Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

A Stochastic Revenue and Cost Model for Determining a BOT Concession Period under Multiple Project Constraints

By Jiang-wei Xu[1], S.M.ASCE, and Sungwoo Moon[2], M.ASC1E

Abstract: The concession period is an important issue in the contractual arrangement of build-operate-transfer (BOT) projects such as highways, bridges, and tunnels. In determining the concession period, it is predominantly the toll revenue and construction costs that affect how long the concessionaire needs to be in operation after the completion of the project. This paper presents a stochastic revenue and cost model to determine a concession period under multiple constraints in planning a BOT infrastructure project. In the study, the stochastic process is converted into an equivalent discrete form, and its parameters are estimated using historical data. Based on the process, a principle-agent problem is addressed as a solution to the conflict between the owner and the concessionaire. This methodology incorporates these stakeholders' interests in terms of 1) incentive constraints, and 2) participation constraints. In a case study, a numerical simulation is carried out to assess the risk of toll revenue and construction costs when applied in practice, and to demonstrate the applicability of the stochastic revenue and cost model.

CD Database Subject Headings: Build-operate-transfer (BOT) project, concession period, stochastic process, incentive constraint, participation constraint

[1] Lecturer, School of Management, Southwest Univ. of Political Science and Law, Chongqing 400037, China. Email: jiangwei-xu@hotmail.com
[2] Professor, Dept. Of Civil and Environmental Engineering, Construction Engrg. Bldg. Room 509, Pusan National Univ., Busan, 609735, Korea (corresponding author). Email: sngwmoon@pusan.ac.kr

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Introduction

The build-operate-transfer (BOT) procurement system is a concession contractual arrangement between the government and the consortium formed by private investors (Tiong 1990). The consortium is usually called a concessionaire, and has a public-private partnership (PPP) with the government (Levy 1996). Under the contractual arrangement, the government grants the concessionaire the rights to build and operate the project for a certain time period, which is called the concession period. After the concession period ends, the concessionaire transfers the project to the concerned government at no cost (Iyer and Sagheer 2012). In this arrangement, the concessionaire can be paid for its investment and also generate profit by operating the built project within the given concession period. The government can develop public infrastructure facilities, such as highways, railways, bridges and municipal water supplies, using the investment from the concessionaire.

A BOT project requires a substantial amount of investment during the construction of infrastructure facilities. Due to the huge amount of investment, the concession period could be very long, spanning over a half a century in some cases (Kumaraswamy and Zhang 2001). The concession period is one of the most critical provisions in a concession contractual arrangement (Cheung and Chan 2011). The concessionaire needs to consider both the recovery of its investment and a reasonable reward. On the other hand, the government is obligated to try and minimize the concession period so that the public can use the infrastructure facility at a lower price. Therefore, the determination of the proper length of the concession period needs to strike a balance between the concessionaire's and the government's interests.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Studies have been done to address the need for balance in determining the concession period. Shen et al. (2002) developed a deterministic model to formulate a concession period that could protect the interests of both the government and private investors. Shen and Wu (2005) extended the deterministic concession model by incorporating risk impact into the concession period. In their study, they used the Monte Carlo simulation technique with input parameters, following a triangular probability distribution and a normal distribution. Zhang and AbouRizk (2006) also developed a risk model for the concession period. As a principle-agent maximization problem, Zhang (2009) proposed a win-win concession period determination in the BOT contractual arrangement. The aforementioned methods mostly estimated a range for the concession period that incorporates both private and public sector benefits, and used the Monte Carlo simulation to quantify and rationalize how risk affects the length of the concession period of a BOT project.

As documented in Zhang (2009), the PPP relationship can be represented as a principal-agent problem, in which the principal is the concerned government and the agent is the concessionaire. In the principal-agent theory, Laffont and Martimort (2001) advocated that both the principal and the agent are rational, and act in a manner that aims to maximize their own benefits. They also argued that the optimal contractual arrangement has to satisfy the two constraints of incentive and participation in order to improve operational efficiency, cost effectiveness, and service quality during construction and operation. In the case of BOT projects therefore, the optimal solution to a principal-agent problem is to design a contractual arrangement so that the agent acts in order to maximize the benefits of the principal, conditional on the agent’s willingness to participate. The problem can be better represented in a stochastic, rather than deterministic, process considering the probabilistic characteristics of toll revenues and construction costs over the project time period.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

