E$ ,  $S$ , and  $P$  is specified, numerical solutions for the probabilities can be obtained.

On the other hand, according to Ho (2005), under the average bidder assumption,  $(H, H)$  will be the equilibrium when  $S < P - 2E$ , and  $(A, A)$  will be the equilibrium when  $S > P - 2E$ . Since  $S < P - 2E$  is often satisfied in practice,  $(H, H)$  will be the most likely equilibrium even when  $S = 0$ . Therefore, Ho argues that, when there are two average bidders, both bidders tend to choose  $H$  and bid compensation is unnecessary. Nevertheless, when there is one strong bidder in a two-bidder

game, the most likely equilibrium is replaced by a mixed strategy NE, in which the probability of the strong bidder choosing  $H$ ,  $u^*$ , is still high but the probability of the regular bidder,  $v^*$ , is reduced significantly. The probability  $v^*$  is small because the likelihood of the regular bidder winning by playing  $H$  is low. Furthermore, this model and Ho's (2005) model on two-bidder games arrive at the common conclusion that bid compensation cannot further increase the probability  $u^*$  and, thus, is not necessary.

### Three-Bidder Games with One Strong Bidder

#### Nash Equilibria and Their Conditions

Following the same reasoning in two-bidder games, the payoffs of three-bidder games with one strong bidder (bidder 1) can be obtained, as summarized by the payoff matrix in Table 2. There are six possible Nash equilibria in which the strong bidder plays a pure strategy. These equilibria include  $(A, 2A)$ ,  $(H, 2H)$ ,  $(A, 2H)$ ,  $(H, 2H)$ ,  $(H, 2HA)$ , and  $(A, 2HA)$ , where  $2A$  (or  $2H$ ) denotes that both regular bidders (bidders 2 and 3) play a pure strategy,  $A$  (or  $H$ ), and  $2HA$  denotes that the regular bidders play mixed strategies. In addition, there will be another mixed strategy, Nash equilibria, denoted by  $(3HA)$ , where no bidders play a pure strategy. By analyzing the conditions for these equilibria, the major concern of the study can be addressed of when or how likely will the strong bidder play  $H$ . Although all the possible equilibria listed above can be solved, here the authors shall solve for those equilibria where the strong bidder may play  $H$ , because the major purpose of offering bid compensation is to encourage the strong bidder to play  $H$ . Thus, in this section, the authors forgo the analyses of equilibria  $(A, 2H)$  and  $(A, 2HA)$  but solve for the solutions of equilibria  $(H, 2H)$ ,  $(H, 2A)$ ,  $(H, 2HA)$ , and  $(3HA)$ . Also, equilibrium  $(A, 2A)$  needs to be solved for because the conditions of  $(A, 2A)$  need to be excluded from the conditions of  $(H, 2H)$  due to the concern of cheap talk discussed later.

First, consider the  $(A, 2A)$  equilibrium. For the strong bidder not to deviate from  $[(A, 2A); (P, S/2, S/2)]$  to  $[(H, 2A); (P - E, S/2, S/2)]$ , Eq. (11) must be satisfied

$$P > P - E \rightarrow E > 0 \text{ (for strong bidder)} \quad (11)$$

For the regular bidders, e.g., bidder 2, not to deviate from  $(A, 2A)$  to  $[(A, 1H1A), (P/2 + S/2, P/2 + S/2 - E, 0)]$ , condition (12) must be satisfied

$$S/2 > P/2 + S/2 - E \rightarrow P - 2E < 0 \text{ (for regular bidders)} \quad (12)$$

Conditions (11) and (12) can be summarized by condition (12). Condition (12) stipulates  $E$ 's valid range for  $(A, 2A)$ , that is,  $E > P/2$ .

Second, check the payoffs of  $(H, 2H)$  equilibrium. Similarly, the equilibrium conditions need to guarantee that the equilibrium will not deviate from  $(H, 2H)$  to either  $(A, 2H)$  or  $(H, 2HA)$ . Accordingly, the required equilibrium conditions are

$$P - E > S/3 + P/3 \rightarrow S < 2P - 3E \text{ (for strong bidder)} \quad (13)$$

$$S/2 - E > 0 \rightarrow S > 2E \text{ (for regular bidders)} \quad (14)$$

The two conditions can be rewritten as in (15)

$$2E < S < 2P - 3E \quad (15)$$

Note that in three-bidder games if  $S$  satisfies conditions (12) for  $(A, 2A)$  and (15) for  $(H, 2H)$  at the same time,  $(A, 2A)$  will be the equilibrium because the payoffs of  $(A, 2A)$ ,  $(P, S/2, S/2)$ , dominate the payoffs of  $(H, 2H)$ ,  $(P - E, S/2 - E, S/2 - E)$ , provided that a consensus among the bidders to play  $A$  can be reached. The process of reaching the Pareto optimal equilibrium is called *cheap talk*, where the agreement is beneficial to all players. In projects with limited potential bidders, it is reasonable to believe that cheap talk can occur. Therefore, for  $(H, 2H)$  to be the equilibrium solution, the conditions for  $(A, 2A)$  must not hold. Thus, to exclude condition (12), condition (16), which is the negation of (12), must be satisfied

$$P - 2E > 0 \rightarrow E < P/2 \quad (16)$$

Therefore, the conditions for  $(H, 2H)$  become inequalities (15) and (16). However, inequality (15),  $2E < S < 2P - 3E$ , implies that  $2P - 5E > 0 \rightarrow E < 2P/5 < P/2$ , which in turn implies that inequality (16) is satisfied. Thus, condition (15) is sufficient to guarantee the  $(H, 2H)$  equilibrium.

