Multiobjective Bayesian Network Model for Public-Private Partnership Decision Support

Jingzhu Xie[1] and S. Thomas Ng[2]

Abstract: To improve the chance of success of a public-private partnership (PPP) scheme, it is essential to consider the feasibility of the scheme both from the economical and noneconomical perspectives according to the interests of all three key stakeholders, namely the government, the private investor, and end-users. Acknowledging the diverse and sometimes conflicting interests of the stakeholders, decision makers must identify a viable scheme that could satisfy public accountability, commercial interests, and social consideration of the government, investor, and community, respectively. However, because each decision item could have several possible values or states, it is difficult for decision makers to come up with different PPP schemes by adopting the conventional analytical methods. This paper proposes the use of Bayesian network (BN) techniques to imitate human reasoning and conduct multiobjective decision making. By establishing a decision network that connects the decision items, evaluating criteria, and the ultimate objectives (i.e., the satisfaction of the three main stakeholders), evaluation can be conducted through the BN and the noisy-OR gate concepts. A weighted score approach is applied to combine the objectives of the three stakeholders into a single value. This enables decision makers to evaluate and compare different PPP alternatives and identify a suitable strategy that could minimize the conflict, thereby ultimately increasing the chance of success of a PPP scheme.
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Introduction

Public-private partnership (PPP) has emerged as a pragmatic mechanism to improve the operational efficiency and/or to ensure that essential public facilities or services are delivered when the government lacks the required budget to pay for the capital cost. According to World Bank (2009), developing countries committed to investing $154 billion in infrastructure projects through private participation. The trend of using PPPs is particularly obvious in some emerging economies such as Brazil, Chile, China, and South Africa because sharp economic growth has resulted in a strong demand for infrastructure and construction facilities [Organisation for Economic Co-operation and Development (OECD) 2008]. Public-private partnerships have also been widely used in advanced countries like Australia, Canada, Japan, Korea, Spain, United Kingdom, and the United States. In the United Kingdom, approximately £56.6 billion worth of public infrastructure and construction projects were commissioned through private finance initiative (a commonly used PPP arrangement in the United Kingdom) (HM Treasury 2010). It is likely that PPPs will remain a prevalent project delivery method in many countries for years to come.

Inviting private companies to invest in public facilities would inevitably entail commercial considerations (Abdel Aziz 2007), and this may not be in line with end-users' expectations and the well-being of society at large. Without carefully scrutinizing and balancing the interests of all the stakeholders involved in a PPP project, the government may have to contend with political ramifications arising from social discontentment, and ultimately bear the consequence of project failure [cf. West Kowloon Cultural District (WKCD) (2007)]. In view of this, the government should make every endeavor to uncover the concerns of various stakeholder groups and evaluate the financial viability, social acceptance, environmental impact, and political sentiment of a PPP scheme at the feasibility stage (Heinke and Wei 2000; Ozdoganm and Birgonul 2000; Zhang 2004). By doing so, the public agency can identify a series of feasible PPP scenarios that would satisfy the concerns of all key stakeholders before entering into any negotiations with the private investor(s).

Over the years, researchers have been attempting to compile a list of critical success factors (CSF) and best-value contributing factors (BVCF) for evaluating the performance of a PPP project at the implementation stage (Zhang 2006). These CSFs and BVCFs can in turn be transformed into relevant decision criteria to facilitate the assessment. However, limited attention has been directed to investigate the CSFs from the perspective of each participant during the feasibility stage of a PPP scheme. For instance, the end-users may regard the tariff level and service quality as the key to success of a PPP scheme (Ozdoganm and Birgonul 2000; Wong, unpublished thesis, 2006), whereas a high return on investment and low maintenance cost upon facility transfer could be the CSF set by the investor and public agency, respectively. Without thorough understanding of the diverse preferences (as represented by CSFs and BVCFs) of all the stakeholders, it is difficult for the government to reach a mutually beneficial and acceptable agreement to warrant success.

As the number of CSFs and BVCFs to be considered by each stakeholder grows, the evaluation could become extremely complicated. The problem is aggravated when there are at least

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several possible states or values for every evaluation criterion. A decision-making system that can support government officials in making decisions regarding a PPP scheme at the early feasibility stage would be helpful (Ng et al. 2007b). Given the extraordinary computational time and resources required by conventional multicriteria analysis approaches, they may not serve well in the current problem domain. In this paper, a Bayesian reasoning technique is proposed to facilitate decision makers of the public agency to derive a series of PPP scenarios based on the expectations and interests of key stakeholders. Using the Bayesian reasoning techniques, a decision network can be devised to portray the relationships between the CSFs, decision criteria, and stakeholder satisfaction. The decision network can then be used for constructing a multiobjective optimization model. The Bayesian network (BN) model enables a list of noninferior solutions to be identified. By eliminating any suboptimal solutions, decision makers can focus on the PPP scenarios that have a higher probability of satisfying the diverse interests of stakeholders and use them as a basis to negotiate with the investor(s) at a later stage.

This paper begins by introducing the decision network, and the potential of applying the Bayesian reasoning techniques to solve the identified problem is then exemplified. A multiobjective BN model is then developed to distinguish the noninferior PPP solutions. Finally, the operation of the developed BN model is demonstrated through a hypothetical case.

Decision Network

It is likely that the public and private partners of a PPP scheme would not share the same objectives, because the goal of a private investor is to maximize its profit, whereas that of the government is to ensure all the essential facilities and services are delivered according to their agenda (Abdel Aziz 2007). Although public facilities are built to serve the community, it is essential to ensure that the requirements of end-users are catered for at the early stages of the decision process. A successful partnership can only be realized when a partner possessing different objectives agrees to align with the others' expectations. As a result, the government should delineate the diverse stakeholders' interests, both economical and noneconomical, and try to minimize any disagreement before a PPP assignment is commissioned (Akintoye et al. 2003; Wong, unpublished thesis, 2006). A decision network capable of showing the relationships between the evaluating criteria, stakeholders' expectations, and the potential PPP options would be indispensible because it can provide decision makers a road map of what to focus.

Decision Items Pertinent to PPP Schemes

When the public agency determines whether to adopt PPPs or not, they should have a clear idea about the service specifications, financial viability, and legal implications of the scheme (HM Treasury 2007; Efficiency Unit 2008). Apart from general items such as statutory requirements, there are some essential items to be determined by decision makers according to the project characteristics and stakeholders' interests. To gain a better understanding of the decision items, PPP literature and guidelines produced by various government agencies [e.g., Canada Council for PPPs (2004), HM Treasury (2007), Efficiency Unit (2008)] and financial institutions [Asian Development Bank (ADB) 2008; World Bank 2007] were reviewed. Consequently, seven decision items commonly considered by stakeholders of a PPP scheme were drawn up, namely the technical scheme, service requirement, tariff/toll level, financial option, PPP type, risk allocation, and concession period.