This paper uses a stochastic revenue and cost process to model the toll revenue and construction costs. This process can be converted into an equivalent separate form for the purpose of estimating the parameters in the model, and can be applied in practice. Based on the stochastic process, the net present value (NPV) is calculated to obtain the present value of the toll revenue and construction costs. This paper then addresses the relationship between the concerned government and the concessionaire as a principle-agent maximization problem, and develops a method to resolve the conflict between the owner and the concessionaire. The two constraints, incentive and participation, are considered in the model development. Finally, a numerical simulation of a hypothetical highway project is presented to demonstrate the method in practice, and to illustrate how it accommodates the uncertainties of the toll revenue and construction costs.

Stochastic Process for Toll Revenue

BOT projects such as highways, bridges and tunnels usually require a large investment. The risk associated with the project should be controlled so as to deliver the project within budget and to gain the expected revenue. For BOT projects, the toll revenue is one of the most important considerations for both the concerned government and the private concessionaire. The toll revenue changes over the project time period and has probabilistic characteristics. The uncertainty involved in the process makes it more advantageous to model the process stochastically.

Toll Revenue as a Geometric Brownian Motion

The biggest challenge in modeling the toll revenue is the large uncertainty regarding traffic volumes, because the toll structure needs to be determined in advance. Moreover, the toll rate is usually regulated by the concerned government because a high toll level or an increase in the level often

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

generates strong public opposition (Zhang and AbouRizk 2006). Simulating the large uncertainty is usually done using the stochastic process. According to Brandao and Saraiva (2008) and Damnjanovic et al. (2010), the toll revenue is assumed to evolve stochastically over time according to the Geometric Brownian motion with drift:

$$dR_t = \mu R_t dt + \sigma_R R_t dw_t \quad \textbf{(1)}$$

where R_t is the toll revenue, μ is the drift parameter, σ_R is the standard deviation of the revenue increment dR_t, and dw_t is the increment of a standard Wiener process w_t.

In the standard Wiener process, $dw_t = \varepsilon_t \sqrt{dt}$, ε_t is a normally distributed random variable with a mean of 0 and a standard deviation of 1. The variables such as R_t and w_t with the subscript *t* are the functions of time *t*, and sometimes written as $R(t)$ and $w(t)$.

The Geometric Brownian motion with drift can be simplified by logarithmic transformation. A change in the natural logarithm of toll revenue $R(t)$ can be written as the simple Brownian motion with drift (Øksendal 2007):

$$d \ln R_t = \left(\mu - \frac{1}{2}\sigma_R^2\right) dt + \sigma_R dw_t \quad \textbf{(2)}$$

For $\tau > 0$, every increment $\ln R_{t+\tau} - \ln R_t$ is normally distributed with a mean $\left(\mu - \frac{1}{2}\sigma_R^2\right)\tau$ and variance $\sigma_R^2 \tau$. For every pair of disjoint time intervals, such as $[t_1, t_2]$, $[t_3, t_4]$, where $t_1 < t_2 \le t_3 < t_4$, the increments $\ln R_{t_2} - \ln R_{t_1}$ and $\ln R_{t_4} - \ln R_{t_3}$ are independent and normally distributed. This

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

property implies that it is very difficult to forecast the toll revenue, which is consistent with the fact that traffic forecasts on toll roads are usually highly inaccurate (Flyvbjerg et al. 2006, Bain 2009). Let T_C denote the completion time for the BOT project. Integrating both sides from completion time T_C to operating time *t*, the simple Brownian motion (2) has the analytical solution given the initial toll revenue $R\left(T_C\right)$

$$\ln R(t) = \ln R\left(T_C\right) + \left(\mu - \frac{\sigma_R^2}{2}\right)\left(t - T_C\right) + \sigma_R \sqrt{\left(t - T_C\right)}\varepsilon_t \qquad \textbf{(3)}$$

Clearly toll revenue is log-normally distributed, and its uncertainty increases with time *t*. This equation explains the difficulty in forecasting future traffic volumes.