Third, check the equilibrium  $(H, 2A)$ . For the strong bidder not to deviate from  $H$  to  $A$ ,

$$P - E > P \rightarrow E < 0 \rightarrow \text{not possible (for strong bidder)} \quad (17)$$

is needed.

Therefore, the  $(H, 2A)$  equilibrium cannot exist.

Fourth, check the equilibrium  $(H, 2HA)$ . Since the regular bidders will play a mixed strategy, the authors should solve for the regular bidders' equilibrium probabilities of playing  $H$  given that the strong bidder plays  $H$ . The equilibrium probability can be obtained by balancing any regular bidder's payoffs, say, bidder 3's as shown below:

$$(S/2 - E)v^* + (S - E)(1 - v^*) = (0)v^* + (S/2)(1 - v^*) \quad (18)$$

where  $v^*$  is bidder 2's equilibrium probability of playing  $H$ . However, since Eq. (18) cannot produce a unique equilibrium probability,  $(H, 2HA)$  does not exist.

Last, check the equilibrium  $(3HA)$ , where all bidders play mixed strategies. Under the mixed strategy equilibrium, two of the bidders' equilibrium probabilities will make the third bidder indifferent to choosing  $H$  and  $A$ . Accordingly, three equations can be obtained following the equilibrium requirements. First, given the two regular bidders' equilibrium probabilities  $v^*$  and  $w^*$ , the strong bidder's payoffs from choosing  $H$  (LHS of equation) and from choosing  $A$  (RHS of equation) will be the same, which leads to Eq. (19). Similarly, given probabilities  $u^*$  and  $w^*$ , Eq. (20) should be satisfied for bidder 2 and, given  $u^*$  and  $v^*$ , Eq. (21) should be satisfied for bidder 3

**Table 2.** Payoff Matrix of Three-Bidder Games with One Strong Bidder

Bidder 3	Bidder 2			
	H		A	
Bidder 1 (strong)	H		A	
	H	A	H	A
H	$(P - E, S/2 - E, S/2 - E)$	$(P - E, 0, S - E)$	$(P - E, S - E, 0)$	$(P - E, S/2, S/2)$
A	$(P/3 + S/3, P/3 + S/3 - E, P/3 + S/3 - E)$	$(P/2 + S/2, 0, P/2 + S/2 - E)$	$(P/2 + S/2, P/2 + S/2 - E, 0)$	$(P, S/2, S/2)$

$$\begin{aligned}
 &(P - E)w^*v^* + (P - E)w^*(1 - v^*) + (P - E)(1 - w^*)(v^*) \\
 &+ (P - E)(1 - w^*)(1 - v^*) = (S/3 + P/3)w^*v^* \\
 &+ (S/2 + P/2)w^*(1 - v^*) + (S/2 + P/2)(1 - w^*)(v^*) \\
 &+ P(1 - w^*)(1 - v^*) \tag{19}
 \end{aligned}$$

$$\begin{aligned}
 &(S/2 - E)u^*w^* + (S/3 + P/3 - E)(1 - u^*)w^* + (S - E)u^*(1 - w^*) \\
 &+ (S/2 + P/2 - E)(1 - u^*)(1 - w^*) = (0)u^*w^* + (0)(1 - u^*)w^* \\
 &+ (S/2)u^*(1 - w^*) + (S/2)(1 - u^*)(1 - w^*) \tag{20}
 \end{aligned}$$

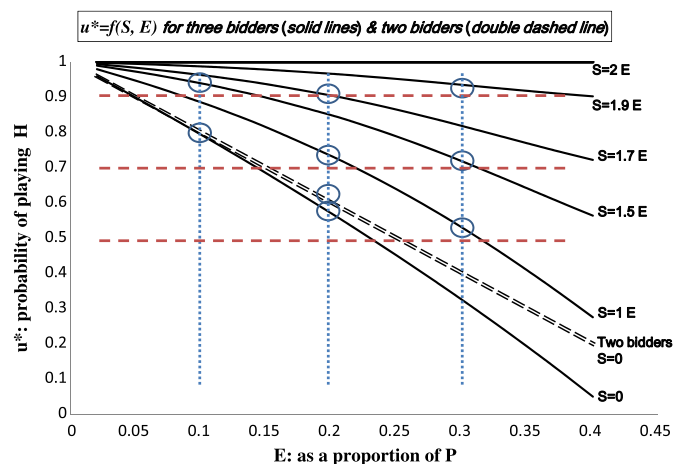
$$\begin{aligned}
 &(S/2 - E)u^*v^* + (S/3 + P/3 - E)(1 - u^*)v^* + (S - E)u^*(1 - v^*) \\
 &+ (S/2 + P/2 - E)(1 - u^*)(1 - v^*) = (0)u^*v^* + (0)(1 - u^*)v^* \\
 &+ (S/2)u^*(1 - v^*) + (S/2)(1 - u^*)(1 - v^*) \tag{21}
 \end{aligned}$$

Note that, as will be shown in next section, since  $S$  and  $E$  can be expressed in terms of  $P$ , numerical solutions can be obtained for the mixed strategy probabilities.