1. Technical scheme: the technical merit of a PPP scheme is an important consideration at the feasibility stage (ADB 2008; European Commission 2003). To increase the prospect of gaining acceptance from the general public and concern groups, the public agency should specify an environmentally friendly and less disruptive technical solution (Efficiency Unit 2008). However, this may lead to a significant increase in construction cost and thus affect the investment return of the scheme.
2. Service requirements: according to HM treasury (2007), the performance requirement is a key element of risk transfer. Although a high service requirement may result in greater end-user satisfaction and minimize the risk of the project proponent, it could increase the operation and management costs. Therefore, the service specifications prepared by the public sector must be agreeable to the other stakeholders and tested in the market (Efficiency Unit 2008).
3. Tariff/toll level: the level of tariff or toll is a sensitive issue in a concession-based PPP scheme. The prime objective of adopting PPPs for facility provision is to provide affordable service and encouraging usage while providing the private investor with sufficient revenue to warrant commercial viability (ADB 2008). Hence, the tariff scheme should simultaneously be acceptable to the community and attractive to the investor.
4. Provision of subsidies: the nature of a PPP project is to provide the required public facility or service to the community. In anticipation of substantial resistance from society when the private investor is permitted to raise the tariff to attain the guaranteed revenue, the public agency may consider contributing partially to the capital investment cost, or even running cost if necessary. A capital investment or subsidy (i.e., a viability gap financing) of any kind may be perceived by society as a transfer of interests to the commercial sector, which could result in political backlash.
5. PPP type: there are many different types of PPPs including build-own-operate-transfer, build-operate-transfer, build-own-operate, buy-build-operate, design-build-finance-operate, design-build-operate, and operation and maintenance. The European Commission (2003) and ADB (2008) proposed considering (1) the available options; (2) objectives and needs of the project and the community; (3) technical, legal, and financial constraints; and (4) market attraction to private sectors when determining the type of PPP mechanism to be adopted and the degree of private sector involvement required. This would minimize the chance of project failure during the operational stage.
6. Risk allocation: although the risk to be borne by various parties is largely governed by the chosen PPP type, different contractual terms and incentive or penalty schemes can be introduced to tailor the exact risk profile. The European Commission (2003) advised that the degree of risk being transferred to the private investor should vary from one project to another, and it is unreasonable to have the private sector bearing the majority of the risks because the additional costs will eventually be transferred to the end-users through a higher tariff scheme. The guiding principle of risk allocation is that risk should be borne by the party who is best able to manage it (European Commission 2003; Efficiency Unit 2008). Consequently, appropriate risk allocation between the private and public sectors is essential for cost-effectiveness and efficient project delivery (HM Treasury 2008).
7. Concession period: the factors to be taken into account when deciding on the concession period include (1) the expected life

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span of the assets; (2) possible residual value; (3) investors' willingness to bid; (4) ability for the potential investor to recover its initial cost; (5) incentives to perform; and (6) continuity in service delivery (HM Treasury 2007). From the project proponent and end-users' perspective, a shorter concession period would be desirable because it could result in better service quality and lower tariff (Ng et al. 2007a).

Evaluation Criteria

Achieving the best value while maximizing the satisfaction of the three key PPP stakeholders (i.e., the government, the private investor, and end-users) is the desired goal of the government when using a PPP scheme to provide essential facilities. Therefore, a set of evaluation criteria should be formulated to measure the effectiveness of a PPP solution in achieving the satisfaction of the three key stakeholders. Many researchers have examined the drivers or factors that could contribute to the success or result in the best value of PPP projects. Although the CSFs and BVCFs as identified in literature (Chan et al. 2004; Li et al. 2005a, b; Zhang 2005; Chen and Chen 2007) are primarily related to the operational and implementation stages of PPP projects, these factors will also help reveal the reasons leading to the success or failure of PPP projects at the early planning stage. A list of criteria for evaluating a PPP scheme was drawn from these CSFs and BVCFs. Using the grouping regime proposed by Ozdoganm and Birgonul (2000), the CSFs and BVCFs related to PPP projects can be classified systemically according to the characteristics of various aspects of risks involved in a PPP project. According to Han and Diekmann (2001), decision makers should carefully examine the risks associated with the technical, financial, economic, social, environmental, political, and legal issues. Because a successful PPP scheme should be economically viable, socially acceptable, politically reliable, and environmentally friendly (Heinke and Wei 2000; Zhang 2004), the evaluation criteria are categorized under technical criteria, financial and economical criteria, social and environmental criteria, political and legal criteria, and other criteria. Table 1 outlines the detailed list of evaluation criteria with their sources of reference provided.

Establishing a Decision Network

Because every PPP scheme is unique, decision makers should identify the evaluation criteria, collect the necessary data, and construct a decision network to represent the case in question each time a PPP scheme is initiated. However, in the absence of a real project, a series of semistructured interviews was conducted to solicit the preliminary data and unveil the generic relationships among the decision items, evaluating criteria, and satisfaction levels of various stakeholders based on a hypothetical PPP-based highway scheme. Samples were drawn by referring to their experience and knowledge in PPPs, and only those who were at senior management level were selected for this study. Of the selected samples, six agreed to take part in the semistructured interview. The interviews consisted of two government officials, three consultants, and one person who worked for a development bank. Because the interviewees had ample experience in PPPs and had participated in the feasibility stage of this type of project, their opinions should be reliable for the establishment of a preliminary decision network.

Voice recordings were made during the interviews, and transcripts were subsequently prepared. This enabled the researchers to extract the most important factors leading to the satisfaction of different participants. Although the opinions of the experts interviewed are not identical, a decision network as shown in Fig. 1 was established by referring to the majority's views. The figure highlights the relationships between the CSFs and decision criteria. For example, tariff/toll level can be influenced by several decision criteria, viz service level, financial viability, and acceptable tariff/toll level. Furthermore, the figure also portrays how the decision criteria would affect the satisfaction of each PPP stakeholder. Although the government is accountable to the general public, its satisfaction of a PPP scheme depends on the possibility to reduce the public budget, risk transfer, service quality, political support, and environmental sustainability. As for the private investors, a good financial viability, low construction and operational cost, effective resources utilization, and early project completion would increase their satisfaction. In contrast, end-users would like to enjoy good service quality, early opening of the facility, and an acceptable tariff/toll level. Further details about the decision network are available in Xie (unpublished thesis, 2010).