This analytical expression provides the convenience of calculating the expected value of the toll revenue, which is usually involved when discounted in the project appraisal. Using the fact that the expectation of e^x is $E\left[e^x\right] = e^{E[x]+\frac{1}{2}Var[x]}$ for any normal variable x with the expectation $E[x]$ and variance $Var[x]$, the expected value of the toll revenue can be calculated as:

$$\overline{R}(t) = E\left[R(t)\right] = R\left(T_C\right)e^{\mu\left(t-T_C\right)} \qquad \textbf{(4)}$$

It is worth noting that, although the initial toll revenue is given, it is essentially stochastic due to the uncertainty of the construction duration. Therefore, the toll revenue is log-normally distributed in the sense of conditional probability. At the same time, the variance of the toll revenue is as follows (Øksendal 2007):

$$Var\left[R(t)\right] = \left(R\left(T_C\right)\right)^2 e^{2\mu\left(t-T_C\right)}\left(e^{\sigma_R^2\left(t-T_C\right)} - 1\right) \qquad \textbf{(5)}$$

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

The variance is very useful in assessing the risk associated with toll revenue. For example, a decision-maker with high risk tolerance may set the toll revenue at the 68th percentile of toll revenue, namely, mean plus one standard deviation $\overline{R}(t)+\sqrt{Var\left[R(t)\right]}$, while the decision-maker of low risk tolerance may adopt the mean minus one standard deviation $\overline{R}(t)-\sqrt{Var\left[R(t)\right]}$. They are both the functions of time *t*.

Calibrating Toll Revenue Parameters

In order to calibrate the parameters in equation (1) and to investigate its rationality, equation (2) is first converted into the equivalent discrete form at monthly frequencies:

$$\ln R_t - \ln R_{t-1} = \mu - \frac{1}{2}\sigma_R^2 + \sigma_R \varepsilon_t \qquad \textbf{(6)}$$

As the equation shows, the change in the natural logarithm of toll revenue follows the normal distribution with a mean $\mu-\frac{1}{2}\sigma_R^2$ and a variance σ_R^2. The actual monthly toll revenue data is then used to test whether the change in the natural logarithm of toll revenue follows the normal distribution, or the series ε_t is a white noise process (Greene 2002).

Some actual data regarding toll revenues and traffic volumes in China has been collected. Fig. 1 shows the monthly toll revenues of the Guang-Shen highway in South China. This highway connects Guangzhou city to Shenzhen city, which are the most developed cities in China. It is the first highway under a BOT contract arrangement in China, and began operating in July 1994. The data was collected from the company website (Hopewell Highway Infrastructure Ltd. 1994-2010). The data is adjusted seasonally using the Census X11 multiplicative method (Dagum 1988), which is the standard method used by the U.S. Census Bureau, because the actual monthly toll revenue fluctuates cyclically, particularly in February, which has either 28 or 29 days, and during which the most important spring festival in China is held.

The data in Fig. 1 has been converted into the change of natural logarithm. Fig. 2 shows the change of toll revenue. In order to determine whether or not the series are mutually independent, the autocorrelation coefficients are calculated. Let S_t denote the sample series in Fig. 2, then the lag n autocorrelation coefficient $\rho_n = Cov\left(S_t, S_{t-n}\right) \Big/ \sqrt{Var\left(S_t\right) Var\left(S_{t-n}\right)}$, where $Cov\left(S_t, S_{t-n}\right)$ is the covariance and $Var\left(S_t\right)$ is the variance. The autocorrelation coefficients ρ_n for $n = 1, 2, \cdots, 6$ are, respectively, -0.218, 0.086, 0.026, -0.003, -0.020, and -0.009. This result shows that the autocorrelation is very weak in the series S_t. In the case of the normality of series S_t, the most commonly used statistic for the test of normality, Jarque-Bera, rejects the null hypothesis at 5% significant level.

According to the central limit theory, the data of the toll revenue on the Guang-Shen highway supports equation (3). The five highways on which other data has been collected include Hu-Ning, Cheng-Yu, Guang-Jing, Xi-Xian, and Xi-Cheng highways from China, and three tunnels, namely, Cross Harbour Tunnel, Eastern Harbour Crossing, and Western Harbour Crossing from Hong Kong. The traffic volume data has been collected because the toll revenue data is usually unavailable. The movement of the traffic volumes is consistent with toll revenues because the toll rate is usually regulated by the government, and can only be adjusted every few years (Zhang and AbouRizk 2006). Therefore, the traffic volumes can be used as a proxy for investigating the rationality of equation (3).