### Impact of Bid Compensation

To analyze the impact of bid compensation, the authors focus on the relationship between  $u^*$ ,  $S$ , and  $E$  by regarding the strong bidder's probability of playing  $H$ ,  $u^*$  as a function of  $S$  and  $E$ . Fig. 1 illustrates the numerical solutions of  $u^* = f(S, E)$  and identifies with circles the thresholds of  $S$  for different levels of desired  $u^*$  in three-bidder and two-bidder games. The horizontal dashed lines in Fig. 1 indicate three levels of desired  $u^*$ : 0.9, 0.7, and 0.5. In this paper, a  $u^*$  greater than 0.9, 0.7, or 0.5 is defined as being "very high," "high," or "medium," respectively. The vertical dotted lines in Fig. 1 indicate three levels of  $E$ , where  $0.1P$ ,  $0.2P$ , and  $0.3P$  are considered *small*, *medium*, and *large*, respectively.

First, consider the three-bidder games. According to Fig. 1, when the  $u^*$  desired by the owner is *very high*, i.e.,  $u^* > 0.9$ , the thresholds for bid compensations to be effective for small, medium, and large  $E$ s are  $S = 1.5E$ ,  $S = 1.7E$ , and  $S = 1.9E$ , respectively. If the desired  $u^*$  is *high*, i.e.,  $0.7 < u^* < 0.9$ , the thresholds of  $S$  for small, medium, and large  $E$ s drop to  $S = 0$ ,  $S = 1E$ , and  $S = 1.5E$ , respectively. If the desired  $u^*$  is only *medium*, i.e.,  $0.5 < u^* < 0.7$ , the thresholds of  $S$  for the three levels of  $E$  further drop to  $S = 0$ ,  $S = 0$ , and  $S = 1E$ , respectively.



**Fig. 1.** Numerical solutions of  $u^*$  and the required  $S$  for different levels of desired  $u^*$  in three-bidder and two-bidder games with one strong bidder

If it is assumed that, as often observed in practice,  $E$  is not too large, the discussion can then be focused on small and medium levels of  $E$ . By examining the probability lines/curves crossing  $E = 0.1P$ , it is found that, when  $E$  is small, the strong bidder will play  $H$  with  $u^* > 0.8$  even without bid compensation; however, to ensure that  $u^* > 0.9$ , the required bid compensation rises to  $S = 1.5E$  or higher. Thus, whether or not to use bid compensation in projects with small  $E$  mainly depends on how critical it is to have a very high level of  $u^*$ . On the other hand, by examining the probability curves crossing  $E = 0.2$ , it is found that when  $E$  is medium, the strong bidder will play  $H$  with  $u^* > 0.55$  without bid compensation. In this case, if it is assumed that the owner desires  $u^*$  to be at least high, the use of bid compensation becomes necessary. Fig. 1 also shows the solutions for two-bidder games, as indicated by the double line. As discussed earlier, in two-bidder games the offering of bid compensation can only reduce  $u^*$ . Therefore only the probability solutions for  $S = 0$  are shown. By examining the double line in Fig. 1, it is found that that it is impossible to ensure  $u^* > 0.9$  in two-bidder games. Also, when  $E$  is medium, the highest  $u^*$  is 0.6 only. Therefore, when it is critical to have a high or very high level of  $u^*$  for projects with a medium level of  $E$ , the owner's objective will be very difficult to achieve when there is only one strong bidder and one regular bidder.

### Four-Bidder Games with One Strong Bidder

#### Nash Equilibria and Their Conditions

The procedure for solving the equilibrium conditions for this game is similar to that for the three-bidder games discussed earlier. The payoff matrix of this game is similar to that in Table 2. Due to the concern of the length of this paper, the entire table will not be presented. Instead, only the payoffs relevant to the equilibria of concern during the solution derivation process will be listed. Similarly, in terms of the derivation of equilibrium solutions, only the general mixed strategy NE, including  $(H, 3HA)$  and  $(4HA)$  will be presented.

First, check the equilibrium  $(H, 3HA)$ . Similar to the equilibrium  $(H, 2HA)$  in the three-bidder games, the first condition for  $(H, 3HA)$  can be obtained by checking when the regular bidders will be indifferent to choosing  $H$  and  $A$ . The equilibrium probability can be obtained by simply balancing the payoffs of any regular bidders, say, bidder 4. First, consider the symmetry among regular bidders and let  $v^*$  be the equilibrium probability of playing  $H$  for each regular bidder. Second, balance the payoffs of bidder 4. In this case, bidder 4 plays against one strong bidder who plays  $H$  (i.e.,  $u^* = 1$ ) and two regular bidders who play mixed strategies. Accordingly, the balancing equation can be expressed as

$$\begin{aligned}
 &(S/3 - E)v^{*2} + (S/2 - E)v^*(1 - v^*) + (S/2 - E)(1 - v^*)v^* \\
 &+ (S - E)v^{*2} = (S/3)(1 - v^*)^2 \tag{22}
 \end{aligned}$$