However, it should be stressed that the identified decision network is by no means universal for various PPP schemes because each project would have its distinctive characteristics and requirements. Therefore, the proposed decision network will only be regarded as a reference when applied to other PPP schemes. Acknowledging the challenge of identifying relevant and representative stakeholder groups and the problem in capturing and aggregating the group preferences (de Neufville 1990; Goodwin and Wright 2004), the government should form a panel that consists of representatives from the affected citizens, nongovernment organizations, professional institutions, trade associations, and concern groups to determine which stakeholder groups can best represent the interests of different parties every time a PPP scheme is commissioned. Focus group meetings should then be organized with each of the identified stakeholder groups to establish the CSFs and decision criteria with which they would be most concerned. This would help ensure that the decision network can meet the features of the PPP project in question and the concerns of the groups being affected by the specific scheme.

Bayesian Reasoning Techniques

Analyzing the possible solutions based on the identified decision network is never an easy task, not only because there are different decision items and evaluation factors, but also because of the availability of different possible states or values. Coping with the interactions between decision variables, interim events, and final objectives would further complicate the decision process, which renders the computation difficult without the help of suitable analytical approaches. Blecic et al. (2007) pointed out that multiobjective decisions require decision makers to (1) simultaneously undertake actions to make decisions on different items/variables; (2) achieve a trade-off among different and often conflicting objectives that are probabilistically dependent on simultaneous actions; and (3) provide the knowledge such that a realistic model can be built. However, because the objectives of a multiobjective problem can be conflicting, it is difficult to reach an optimal solution. Therefore, one would strive to search for a noninferior solution—one in which no other solution can improve an objective while not damaging the others. With a list of noninferior solutions, decision makers can then transform the multiobjective problems into a single objective problem by assigning a weighting to each objective. An ultimate decision can be reached by choosing a noninferior solution that has the optimal expected objective value [cf. Evans (1984)].

Bayesian Networks

A BN is a graphical probabilistic model adopted to provide normative decision support. The BN technique has been widely applied to

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Table 1. Summary of Successful/Best Value Criteria for Evaluating the PPP Scheme

Criteria	Abdel Aziz (2007)	Akintoye et al. (2003)	Ashley et al. (1998)	Chan et al. (2004)	Efficiency Unit (2008)	Grasman et al. (2008)	HM Treasury (2004)	Ozdoganm and Birgonul (2000)	Sobhiyah et al. (2009)	NCPPPs (2002)	Wong (unpublished thesis, 2006)	Zhang (2005)	Zhang (2006)
Technical criteria													
Available potential private sectors					x	x	x	x			x	x	
Utilization of resources, managerial skills, and technologies of private and stimulation of innovation	x	x		x	x	x	x		x	x	x	x	x
Good service quality	x	x		x	x		x		x	x	x		
Long project life span					x				x				x
Modular and repeatable design/construction					x								x
Financial criteria													
Good financial viability/value for money	x	x	x		x	x	x	x	x	x	x	x	
Risks transfer	x	x	x	x	x	x	x	x	x		x	x	x
Reduce public off-balance budget						x	x	x	x		x		x
Reduce public administrative costs		x											x
Reduce transaction cost of procurement process		x					x						
Early project completion/product or service delivery		x		x	x								x
Acceptable tariffs/tolls	x		x		x				x		x	x	x
Low project life cycle costs													x
Low construction, operation, and maintenance costs		x						x				x	x
Acquisition of a fully completed and operational facility			x										x
Benefits to local economy											x		x
Political criteria													
Political sensitivity of the project											x		
Political support to the project	x		x					x			x	x	
Social and environmental criteria													
Acceptable to the interest group and general public					x			x			x	x	
Environmentally sustainable			x		x		x	x			x	x	x
Other criteria													
More job opportunities					x					x	x		
Fairness to staff							x				x		
Supportiveness and commitment of staff											x		

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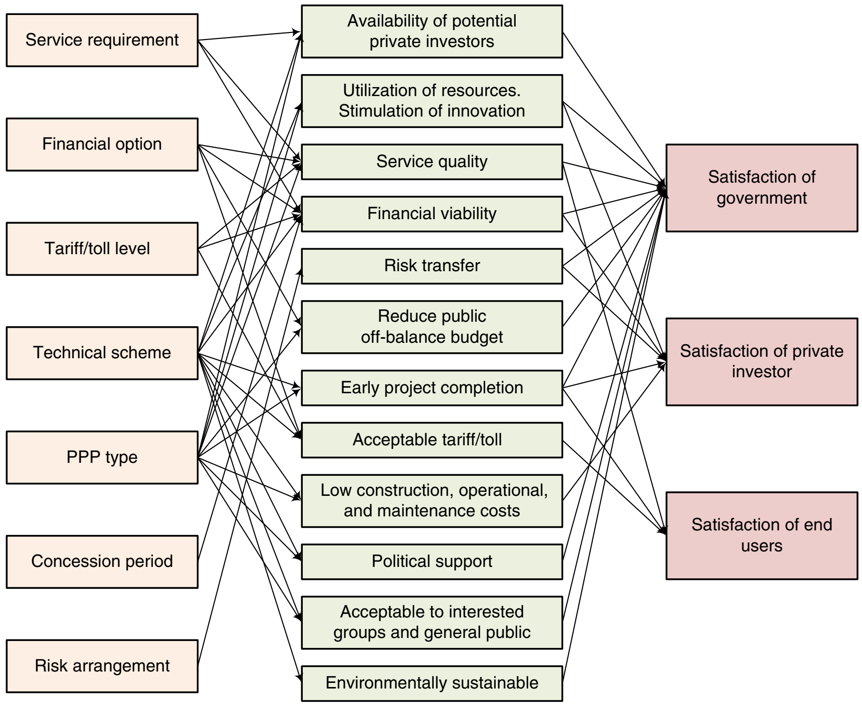

Fig. 1. Decision support network for evaluating PPP scheme

solve diagnostic and decision problems, including those in the medical, economical, military, and engineering domains (Russell and Norvig 2003; Diehl and Haimes 2004; Dorner et al. 2007). According to Borsuk et al. (2003), a BN is a formal statistical modeling framework that facilitates the analysis of relationships using the Bayes theorem in which predictions are represented probabilistically using a confidence interval. The BNs usually include (1) the decision variables (i.e., actions) that are under the decision makers' control; (2) objective variables to express the decision makers' preferences; and (3) some interim variables that connect the decision variables and the objective variables. The network is constructed by a series of nodes that represent the variables, and the nodes are connected according to the reasoning direction of decision makers (Kjaerulff 2008). The relationship between each pair of connected nodes is expressed in the form of probability distribution that encapsulates the decision makers' experience (Kjaerulff 2008). Once the model is built, the BN can assist decision makers in identifying the actions through the decision theory and according to the specified preferences (Blecic et al. 2007).