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Since the actual data supports equation (1), the data in Fig. 2 has been used to calibrate the parameters. The mean and standard variance of the series S_t are, respectively, 0.0101 and 0.03974. Let $\mu - \frac{1}{2}\sigma_R^2 = 0.0101$ and $\sigma_R = 0.03974$ give the value of parameters, namely, $\mu = 0.0109$ and $\sigma_R = 0.03974$. The toll revenue of the Guang-Shen highway follows the stochastic process, $dR_t/R_t = 0.0109dt + 0.03974dw_t$, given the initial toll revenue of 42 million RenMingBis (RMBs) (or Chinese yuans).

Stochastic Process for Construction Costs

As discussed in the previous section in respect of the stochastic process for toll revenue, BOT projects have stochastic characteristics which relate to the contractual goals and requirements. Construction costs are an important consideration for both the government and the concessionaire in terms of project delivery. This section proposes using a controlled stochastic process to model the construction costs using the data from an actual BOT project.

Construction Cost

Consider an investment in a project which involves a sequence of investment outlays, corresponding to the specific steps in construction. Let $C(t)$ denote the total amount of investment in the implementation of the project in time *t*, and T_C is the total duration of construction until completion. The actual total cost of completion is then $C(T_C)$, and $C(T_C) - C(t)$ is the total investment required until completion. Although the actual cost of completion, $C(T_C)$, and the duration, T_C, are both random variables, the sequence of investment outlays or the speed of investment, as defined by Dixit

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

and Pindyck (1994), can be controlled by the owner. This is referred to as the investment rate and is denoted by $I(t)$. This rate is subject to the constraint $I_{\min} \le I(t) \le I_{\max}$, where the minimum investment rate is usually greater than zero because the project is public and irreversible.

In order to model the uncertainty of the construction costs, the total amount of investment in the implementation of the project in time *t*, $C(t)$, follows a controlled diffusion process:

$$dC_t = I_t dt + \sigma_C dz_t \qquad \textbf{(7)}$$

where, as in equation (1), C_t, I_t, and z_t are the functions of time *t*, dz_t is the increment of a standard Wiener process $z(t)$, namely $dz_t = \varepsilon_t \sqrt{dt}$. Also, where ε_t is a normally distributed random variable with a mean of 0 and a standard deviation of 1, σ_C is the standard deviation of the change of investment.

The stochastic differential equations (1) and (7) are similar in form. The state variable in equation (7) is the total amount of investment required for implementation $C(t)$, while that in equation (1) is the toll revenue $R(t)$. These two equations however, have some crucial differences. Equation (7) contains a controlled variable $I(t)$ by which the concessionaire can control the progress of construction. Meanwhile, equation (1), which does not have a controlled variable, reflects the fact that the toll revenue or traffic volumes are not controlled by the government or the concessionaire given the toll structure. Traffic volumes are largely driven by factors relating to the national economic situation and the level of economic development.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Calibrating Construction Cost Parameters

Similarly, to set the parameters, equation (7) is first converted into the equivalent discrete form at a monthly frequency:

$$C_t - C_{t-1} = I_t + \sigma_C \varepsilon_t \quad \textbf{(8)}$$

The cost increment $C_t - C_{t-1}$ is the actual increment of investment in month *t*. The cost increment is also affected by uncontrolled factors or uncertainty. The standard deviation σ_C and the random variable ε_t are at a monthly frequency. In addition, it is necessary to define $C_0 = 0$ at the beginning of the BOT project such that $C_1 = I_1 + \sigma_C \varepsilon_1$ in the first month. Equation (8) has a recursive structure, and the variance of C_t in month *t* can be calculated as $t\sigma_C^2$. From the discrete form, the total cost of completion $C(T_c)$ is obviously normally distributed. For some specific BOT projects, the expected value or budget and standard deviation may be constructed in advance based on the design, planning, and other available information, which are respectively denoted by $\overline{C}$ and σ_C. The investment rate I_t, which represents the desired amount of investment in month *t*, the maximum investment rate $I_{\max}$, and the minimum investment rate $I_{\min}$ can all be determined according to the construction plan.