where the LHS is the payoff of bidder 4 for playing  $H$  and the RHS is the payoff for playing  $A$ . The solution for Eq. (22) is

$$v^* = (2S - 3E)/S \tag{23}$$

Note that this equation implies that  $2S - 3E > 0 \rightarrow S > 1.5E$ ; that is, equilibrium  $(H, 3HA)$  can only exist when  $S > 1.5E$ . The second condition for  $(H, 3HA)$  is for the strong bidder to play the pure strategy  $H$ . This can be obtained by letting the strong bidder's payoff of playing  $H$  be greater than that of playing  $A$ . Accordingly, inequality (24) must be satisfied for equilibrium  $(H, 3HA)$



$$P - E > (P/4 + S/4)v^3 + 3[(P/3 + S/3)v^2(1 - v^*)] + 3[(P/2 + S/2)v^*(1 - v^*)^2] + P(1 - v^*)^3 \quad (24)$$

As a result, Eq. (23) and inequality (24) are the conditions for (H, 3HA). Again, numerical solutions can be obtained by trials and errors for any given relations between  $P$ ,  $S$ , and  $E$ . For example, Fig. 2 shows that when  $S = 2E$  the probability  $u^*$  equals 1, indicating that the game equilibrium is (H, 3HA). Fig. 2 also illustrates that when  $S = 1.7E$  and  $E < 0.2P$  the game equilibrium is also (H, 3HA); however, when  $E > 0.2P$  the strong bidder stops playing the pure strategy H and the equilibrium becomes (4HA).

Second, check the equilibrium (4HA). The first condition for (4HA) can be solved by checking when the strong bidder will be indifferent to choosing H and A. To obtain this condition, the inequality (24) is changed into the following equation:

$$P - E = (P/4 + S/4)v^3 + 3[(P/3 + S/3)v^2(1 - v^*)] + 3[(P/2 + S/2)v^*(1 - v^*)^2] + P(1 - v^*)^3 \quad (25)$$

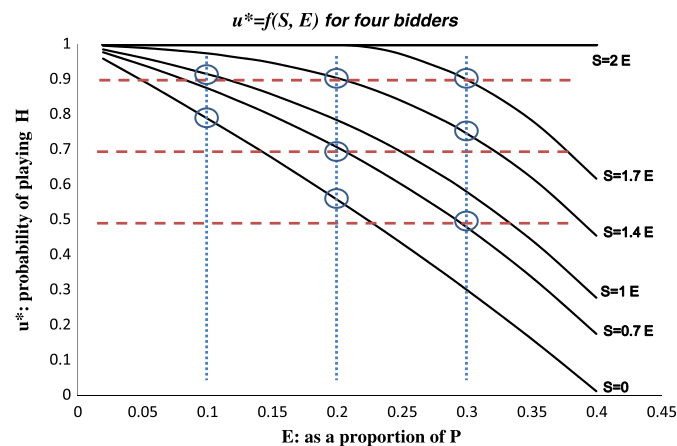
Again, given any relations between  $P$ ,  $S$ , and  $E$ ,  $v^*$  may be solved for numerically. The second equilibrium condition can be obtained by letting every regular bidder's payoff of playing H be equal to that of playing A. Consider the payoff of bidder 4

$$u^*[(S/3 - E)v^2 + 2(S/2 - E)v^*(1 - v^*) + (S/2 - E)v^*(1 - v^*)] + (S - E)(1 - v^*)^2 + (1 - u^*)[(P/4 + S/4 - E)v^2 + 2(P/3 + S/3 - E)v^*(1 - v^*) + (P/2 + S/2 - E)(1 - v^*)^2] = u^*(S/3)(1 - v^*)^2 + (1 - u^*)(S/3)(1 - v^*)^2 \quad (26)$$

Note that  $u^*$  can be solved numerically by substituting the value of  $v^*$  obtained from Eq. (25) into Eq. (26). Fig. 2 shows that, given a specific relation between  $S$  and  $E$ , the probability  $u^*$  may be obtained as a function of  $E$ , where  $E$  is expressed as a proportion of  $P$ .

### Impact of Bid Compensation

Fig. 2 illustrates the numerical solutions of  $u^* = f(S, E)$  and identifies with circles the thresholds of  $S$  for different levels of desired  $u^*$  in four-bidder games. According to Fig. 2, when the desired  $u^*$  is very high, i.e.,  $u^* > 0.9$ , the thresholds for bid compensations to be effective for small, medium, and large  $E$ s are  $S = 1E$ ,  $S = 1.4E$ ,



**Fig. 2.** Numerical solutions of  $u^*$  and the required  $S$  for desired  $u^*$  in four-bidder games with one strong bidder

and  $S = 1.7E$ , respectively. Note that these thresholds are significantly lower than those in three-bidder games identified earlier. This is because, with one more regular bidder, the higher degree of competition among regular bidders to win the bid compensation drives the strong bidder to play H with a higher probability. According to Fig. 2, if the desired  $u^*$  is high, i.e.,  $0.7 < u^* < 0.9$ , the thresholds of  $S$  for small, medium, and large  $E$ s drop to  $S = 0$ ,  $S = 0.7E$ , and  $S = 1.4E$ , respectively. If the desired  $u^*$  is only medium, i.e.,  $0.5 < u^* < 0.7$ , the thresholds of  $S$  for the three levels of  $E$  further drop to  $S = 0$ ,  $S = 0$ , and  $S = 0.7E$ , respectively. An interesting finding is that the solution curve of  $u^*$  for  $S = 0$  in Fig. 2 is almost identical to that in three-bidder games shown in Fig. 1, indicating that the competition pressure from the regular bidders is small in both three-bidder and four-bidder games due to the lack of bid compensation. Therefore, similar to the situations in three-bidder games, when  $E$  is medium and if it is assumed that the owner desires  $u^*$  to be at least high, the use of bid compensation is necessary.