Using an acyclic directed graph to model the probabilistic dependencies and independencies among variables, the BNs can be derived to (1) represent the variables by nodes; (2) give directed links between related nodes; and (3) define a factorization of joint probability distribution over variables/nodes (Kjaerulff 2008). Therefore, for a BN, $B = (V, E)$, where V denotes a set of nodes (or variables), and E denotes a set of directed links between pairs of the nodes, a joint probability distribution that can be factorized as

$$\Pr(X_V) = \prod_{v \in V} \Pr(X_v | X_{pa(v)}) \tag{1}$$

where X_V = set of variables indexed by V; X_v = variable for each node $v \in V$; and $X_{pa(v)}$ = set of parent variables of X_v, $pa(v) \in V$.

Each conditional probability distribution for node v, $\Pr(X_v | X_{pa(v)})$, consists of a series of conditional probability p_i

$$p_i = \Pr(x_v | x_{pa(v)}), \qquad (i = 1, 2, \ldots, I) \tag{2}$$

where x_v and $x_{pa(v)}$ = value assigned to X_v and a vector of values assigned to $X_{pa(v)}$, respectively; and p_i = probability that $X_v = x_v$ when $X_{pa(v)} = x_{pa(v)}$. Assuming that X_v has n parent variables and X_v itself and its parent variables have m possible values for each, then the number of p_i, i.e., i, is equal to m^{n+1}. Specifying all the conditional probabilities for a given node would enable a $m^n \times m$ table, i.e., a conditional probability table (CPT), to be built.

Noisy-OR Gate

The largest limitation of normative BN lies in the definition of CPT for each node. Its complexity increases exponentially with the number of parent nodes and that of the possible values of these nodes. For example, in a binary BN, there are 2^{n+1} conditional probabilities in the CPT for one single node, where n is the number of parent nodes. This makes the definition of CPT an onerous task for decision makers when the number of parent nodes (n) increases, especially for the nonbinary BN.

Some types of conditional probability distributions can be approximated by canonical interaction models that require fewer parameters. The approximate degree can usually be sufficiently good and can significantly reduce the effort of building a BN (Oniśko et al. 2001). One widely used technique is the noisy-OR gate (Pearl 1988; Oniśko et al. 2001; Diez and Druzdel 2007; Antonucci 2011). The idea of the noisy-OR gate was originally proposed by Pearl (1988), who believed that if the parents of a variable can be regarded as independent and have sufficient causes of it, an OR gate may provide a parameter-free quantification of the CPT, with the conditional probability functions assigned all the mass to the state corresponding to the Boolean disjunction of the parents. Assuming that each cause (parent) has a nonnegligible probability of being inhibited, i.e., even if a sufficient cause is active (true), it may not be able to trigger the effect (its child), and the CPT can be quantified only on the basis of the inhibition probabilities for each parent, resulting in a linear instead of exponential number of parameters (Oniśko et al. 2001; Antonucci 2011).

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Here, the binary noisy-OR gate is used to highlight the concept. Suppose there is a variable X_0, and it has several possible causes (shown as parent nodes in BN), which are $X_1, X_2, \ldots, X_n$, assuming (1) each node $X_i (i = 1, 2, \ldots, n)$ has a probability p_i of being sufficient to produce an impact on X_0 in the absence of all other causes; and (2) the ability of each cause being sufficient is independent of the presence of the other causes. Then, p_i presents the probability that X_0 will be true if the cause X_i is present and all other causes are absent

$$p_i = \Pr(x_0|\bar{x}_1, \bar{x}_2, \ldots, x_i, \ldots, \bar{x}_{n-1}, \bar{x}_n) \tag{3}$$

The probability that X_0 is true, given that the causes in a subset X_P of $X_i s$ are all presented, can be represented by the following formula:

$$\Pr(x_0|x_P) = 1 - \prod_{i:X_i \in X_P} (1 - p_i) \tag{4}$$

Eq. (4) can be used to derive the complete CPT of X_0 conditional on its parents $X_1, X_2, \ldots, X_n$. The model can also be extended to non-Boolean variables as demonstrated by Diez (1993), Srinivas (1993), Diez and Druzdel (2007), and Antonucci (2011).

Applying Bayesian Networks for Multiobjective Decisions

A BN can be applied to solve the decision problems by (1) representing the decision variables, interim variables, and objective variables through the nodes D_N, C_M, and O_Q ($N = 1, 2, \ldots, n$; $M = 1, 2, \ldots, m$; $Q = 1, 2, \ldots, q$), respectively; (2) connecting the related nodes to show the logical relationship; and (3) defining the CPTs for each node. The CPT relationship between each pair of connected nodes is expressed in the form of a probability distribution that contains the statistical information of the decision makers' experience. The structure is shown in Fig. 2.

After assigning a set of values to the decision variables, the Bayesian rules and noisy-OR gate techniques can be applied to deduce the values or expected values of the objective variables. Given a set of values $(d_1, d_2, d_3, \ldots, d_n)$ for the decision items, which construct a possible solution, the objective value can be derived step-by-step along the direction of the arrows using the noisy-OR gate technique. Eqs. (5)–(8) illustrate how the expected value of an objective O_2 is calculated. The conditional probabilities of nodes C_1 and O_2 can be determined by Eqs. (5) and (6) according to Eqs. (3) and (4). Eq. (7) derives the probability that $O_2 = o_2$,

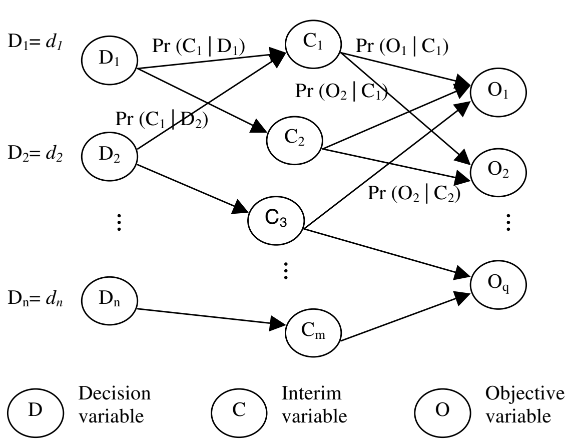

Fig. 2. BN model for multiobjective decision

whereas Eq. (8) quantifies the expected objective value. Similarly, the expected values of all the objectives can be derived.

$$\begin{aligned}\Pr(C_1 = c_1|D_1 = d_1; D_2 = d_2) &= 1 - \Pr(C_1 = c_1|D_1 \neq d_1) \\ &\quad \times \Pr(C_1 = c_1|D_2 \neq d_2) = 1 - [1 - \Pr(C_1 = c_1|D_1 = d_1)] \\ &\quad \times [1 - \Pr(C_1 = c_1|D_2 = d_2)]\end{aligned} \tag{5}$$

$$\begin{aligned}\Pr(O_2 = o_2|C_1 = c_1; C_2 = c_2) &= 1 - \Pr(O_2 = o_2|C_1 \neq c_1) \\ &\quad \times \Pr(O_2 = o_2|C_2 \neq c_2) = 1 - [1 - \Pr(O_2 = o_2|C_1 = c_1)] \\ &\quad \times [1 - \Pr(O_2 = o_2|C_2 = c_2)]\end{aligned} \tag{6}$$

$$\Pr(O_2 = o_2) = \sum_{c_1}\sum_{c_2} \Pr(O_2 = o_2|C_1 = c_1; C_2 = c_2) \tag{7}$$

$$E(O_2) = \sum_{o_2} o_2 \Pr(O_2) \tag{8}$$

These steps are repeated to evaluate the objective values for every possible solution. By testing all the possible value sets of the decision variables and comparing their objective values, noninferior solutions can be derived.