Maintenance Cost

The maintenance costs usually include toll operation expenses, resurfacing expenditure, administrative expenses, interest, and even income tax. Let $\lambda(t)$ represent the proportion of the maintenance cost to the toll revenue in month *t*, which is also subject to the constraint

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

$\lambda_{\min} \le \lambda(t) \le \lambda_{\max}$. Although it usually increases rapidly at the end of the designed economic operation life, the maintenance cost may be flat during the concession period, particularly for infrastructure projects. Zhang and AbouRizk (2006) assumed that the annual maintenance cost is proportional to the yearly revenue. This paper follows the same assumption. Using the data of the Guang-Shen highway from 2001 to 2007, the proportion $\lambda(t)$ can be calculated as shown in Table 1.

Here, the maintenance cost includes five terms: the resurfacing cost (RC), toll operation expenses (TOE), general and administrative expenses (GAE), finance costs (FC), and income tax expenses (ITE), which are listed in the annual reports of Hopewell Highway Infrastructure Ltd. (2001-2008). Since FC are related to debt, and the government usually grants infrastructural facilities preferential policies, such as a reduction or waiver on taxation, the FC and ITE differ greatly according to time and region. In Table 1, two parameters λ_2, including all the five terms, and λ_1, including RC, TOE, and GAE, are computed. λ_1 ranges from 9% to 14%, and λ_2 ranges between 13% and 23%. Setting $10\% \le \lambda(t) \le 20\%$ seems to be reasonable during the concession period.

Calculation of Net Present Value

Once the stochastic processes have been set up for the revenue and costs, the NPV can be computed from equations (1) and (7). When calculating the NPV, the discount rate should also be continuous in order to gear with the continuous stochastic processes. In this situation, the discount rate changes over time and should be considered as probabilistic.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013; posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Let the discount rate be a function of time, which is denoted by $r(t)$. The continuous compounding rate implies that the discount factor equals $e^{-\int_0^t r(\tau)d\tau}$ from time 0 to time *t* (Watsham and Parramore 1997). The discount rate is taken on the expected value, namely, $\bar{r} = E\left[r(\tau)\right]$. It can then be simplified as $e^{-\bar{r}t}$. In fact, according to the standard textbook (Ross et al. 2008), the cash flow and discount rate should all adopt their expected values. The expected discount rate usually includes the interest rate and inflation rate, as documented in extant studies (Shen et al. 2002, Shen and Wu 2005, Zhang and AbouRizk 2006).

The present value of cash flow in an operating period needs to be calculated first. In time *t*, the expected cash flow equals $\left(1-\lambda(t)\right)\overline{R}(t)$, and its discount present value is $\left(1-\lambda(t)\right)\overline{R}(t)e^{-\bar{r}t}$. Summarizing from the expected beginning of the operation $\overline{T_C} = E\left[T_C\right]$ to the end of the concession period gives the following net present value:

$$PVT = \int_{\overline{T_C}}^{T}\left(1-\lambda(t)\right)\overline{R}(t)e^{-\bar{r}t}dt = \int_{\overline{T_C}}^{T}\left(1-\lambda(t)\right)R\left(T_C\right)e^{\mu\left(t-T_C\right)}e^{-\bar{r}t}dt \qquad \textbf{(9)}$$

Here, the initial toll revenue $R\left(T_C\right)$ essentially denotes the expectation of the initial toll revenue. Therefore, the initial toll revenue is also a random variable. More specifically, the revenue can be set as $R_0 e^{T_C-\overline{T_C}}$, which is the increasing function of the completion duration in the neighborhood of its expected value.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Since the expectation of dC_t is $I_t dt$ according to equation (7), the present value of the construction cost can be calculated as follows:

$$PVC = \int_0^{\overline{T_C}} I(t) e^{-\bar{r}t} dt \qquad \textbf{(10)}$$

Thus, the net present value during the concession period equals equation (9) minus equation (10), which is given by

$$NPV = \int_{\overline{T_C}}^{T} \left(1-\lambda(t)\right) R(T_C) e^{\mu(t-T_C)} e^{-\bar{r}t} dt - \int_0^{\overline{T_C}} I(t) e^{-\bar{r}t} dt \qquad \textbf{(11)}$$

Since all of the random variables are taken as the expected values, this expression suggests that the project evaluation is carried out in a risk-neutral condition without considering the degree of the decision-maker's risk tolerance, which was discussed in equation (5).

Usually the concerned government will try to minimize the NPV while, on the other hand, the concessionaire will try to maximize its profit or NPV. This contradiction results in a conflict of interest. The following section will discuss the problem and its possible solution by addressing the relationship between the government and the concessionaire as a principal-agent problem.