### Comparative Analysis of the Effectiveness of Bid Compensation in Games with One Strong Bidder

The comparative analysis can be performed by reorganizing the results shown in Figs. 1 and 2. For example, Table 3 summarizes the required magnitudes of bid compensation for different levels of expected  $E$  and desired  $u^*$  in games with one strong bidder. Note that the lower the magnitude of bid compensation is for the desired  $u^*$  given a specific  $E$ , the higher the effectiveness of bid compensation is. Three major findings can be concluded by analyzing Figs. 1 and 2 and Table 3.

**Finding One:** In three-bidder and four-bidder games, the offering of bid compensation is necessary to ensure a very high level of  $u^*$ . This finding provides the theoretical foundation for supporting the offering of bid compensation in projects that desire a very high level of  $u^*$ .

**Finding Two:** To achieve the desired level of  $u^*$ , the required magnitude of bid compensation in four-bidder games is lower than that in three-bidder games. In other words, the effectiveness of bid compensation is higher in four-bidder games. This is because the more intensive competition among regular bidders stimulated by the bid compensation drives the strong bidder to play H with a higher probability. By this logic, it is reasonable to infer that the required magnitude of bid compensation for a desired level of  $u^*$  will decrease when the number of bidders increases, and vice versa.

**Finding Three:** According to Figs. 1 and 2, while games with more regular bidders are preferred in bidding when bid compensation is offered, surprisingly, games with three or four bidders will be outperformed by two-bidder games in terms of the probability  $u^*$  when bid compensation is not offered. This is best illustrated in Fig. 1, where the probability curve of  $S = 0$  for three-bidder games lies below the probability curve of  $S = 0$  for two-bidder games. The strategic implication is that if bid compensation is not offered, having more regular bidders can be countereffective in terms of  $u^*$ .

In addition, it is not until now that the authors may better explain the equilibria of two-bidder games that offering bid compensation is countereffective by the reduction of  $u^*$ . In fact, this is consistent with the previous finding that as the number of regular bidder decreases, the effectiveness of bid compensation decreases; that is, the required bid compensation will be higher with less regular bidders. In the extreme case where there is only one regular bidder, even if the bid compensation is infinitely large, the desired probability  $u^*$  cannot be reached. As for why offering bid compensation is countereffective only in two-bidder games, it can be intuitively

**Table 3.** Effective Magnitude of Bid Compensation for Different Levels of Expected  $E$  and Desired  $u^*$ 

Owner's desired $E$ for projects	Owner's desired probability level, $u^*$	Effective magnitudes of bid compensation, $S$		
		2 Bidders	3 Bidders	4 Bidders
Small $E$ ; $E \approx 0.1P$	Very high ( $u^* > 0.9$ )	Impossible	$S \geq 1.5E$	$S \geq 1E$
	High ( $u^* > 0.7$ )	$S = 0$	$S \geq 0$	$S \geq 0$
	Medium ( $u^* > 0.5$ )	$S = 0$	$S \geq 0$	$S \geq 0$
Medium $E$ ; $E \approx 0.2P$	Very high	Impossible	$S \geq 1.7E$	$S \geq 1.4E$
	High	Impossible	$S \geq 1E$	$S \geq 0.7E$
	Medium	$S = 0$	$S \geq 0$	$S \geq 0$
Large $E$ ; $E \approx 0.3P$	Very high	Impossible	$S \geq 1.9E$	$S \geq 1.7E$
	High	Impossible	$S \geq 1.5E$	$S \geq 1.4E$
	Medium	Impossible	$S \geq 1E$	$S \geq 0.7E$

explained through the *proof by contradiction* logic, a form of proof that establishes the truth or validity of a proposition by showing that the contrary of the proposition would imply a contradiction. Here, the negation of the proposition is that offering bid compensation is effective in two-bidder games. If so, the regular bidder's chance to win will be further lowered. Moreover, as discussed earlier, if bid compensation can be effective in two-bidder games, the required bid compensation must be larger than those required in three-bidder and four-bidder games. If so, in two-bidder games, the worst scenario for the regular bidder is that the bidder is guaranteed to earn a relatively *large* amount of bid compensation. Given that the regular bidder's chance of winning is lowered but the regular bidder's worst payoff is to be well compensated, it would be better off for the regular bidder to be further away from playing  $H$  but anticipate the bid compensation; i.e., the regular bidder's probability of playing  $H$  will be lowered. However, the reduced competition pressure from the regular bidder should induce the strong bidder to play  $H$  with a *lower* probability. This would be in conflict with the negated proposition, where the strong bidder will play  $H$  with a *higher* probability. Note that this argument is not as rigorous as mathematical derivation and is only for the purpose of providing intuitive insights.