Modeling Procedure

In this paper, a multiobjective BN model for determining the most appropriate PPP solution was established through an eight-step decision procedure, as shown in Fig. 3.

1. Construct a solution by determining a set of values for the decision items. The decision items and their possible values are predefined by decision makers. As discussed previously, the decision items of PPPs shall include (1) the technical scheme; (2) service requirement; (2) tariff/toll level; (4) financial option; (5) PPP type; (6) risk allocation; and (7) concession period.
2. Evaluate the solution under the criteria package. The evaluation criteria derived from the CSFs or BVCFs are categorized under five aspects, namely, technical, financial, political, social and environmental, and others.
3. Calculate the objective values for the solution by using the Bayesian reasoning techniques. Although there are three conflicting objectives (i.e., satisfying the government, the private investor, and end-users), it is necessary to compute the three objective values for the current solution based on the evaluation results in step 2.
4. Establish a noninferior solution pool in the first iteration. The first solution is sent to this pool as a temporary one, and in the subsequent iterations, the objective values of the current solution are compared with those in the noninferior solution pool.
5. Determine whether the current solution is a noninferior solution. If it is a noninferior solution, the process will continue to step 6. Otherwise, it will proceed to step 7.
6. Adjust the noninferior solution pool. The current solution is sent to the pool, whereas any solution shown to be inferior in all three objective values simultaneously during the comparisons will be removed.
7. Check whether all the possible solutions have been evaluated. If so, go to step 8. Otherwise, return to step 1 to begin another iteration.
8. Further decisions shall be made to determine an appropriate solution based on the noninferior solutions. Because there

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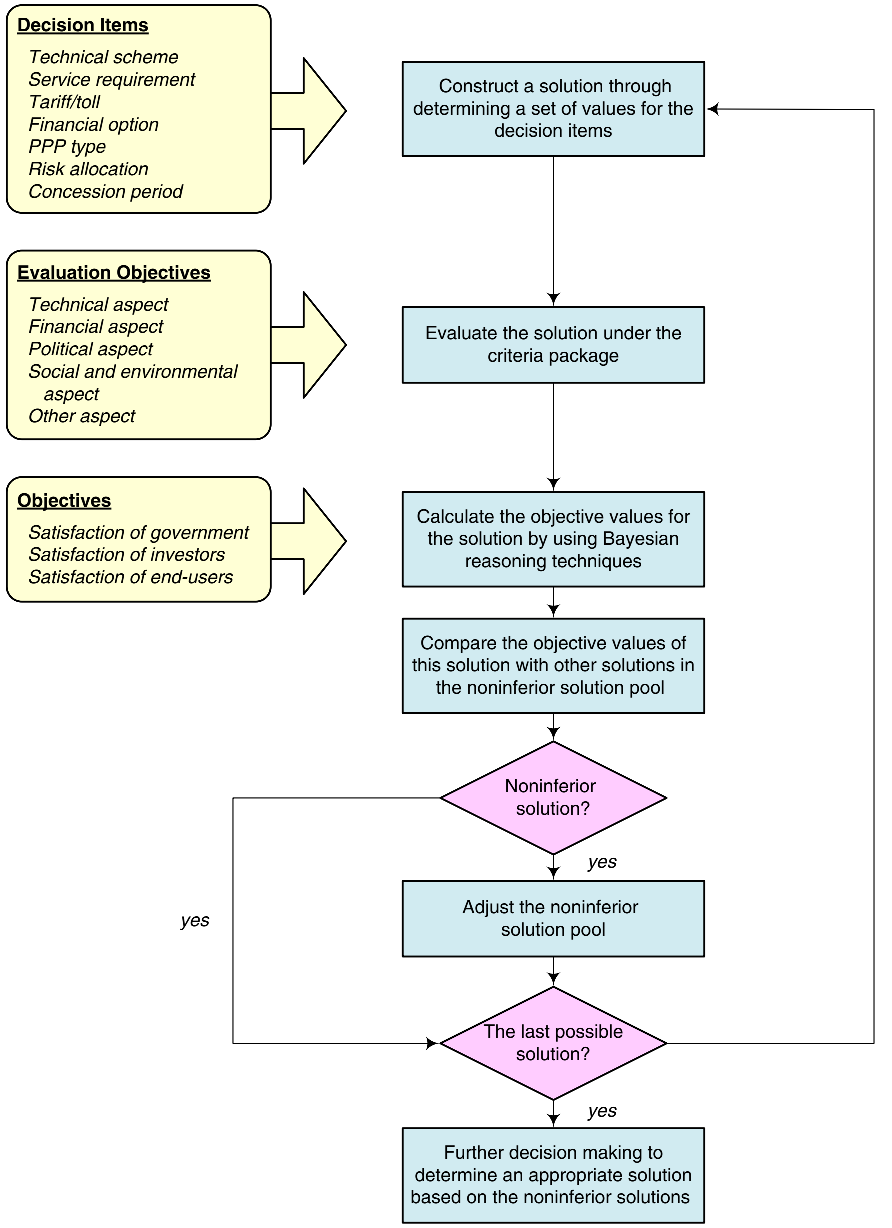

Fig. 3. Conceptual framework of the multiobjective BN model for determining appropriate PPP solution

is seldom a best solution for a multiobjective problem, one way to obtain an optimized solution is to assign weightings to the objectives and to compute a weighted average objective value for each solution in the noninferior solution pool, through which an optimal solution can be derived.

The result generated by the proposed multiobjective BN model should provide a pool of noninferior solutions in which no other solutions can simultaneously improve the three objective values. This can support the government in determining an appropriate solution to balance the interests of different PPP stakeholders and achieve success and best value in the end.

Bayesian Networks for PPP Modeling

A prototype multiobjective BN decision-making model was developed using Microsoft Access, Matlab, and Delphi. The structure of this prototype is shown in Fig. 4.