### Games with Two Strong Bidders

In practice, it is not unusual to have two strong bidders participating as qualified bidders. While the games with two strong bidders and some regular bidders may appear to be more complicated, the solutions are much simpler and can be generalized to  $n$ -bidder games.

### Three-Bidder Games with Two Strong Bidders

Since the owner's major concern is when the strong bidders will play the pure strategy  $H$ , the authors will focus on the equilibria where both strong bidders play  $H$ . The analysis begins by examining equilibrium  $[(2H, A); (P/2 + S/2 - E, P/2 + S/2 - E, 0)]$ . For the strong bidders (e.g., bidder 1) not to deviate from  $(2H, A)$  to  $[(A, H, A); (S, P - E, 0)]$ ,  $P/2 + S/2 - E > S \rightarrow S < P - 2E$  is needed, which will be satisfied when  $P - 2E > 0$ , i.e.,  $E < P/2$ . For the regular bidder not to deviate from  $(2H, A)$  to  $[(2H, H); (P/2 + S/2 - E, P/2 + S/2 - E, -E)]$ ,  $0 > -E$  is needed, which is always true. To summarize, as long as  $E < P/2$ , the equilibrium conditions for  $(2H, A)$  can always be satisfied even when  $S = 0$ . Since  $E < P/2$  is valid for most projects, it can be concluded that, for most projects, the two strong bidders will automatically play the pure strategy  $H$  even without any bid compensation; that is, the offering of bid compensation is not necessary. Having obtained this important result, the authors do not need to solve for other possible game equilibria.

### Four-Bidder and $n$ -Bidder Games with Two or Three Strong Bidders

Again, the analysis here will focus on the equilibria in which both strong bidders play  $H$ . The analysis begins by examining the equilibrium  $[(2H, 2A), (P/2 + S/2 - E, P/2 + S/2 - E, 0, 0)]$ . For the strong bidders (e.g., bidder 1) not to deviate from  $(2H, 2A)$  to  $[(A, H, 2A); (S, P - E, 0, 0)]$ ,  $P/2 + S/2 - E > S \rightarrow S < P - 2E$  is needed, which will be satisfied when  $P - 2E > 0$ , i.e.,  $E < P/2$ . This condition is identical to the condition for three-bidder games with two strong bidders. For the regular bidder not to deviate from  $(2H, 2A)$  to  $[(2H, H, A); (P/2 + S/2 - E, P/2 + S/2 - E, -E, 0)]$ ,  $0 > -E$  is needed, which is always true. Therefore, the equilibrium condition for  $(2H, 2A)$  is identical to that for  $(2H, A)$  in the three-bidder games.

By examining the strong bidder's payoff patterns in three-bidder and four-bidder games discussed above, the authors find that the patterns remain the same for five-bidder games with two strong bidders. In fact, it can be easily shown that for  $n$ -bidder games with two strong bidders, the equilibrium condition for  $[2H, (n - 2)A]$  is identical for any  $n$ . Therefore, it can be concluded that, given  $E < P/2$  (as seen in most projects), and when there are two strong bidders, the strong bidders will play the pure strategy  $H$  no matter how many regular bidders there are. Note that this equilibrium is identical to that in Ho's (2005) two-bidder games with homogeneous bidders. In other words, the regular bidders in games with two strong bidders have no impact on how the strong bidders will play, and thus the strong bidders behave as if in a two-bidder game with homogeneous bidders. This is because the regular bidders are in a relatively inferior position to compete with the strong bidders and, when there are two strong bidders, they have little chance of winning the bid compensation even by playing  $H$ . By the same token, when there are three strong bidders, the regular bidders will be in an even weaker position to compete with the strong bidders. As a result, the solutions of games with more than three strong bidders are also very close to that of games with homogeneous bidders as discussed in Ho (2005). Following Ho's (2005) results, bid compensation is not necessary either for games with three strong bidders. Last, for games with four or more strong bidders, the regular bidders should have no intention of participating in the bidding; thus, the bid compensation strategies should also follow those in Ho (2005).

To summarize, in  $n$ -bidder games with two or three strong bidders, the strong bidders will play high efforts with very high probability even without any bid compensation. Due to this desired result, project owners should strive to encourage at least two strong bidders to compete for the project, so as to promote high-quality bid preparation. As for how to encourage more strong bidders to participate in project tendering, this is beyond the scope of this study.



## Proposed Bid Compensation Strategies and Model Limitations

Based on the modeling and analysis performed in this study, some applicable bid compensation strategies are proposed. The authors first propose the strategies that may improve information completeness. Then, the strategy that the use of bid compensation be considered together with the design or characteristics of the tendering process is proposed. Specifically, the scenario where there is a short-listing process and the owner's decision on whether or how to offer bid compensation will be made ex post after the composition of bidders can be closely estimated (i.e., after short-listing) is discussed. Next, the authors discuss the scenario where the bid compensation clause has to be decided ex ante and disclosed in the request for proposal (RFP) before the composition of bidders can be estimated. Readers may refer to the notation list provided at the end of the paper for quick reference of the notations.

### Strategies for Improving Information Completeness

Information completeness is a prerequisite for the success of offering bid compensation. The validity of the prediction about bid compensation effectiveness and the proposed strategies is based on the assumption that all bidders know what the extra efforts for playing  $H$  are and that the extra efforts will be identified and preferred by the owner. In this perspective, strategies concerning the use of bid compensation may include the following:

- The owner should explicitly specify in the RFP what the owner's expectation on the *high efforts* is, what the estimated extra costs are, and how the high efforts will be rewarded in terms of tendering preference and bid compensation. It is suggested that the owner fully compensate the costs of desired extra efforts,  $E$ , i.e., to let  $S = 1E$ , so that the amount of bid compensation can be used as a valid communication signal for  $E$ .
- Given the strategy that  $S = 1E$ , according to Table 3, if the owner desires the high or very high level of  $u^*$ , the owner should so design the desired high efforts so that the extra costs are within the range between  $0.1P$  and  $0.2P$ .
- Negotiation or open discussion during the early phase of tendering process can help to facilitate the process of reaching the information completeness.

### Strategies for Ex-Post Bid Compensation Decision Making

In practice, especially for private owners, the owners may delay the decision on whether to offer bid compensation until the bidders are short-listed. In this case, the numbers of short-listed strong and regular bidders will be very close to the actual numbers in bidding. As such, the modeling results can be directly applied to derive the bid compensation strategies.

1. Whereas bid compensation is not required for encouraging the bidders' high efforts in homogeneous-bidder games, bid compensation is not necessary either for encouraging the strong bidders' high efforts in heterogeneous-bidder games with at least two strong bidders. As a result, the corresponding bid compensation strategies are
  - When there are no strong bidders to be short-listed, the use of bid compensation is not necessary for encouraging the bidders' high efforts; and
  - When there are at least two strong bidders to be short-listed, the use of bid compensation is not necessary for encouraging the strong bidders' high efforts.
2. In heterogeneous-bidder games with one strong bidder, bid compensation may become a crucial incentive for encouraging

the strong bidder's playing high efforts. As shown in Table 3, while the use of bid compensation is not effective in two-bidder games, in three-bidder and four-bidder games with one strong bidder, by offering appropriate amount of bid compensation, a high or very high level of  $u^*$  can be induced. Accordingly, the following strategies for the use of bid compensation are suggested:

- When there is only one strong bidder to be short-listed, the owner should also short-list at least two regular bidders and then offer an appropriate amount of bid compensation that corresponds to the desired probability  $u^*$ , as indicated by Table 3.
- When there is only one strong bidder to be short-listed and the desired level for  $u^*$  is very high, the owner is suggested to short-list three to four regular bidders and offer bid compensation.
- When there are only one strong bidder and one regular bidder to be short-listed, the use of bid compensation is not suggested as it will reduce the probability  $u^*$ .

### Strategies for Ex-Ante Bid Compensation Decision Making

If the owner has to decide on the bid compensation and disclose the bid compensation clause in the RFP before short-listing, the owner will face the uncertainty of the ex post numbers of strong and regular bidders. The actual numbers of strong and regular bidders could render the bid compensation ineffective or unnecessary and, consequently, cause the bid compensation to be wasted ex post. This leads to an important practical issue: when the offering of bid compensation can be justified under uncertainty. First, the authors argue that when there are *limited* potential strong bidders in the market among a few potential regular bidders and when the benefits due to the strong bidders' high efforts are very high, offering bid compensation could be beneficial because the very high level of  $u^*$  cannot be reached without offering bid compensation if there is only one strong bidder competing with some regular bidders in actual bidding. In other words, offering bid compensation is considered insurance, which can be justified when the payoffs due to the strong bidders' extra efforts are high enough. Second, even when there are many professional and capable (i.e., strong) potential bidders so that the offering of bid compensation seems to be unnecessary, the offering of bid compensation equal to the extra costs of high efforts can still be beneficial. This is because the amount of bid compensation disclosed in the RFP can serve as a good communication signal for game players concerning the desired extra efforts as discussed earlier. Accordingly, the strategy suggested is as follows: If the strong bidders' extra efforts are critical to project performance and the owner has to decide on and disclose the bid compensation clause in the RFP (before short-listing), the offering of bid compensation can be justified and is suggested.

### Model Limitations

This paper reports the results of our theory development concerning bid compensation effectiveness and strategies. The theoretical validity is mainly determined by the modeling process and mathematical correctness of the model. However, there are some limitations concerning this theoretical development. First, in the study, as in many economic modeling studies, some simplifying assumptions are necessary for the theoretic modeling. Therefore, although model solutions are derived mathematically in this study, the insights and qualitative implications of the model are more important and useful than the exact solutions obtained. Second, this study

focuses on theory development, and thus the empirical evidence on the modeling results and strategic implications is not within the scope of this paper. Due to the lack of empirical evidence, the empirical validity of this model and the derived suggestions are subject to be evaluated in future studies. Third, a general model for  $n$ -bidder games is needed for a more complete view of bid compensation strategies. Fourth, since the modeling process assumes fixed numbers of bidders, the model cannot predict whether offering bid compensation can attract more bidders to bid. Therefore, the effectiveness of bid compensation cannot be defined or considered in terms of attracting more bidders.