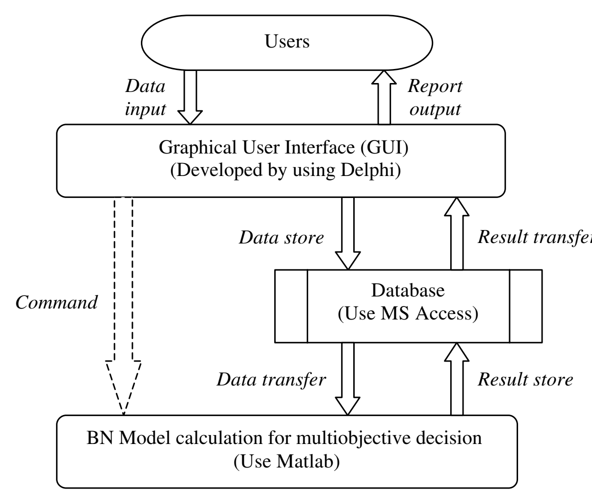

Fig. 4. Structure of the prototype

1. A database was designed using Microsoft Access to store the necessary data such as the information of three types of nodes representing the decision items, criteria, and objectives in the BN; the possible states or values of each node; the CPT data between each related node in the BN; and the results of all possible solutions and noninferior solutions after the BN computation.
2. The model component of Bayesian reasoning was realized by Matlab, a high-level language and interactive environment that enables users to perform computationally intensive tasks.
3. A graphical user interface was developed in Delphi to facilitate easy interaction with the database and background models. With that, decision makers can input the data related to the decision items, criteria, and objectives as required for the BN modeling and for generating the output such as various possible solutions, a collection of different noninferior solutions, and the BN diagram.

Hypothetical Case

To demonstrate the features of the proposed BN model, a hypothetical case was established. Hypothetical cases are often used in model development to demonstrate the performance of a prototype designed as a decision model that often involves some improvements to the existing decision process, and this may require the data being presented in a different form (Shen et al. 2002; Shen and Wu 2005; Zhang 2009). Because the proposed BN model has given rise to a new decision process of involving the private investor and end-users when evaluating a PPP scheme, it is difficult to gather the necessary data from a real case, especially because the concession items can be extremely sensitive. Moreover, because a practicable decision model would require many cycles of improvements, it is also not economical to collect the data from project personnel of a PPP project at the current stage, and these justify the use of a hypothetical case for the prototype development in this study.

For the purpose of model development, it was assumed that the government plans to procure a highway through the PPP mode, and decision makers from a relevant government department has identified some preliminary features for the PPP scheme. Having consulted the general public or end-users and the potential private investors, they found that the interests among the three stakeholders are conflicting. For instance, whereas the end-users prefer the tariff to be kept as low as possible, the private investors consider the scheme as nonprofitable if the tariff is set at the level desired by the end-users.

Before conducting a multiobjective evaluation, the public agency shall invite representatives of various stakeholder groups to solicit opinions from a wider population through internal meetings, forums, focus group meetings, and/or public engagement exercises. Nonetheless, because it was impossible to perform this in a hypothetical case, questionnaires were distributed to several interviewees to obtain analogous data about the decision items, evaluation criteria, and their relationships. All six interviewees were experienced in PPPs, with two of them from the government, three representing the consultants, and one being a financial expert. Because the case is hypothetical, end-users, especially the lay persons, may have difficulties coming up with practical suggestions on decision items and evaluation criteria. Therefore, they were not invited to take part at this stage. Despite that, because the consultants are knowledgeable in the technical aspect and experienced in soliciting feedback from the general public, they should be able to provide some insight about the end-users' concerns. In the questionnaire, the interviewees were asked to tick the most important decision items that the government shall consider when establishing a PPP scheme for this hypothetical case.

Table 2. Decision Items and Their Alternative States/Values

Node number	Decision item	Alternative states/values
1	Technical scheme	Simple: no specific techniques, only list the requirements of the result Common: this specifies the application of a commonly used technique Complex and high design standard: this specifies the application of an advanced and complex technique for the purpose of sustainability and protecting the environment
2	Service requirement	High Moderate Low
3	Tariff/toll level	High: HK$30/unit service Moderate: HK$20/unit service Low: HK$15/unit service
4	Financial options	Much: HK$200 M (finance HK$100M in construction cost and subside in tariff up to HK$100M) Some: HK$100 M Non: 0
5	PPP type	Design, build, finance and operate (DBFO) Build, own and transfer (BOT) Build, own, operate and transfer (BOOT)
6	Risk arrangement	Transfer most risks: transfer most risks including economic, demand, and political risks to private sector Transfer moderate risks: public sector keeps political risks and transfers economic risk to end-users Transfer few risks: public sector keeps political risk, assumes some demand risks with private sector and transfers economic risk to the end-users
7	Concession period	20 years 25 years 30 years

After summarizing their opinions, it was found that all seven items in the questionnaire shall be traded off before calling for a formal submission of proposals. These items (see Table 2) are the key decision items for establishing a BN for this case project. The five groups of evaluation criteria (as shown in Table 1) were also listed in the questionnaire for the respondents to express their perception on the degree of importance in relation to the success of a PPP project through a seven-point Likert scale. Considering the rapid growth in complexity of the BN model when the number of variables increases, the 12 most important criteria with an arithmetical mean greater or equal to five were considered (see Table 3). Besides, the objectives used and their rating scales along with the corresponding node number are shown in Table 4. The respondents were also asked to identify the decision items that may relate to each evaluating criterion, and to delineate the objectives having the greatest influence. The results are shown in Tables 5 and 6. Based on this information, the decision network as illustrated in Fig. 1 can be constructed. It is worth noting that the decision network may vary according to the characteristics and requirements of each PPP scheme, and it is important to form a panel to fine-tune or develop the decision network to suit the particular scheme. In this example, it is assumed that the alternative states or values

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Table 3. Criteria for Evaluating the PPP Solution

Evaluating criteria	Rating scales	Node number	Parent nodes
Technical criteria			
Availability of potential private sectors	Many, some, few	8	1, 5, 6
Utilization of resources, managerial skill, and technologies of the private sector and stimulation of innovation	Effective, moderate, few	9	1, 5
Good service quality	Good, moderate, bad	10	2, 3, 4
Financial criteria			
Good financial viability/value for money	Good, moderate, bad	11	1, 2, 3, 4, 7
Risks transfer	High, moderate, low	12	3, 4, 6
Reduce public off-balance budget	Many, some, few	13	4, 5
Early project completion/product or service delivery	High, moderate, low	14	1, 5
Acceptable tariffs/tolls	High, moderate, low	15	2, 3
Low construction, operation, and maintenance costs	High, moderate, low	16	1, 5
Political criteria			
Political support for the project	Many, some, few	17	4, 5
Social and environmental criteria			
Acceptable to the interest group and general public	Many, some, few	18	1, 5
Environmentally sustainable	Good, moderate, bad	19	1

Table 4. Objectives Used and Their Rating Scales

Objectives	Rating scales (scores)	Node number	Parent nodes
Satisfaction of public sector	High (9) Moderate (5) Low (1)	20	8, 9, 10, 11, 12, 13, 14, 18, 19
Satisfaction of private sector	High (9) Moderate (5) Low (1)	21	9, 11, 12, 14, 16
Satisfaction of end users and general public	High (9) Moderate (5) Low (1)	22	10, 14, 15, 18, 19

of the seven decision items are known and are as listed in the second column of Table 2.