## Conclusions

Understanding the effectiveness of various bid compensation strategies is important. If owners misjudge the effectiveness of the bid compensation, they lose the opportunity of taking other measures to promote higher efforts in project planning, and tendering preparation. Whereas Ho (2005) argues that the use of bid compensation is generally not effective, the author show that, under a more general setting in which the bidders are heterogeneous, the use of bid compensation can be very effective. In this study, equilibrium conditions are solved for the two-bidder, three-bidder, and four-bidder games with one strong bidder and for the games with more than one strong bidder. Numerical solutions for the strong bidders' probabilities of playing the high-effort strategy are obtained and then used to deduce some important strategic implications.

Based on the game theoretic modeling, the effectiveness of bid compensation depends on different scenarios of project tendering. From the structural point of view, the strong bidder(s)' probability of playing high efforts in heterogeneous-bidder games is determined by four major factors: the number of strong bidders, the number of regular bidders, the costs of desired extra efforts, and the amount of bid compensation. The number of strong bidders and that of regular bidders characterize the market in terms of the degree of competition. The costs of desired extra efforts characterize the complexity of the project. In particular, given that there is one strong bidder and at least two regular bidders, the effectiveness of offering bid compensation increases with more regular bidders, lower costs of desired extra efforts, and larger amount of bid compensation. When there are two or more strong bidders, the strong bidders will almost surely play high efforts, and thus the use of bid compensation is not necessary.

Strategic implications and applications are proposed as follows. When there is only one strong bidder to be short-listed, the owner should short-list at least two regular bidders and then offer an appropriate amount of bid compensation that corresponds to the desired probability of playing high efforts. When there are only one strong bidder and one regular bidder to be short-listed, the use of bid compensation is not suggested as it will be countereffective. If the owner has to decide on and disclose the bid compensation clause in the request for proposal (before short-listing) and the strong bidders' extra efforts are critical to project performance, the offering of bid compensation is suggested. The owner should explicitly specify in the request for proposal what the owner's expectation on the *high efforts* is, what the estimated extra costs are, and how the high efforts will be rewarded in terms of tendering preference and bid compensation. It is suggested that the owner compensate in full the costs of desired extra efforts so that the amount of bid compensation can be used as a communication signal for the owner's desired extra costs.

The contributions of this paper are twofold. First, this study offers a more general theory on bid compensation strategies by considering heterogeneity among bidders. Second, this study helps project owners choose better bid compensation strategies for more effective procurements.

*Publisher's Note: This paper as originally published online included egregious errors introduced by ASCE during the composition process. The errors have been rectified in this republished version.*

## Notation

The following symbols are used in this paper:

- $A$  = average efforts;
- $E$  = the extra costs for playing high efforts,  $H$ ;
- $H$  = high efforts;
- $P$  = the profit margins from winning and performing the project;
- $S$  = the bid compensation for the second best bidder;
- $u$  = the strong bidder's probability of choosing  $H$ ;
- $v$  = the first regular bidder's probability of choosing  $H$ ;
- $w$  = the second regular bidder's probability of choosing  $H$ ; and
- $u^*$ ,  $v^*$ ,  $w^*$  = the equilibrium probabilities of choosing  $H$ .

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## References

- Burati, J. L. J., Farrington, J. J., and Ledbetter, W. B. (1992). "Causes of quality deviations in design and construction." *J. Construct. Eng. Manage.*, 10.1061/(ASCE)0733-9364(1992)118:1(34), 34–49.
- Design-Build Institute of America (DBIA). (1996). *Design-build manual of practice document number 201: The design-build process utilizing competitive selection*, Washington, DC.
- Drew, D., and Skitmore, M. (2006). "Testing Vickery's revenue equivalence theory in construction auctions." *J. Construct. Eng. Manage.*, 10.1061/(ASCE)0733-9364(2006)132:4(425), 425–428.
- Gibbons, R. (1992). *Game theory for applied economists*, Princeton University Press, Princeton, NJ.
- Ho, S. P. (2005). "Bid compensation decision model for projects with costly bid preparation." *J. Construct. Eng. Manage.*, 10.1061/(ASCE)0733-9364(2005)131:2(151), 151–159.
- Ho, S. P. (2006). "Model for financial renegotiation in public-private partnership projects and its policy implications: Game theoretic view." *J. Construct. Eng. Manage.*, 10.1061/(ASCE)0733-9364(2006)132:7(678), 678–688.
- Ho, S. P., Hsu, Y., and Lin, E. (2011). "Model for knowledge-sharing strategies: A game theory analysis." *Eng. Proj. Organ. J.*, 1(1), 53–65.
- Levitt, R., Wang, C., Ho, S. P., and Javernick-Will, A. (2013). "Encouraging knowledge sharing in engineering firms—part II: Game theory analysis and firm strategies." *Eng. Proj. Organ. J.*, 3(1), 22–31.
- Mas-Colell, A., Winston, M., and Green, J. R. (1995). *Microeconomic theory*, Oxford University Press, New York.
- Myerson, R. B. (1991). *Game theory: Analysis of conflict*, Harvard University Press, Cambridge, MA.
- Unsal, H., and Taylor, J. (2011). "Modeling interfirm dependency: Game theoretic simulation to examine the holdup problem in project networks." *J. Construct. Eng. Manage.*, 10.1061/(ASCE)CO.1943-7862.0000286, 284–294.