With seven decision items, each having three alternative states or values, the number of possible solutions can be as many as 2,187 (i.e., 3^7). Consequently, it is difficult for decision makers in the government to determine which options are more likely to result in a successful PPP scheme considering the conflicting interests of the three main stakeholders.

To conduct the Bayesian reasoning, it is assumed that the alternative states or values of the 12 evaluating criteria and the three objectives have been identified by the decision maker, and are as listed in the second column of Tables 3 and 4.

Taking the decision items, evaluating criteria, and the satisfaction of the three stakeholders as three different types of nodes in BN, a node number can be assigned to each (refer to the "Node number" column of Tables 2–4), with the relationship between each node and the parent nodes being identified based on the result of Tables 5 and 6. For example, Table 5 shows that the three decision items, namely service requirement (node 2), tariff/toll level (node 3), and public financing (node 4), determine whether the project could achieve good service quality (node 10) in the future, and these three nodes (i.e., nodes 2, 3, and 4) would be regarded as the parents of node 10.

After constructing the BN, the next step is to set up the probabilistic relationships, i.e., CPTs, for each pair of nodes. In the real world, this should be conducted by inviting representatives from each stakeholder group to rate each item under the criterion with which they are most concerned, and the rating can then be aggregated and transferred to the CPT. For example, an end-user would be asked to rate each of the alternative tariff levels under the

Table 5. Relationship between the Decision Items and the Evaluating Criteria

Important criteria	Technical scheme	Service requirement	Tariff/toll level	Public financing	PPP type	Risk arrangement	Concession period
Technical criteria							
Availability of potential private sectors	x	x			x		
Utilization of resources, managerial skill, and technologies of private and stimulation of innovation	x				x		
Good service quality		x	x	x	x		
Financial criteria							
Good financial viability/value for money	x	x	x	x		x	
Risks transfer							x
Reduce public off-balance budget				x	x		
Early project completion/product or service delivery	x				x		
Acceptable tariffs/tolls			x	x			
Low construction, operation, and maintenance costs	x				x		
Political criteria							
Political support to the project	x				x		
Social and environmental criteria							
Acceptable to the interest group and general public	x				x		
Environmentally sustainable	x						

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Table 6. Relationship between the Evaluating Criteria and the Objectives

Important criteria	Satisfaction of public sector	Satisfaction of private sector	Satisfaction of end-users
Technical criteria			
Availability of potential private sectors	x		
Utilization of resources, managerial skill, and technologies of private and stimulation of innovation	x	x	
Good service quality	x		x
Financial criteria			
Good financial viability/value for money	x	x	
Risks transfer	x	x	
Reduce public off-balance budget	x		
Early project completion/product or service delivery	x	x	x
Acceptable tariffs/tolls			x
Low construction, operation, and maintenance costs		x	
Political criteria			
Political support to the project	x	x	
Social and environmental criteria			
Acceptable to the interest group and general public	x		x
Environmentally sustainable	x		x

Table 7. Rating of the Items under a Criterion

	Evaluating criterion		
	Acceptable tariffs/tolls (node 15)		
Decision item	High	Moderate	Low
Tariff level (node 3)			
\$30/journey			x
\$20/journey		x	
\$15/journey	x		

Table 8. Conditional Probability Table for Nodes 2 and 15

	Node		
	Acceptable tariffs/tolls (node 15)		
Parent node	High	Moderate	Low
Tariff level (node 3)			
\$30/journey	0	0.1	0.9
\$20/journey	0.4	0.6	0
\$15/journey	1	0	0

criterion acceptable tariffs/tolls as shown in Table 7. Then, the ratings given by all the representatives within the end-user group will be counted and transferred to a CPT as shown in Table 8.

In the hypothetical case, it is assumed that the CPTs have already been established similar to Table 8. The figures suggest that 40% of the end-users consider \$20 per journey as a highly acceptable tariff level, whereas 60% of them feel it is moderately acceptable. Altogether, there are 48 CPTs for all the pairs of connected nodes.

Model Results

By inputting all the data of CPTs to the prototype and activating the BN program, the three objective values for each possible solution can be computed according to the Bayesian reasoning and noisy-OR gate concept. The solutions are compared with one another based on the three objective values, and the best solutions are derived based on the concept of the noninferior options. Ultimately, 31 noninferior solutions are derived through this multiobjective BN optimization, as shown in Table 9.

In this case, the scores of 9, 5, and 1 represent the degree of satisfaction as being high, moderate, and low, respectively. The result shows that the noninferior solutions help ensure all three stakeholders are attaining a greater satisfaction than usual.

There are two approaches to solve the multiobjective decision-making problems, i.e., the generating approach and the preference-based approach (Gen and Cheng 2000). The former requires the decision makers to select the best option within the pool of noninferior solutions based on their own value judgment. The latter applies a predetermined preference structure of the objectives to derive the best solution. With 31 noninferior solutions in this hypothetical case, it seems difficult to apply the generating approach. Although the weighted-sum method (based on a predetermined weighting structure of the objectives) is widely used among various preference-based approaches because of its ability in compositing different objectives into a single one easily and effectively, its disadvantage lies in the difficulty to determine a creditable weighting structure. In the questionnaire developed for this study, the interviewees were asked to express their views on which stakeholder's satisfaction should bear a higher weighting. Yet, all the respondents agreed that all three are of equal importance, and thus a 1:1:1 weighting structure was assigned to the three objectives. Subsequently, the composite objective value of each noninferior solution can be calculated by the weighted sum of those three objective values.

The solution with the highest composite objective value is highlighted in Table 9 and shown in detail in Table 10. Based on the results, the government should consider procuring the highway through the design, build, finance, and operate approach to maintain a high quality service. As for the toll, it should be set at a common level of \$20 per vehicle, with few risks being absorbed by the private investor while the government provides \$100 million to support the project.

This solution is supposed to be the most appropriate for the PPP scheme in question because (1) it is a noninferior solution, meaning that no other solutions can further simultaneously improve the satisfactions of the three stakeholders; and (2) it is the best one because it can achieve the highest composite objective value among the noninferior solutions.

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Table 9. List of Noninferior Solutions

Solution								Objective values			
Solution identification	Technical scheme	Service requirement	Tariff/toll	Financial option	PPP type	Risk transfer	Concession period	Satisfaction of public sector	Satisfaction of private sector	Satisfaction of end-users	Composite value
1	Common	High	20	200	DBFO	High	20	5.146530	5.461291	5.701590	5.436471
2	Common	High	20	100	DBFO	High	20	5.254614	5.372630	5.692852	5.440032
3	Common	Moderate	30	200	DBFO	Moderate	20	5.152180	5.679690	5.367850	5.399907
4	Common	High	20	200	DBFO	Moderate	20	5.157712	5.599664	5.701590	5.486322
5	Common	High	20	100	DBFO	Moderate	20	5.286169	5.483899	5.692852	5.487640
6	Common	High	20	200	DBFO	Low	20	5.155851	5.609363	5.701590	5.488935
7	Common	High	20	100	DBFO	Low	20	5.229353	5.578131	5.692852	5.500112
8	Common	Moderate	30	200	DBFO	Moderate	25	5.159567	5.706868	5.367850	5.411428
9	Common	High	20	200	DBFO	Moderate	25	5.161710	5.613030	5.701590	5.492110
10	Common	Moderate	20	200	DBFO	Moderate	25	5.170051	5.645322	5.618953	5.478109
11	Common	Moderate	30	100	DBFO	Moderate	25	5.245680	5.656795	5.356873	5.419782
12	Common	High	20	100	DBFO	Moderate	25	5.294097	5.499733	5.692852	5.495561
13	Common	Moderate	20	100	DBFO	Moderate	25	5.298871	5.528440	5.573328	5.466880
14	Common	High	20	200	DBFO	Low	25	5.159765	5.622681	5.701590	5.494679
15	Common	Moderate	20	200	DBFO	Low	25	5.167985	5.654883	5.618953	5.480607
16	Common	Moderate	30	100	DBFO	Low	25	5.242050	5.666401	5.356873	5.421774
17	Common	High	20	100	DBFO	Low	25	5.235969	5.594534	5.692852	5.507785
18	Common	Moderate	20	100	DBFO	Low	25	5.239308	5.624486	5.573328	5.479041
19	Common	High	30	200	DBFO	Moderate	30	5.152140	5.682425	5.424154	5.419573
20	Common	Moderate	30	200	DBFO	Moderate	30	5.167269	5.735106	5.367850	5.423408
21	Common	High	20	200	DBFO	Moderate	30	5.165479	5.625644	5.701590	5.497571
22	Common	Moderate	20	200	DBFO	Moderate	30	5.177136	5.668911	5.618953	5.488333
23	Common	Moderate	30	100	DBFO	Moderate	30	5.257028	5.685736	5.356873	5.433212
24	Common	High	20	100	DBFO	Moderate	30	5.302913	5.517546	5.692852	5.504437
25	Common	Moderate	20	100	DBFO	Moderate	30	5.312105	5.555085	5.573328	5.480173
26	Common	High	20	200	DBFO	Low	30	5.163450	5.635236	5.701590	5.500092
27	Common	Moderate	20	200	DBFO	Low	30	5.174916	5.678378	5.618953	5.490749
28	Common	Moderate	30	100	DBFO	Low	30	5.253162	5.695231	5.356873	5.435088
29	Common	Low	30	100	DBFO	Low	30	5.193757	5.695676	5.102602	5.330678
30	Common	High	20	100	DBFO	Low	30	5.243341	5.612926	5.692852	5.516373
31	Common	Moderate	20	100	DBFO	Low	30	5.250364	5.652128	5.573328	5.491940

Table 10. Solution Having the Highest Composite Objective Value

Decision item	Option
Technical scheme	Common: this specifies the application of a commonly used technique
Service requirement	High
Tariff/toll level	Common: HK$20/journey
Public financing	Some: HK$100 M
PPP type	DBFO
Risk arrangement	Transfer few risks: public sector keeps political risk, assumes some demanded risks with private sector, and transfers economic risk to the end-users
Concession period	30 years

Conclusions

Balancing the conflicting interests of major stakeholders at the feasibility stage is critical to the success of a PPP scheme. In this paper, seven decision items commonly considered by stakeholders of a PPP scheme including the technical scheme, service requirement, tariff/toll level, financial option, PPP type, risk allocation, and concession period have been drawn up. These decision items should be carefully scrutinized by decision makers to determine if there is a significant divergence in perception among the government, investor, and end-users in terms of their interests in these items. Through a series of interviews, the relationships among the decision items, evaluation items, and stakeholders' satisfaction have been identified. Although the decision items and satisfaction could vary depending on the project nature and size and the political environment and social sentiment, the results of this paper should provide decision makers with an initial framework for more rigorous investigations when it comes to real-life application.

Acknowledging that the current practice of relying on subjective judgments for establishing a PPP solution at the feasibility stage is unreliable, this paper has proposed the application of Bayesian reasoning techniques to develop a multiobjective BN model. Through the BN model, a series of noninferior solutions can be derived. This could assist decision makers in determining the most appropriate PPP solution that can satisfy the interests of all the stakeholders to the greatest extent simultaneously. Although the largest challenge of normative BN lies in the definition of a conditional probability table for each node, the noisy-OR gate approach has been employed for model development to allow the conditional probability distributions to be approximated by canonical interaction models that require fewer parameters.

A prototype multiobjective BN decision-making PPP model has been developed using Microsoft Access, Matlab, and Delphi. In addition, a hypothetical case has been established to demonstrate the features of the proposed BN model. The results show that a list of noninferior solutions can be derived by the multiobjective BN optimization model. By associating the degree of satisfaction of different stakeholders with the noninferior solutions identified, decision makers can select the most suitable PPP solution. Despite that, the model should be flexible enough to cater for different environments and requirements with more decision items and evaluation criteria to be included in the model before it can be of practical usage. The flexibility should also be enhanced to allow the impacts of any unforeseeable changes to be reanalyzed. For example, in case of a sudden change in economic environment, the government should invite end-users to determine which tariffs/tolls are more acceptable to them, and a new series of PPP scenarios can then be generated. Moreover, the evaluation should also be extended to nonbinary cases so that decision makers are only required to define the CPT for each pair of related nodes in the BN because the joint conditional probability can then be approximated using the noisy-OR gate technique during the Bayesian reasoning process.

To improve the accuracy of the model, it is necessary to confirm whether the assumptions and findings, including (1) the number of stakeholder groups to be involved in the analysis; (2) who should be the representing groups (e.g., whether it is necessary to delineate the equity investors and debt providers and/or to differentiate the affected citizens and concern groups); (3) the major concerns of each stakeholder group; (4) the relationships between the CSFs and decision items; and (5) whether a series of linear relationships exist between CSFs and decision items are reliable enough through a large-scale questionnaire survey. With a wide variety of PPP modes, the variables leading to stakeholder satisfaction could vary, and this deserves a more comprehensive examination through interviews and questionnaire surveys as well. Although it is difficult to determine the efficacy and credibility of the model by simply testing it with hypothetical cases, it would be desirable to seek decision makers' participation in a longitudinal study by applying the model in a real case so as to determine whether the BN model is of practical usage and if it can indeed fully replace or expedite the negotiation process.

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