Balancing Interests between Stakeholders

In the case of PPPs, the concerned government is the principal and the concessionaire is the agent. According to the principal-agent theory, both the principal and the agent are rational, and behave according to their intention, which is to maximize their own benefits. In this section, the optimal contractual arrangement is discussed with the aim of satisfying the two constraints: incentive and participation. Then, using the NPV model, the concession period can be calculated under multiple constraints.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013; posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Incentive constraint

In BOT projects, the incentive constraint requires that the selected concessionaire volunteer to perform the BOT contract with the intention of achieving the government's goals, which may include continuously improving efficiency, cost effectiveness, and service quality. In incorporating the incentive constraint, this paper mainly considers the factors that the concessionaire can control. The investment rate $I(t)$ and maintenance cost $\lambda(t)$ are chosen as two key factors in the calculation of the NPV. Given the expected total cost, in fact, the expected construction duration can be determined by the investment speed. Therefore, the construction period is essentially also considered as an incentive constraint. The concessionaire makes a decision on the investment rate $I(t)$ and maintenance costs $\lambda(t)$ in order to maximize its profits.

$$\max_{I(t),\ \lambda(t)} NPV\left(I(t),\ \lambda(t)\right) = \int_{\overline{T_C}}^{T} \left(1-\lambda(t)\right) R\left(T_C\right) e^{\mu(t-T_C)} e^{-\bar{r}t} dt - \int_0^{\overline{T_C}} I(t) e^{-\bar{r}t} dt \qquad \textbf{(12)}$$

Given the concession period *T*, on the one hand, the government wants to complete the infrastructure as soon as possible to meet public demand. On the other hand, the greater investment rate suggests that the concessionaire can obtain the toll revenue faster over a longer toll period. The concessionaire should complete the project as early as possible in order to start generating toll revenue sooner. Not to mention, $I_{\max}$ should respectively be the reasonable maximum investment rate under the premise of guaranteeing project quality. In addition, during the operational period, highly efficient operation and low maintenance costs should ideally be achieved for the concessionaire to maximize its profits. The maximum problem, then, can be solved as:

$$NPV\left(I_{\max},\ \lambda_{\min}\right) = \int_{\overline{T_C}}^{T} \left(1-\lambda_{\min}\right) R\left(T_C\right) e^{\mu(t-T_C)} e^{-\bar{r}t} dt - \int_0^{\overline{T_C}} I_{\max} e^{-\bar{r}t} dt \qquad \textbf{(13)}$$

Here, $\lambda_{\min}$ and $I_{\max}$ vary over time in practice, and can be set as a function of time. For simplicity, $\lambda_{\min}$ and $I_{\max}$ are set as constants, which can also be regarded as a mean. The determination of the minimum operating cost $\lambda_{\min}$ can usually be solved by collecting the data of similar existing BOT projects. The highest investment rate $I_{\max}$ can be determined according to the construction plan. Directly solving the integration in (12) gives the following equation:

$$NPV\left(I_{\max},\ \lambda_{\min}\right)=\frac{\left(1-\lambda_{\min}\right)R\left(T_C\right)\left(e^{\left(\mu-\bar{r}\right)T-\mu\overline{T_C}}-e^{-\bar{r}\overline{T_C}}\right)}{\mu-\bar{r}}-\frac{I_{\max}\left(1-e^{-\bar{r}\overline{T_C}}\right)}{\bar{r}} \qquad \textbf{(14)}$$

When solving the maximum problem (12), the concession period is assumed to be known. This calculation uses backward induction, which is widely used in dynamic programming. In backward induction, a sequence of optimal actions is undertaken by reasoning backwards from the optimal end of the problem. This result implies an incentive constraint, and can be considered as a dynamic game between the government and the concessionaire.

Participation Constraint

The participation constraint requires that the concession period be long enough to recover the investment, pay back the loan, and obtain a reasonable return such that the concessionaire has sufficient motivation to invest. Otherwise, the concessionaire will choose to invest where the government can provide more favorable terms. In principal-agent literature (Cachon 2003), this reasonable return is referred to as the reservation profit, below which the agent will not participate in the game. The reservation profit is usually assumed to be independent of the contract negotiation process, and should include the return on investment as well as compensation associated with the project risk. Let $NPV_{\min}$ denote the reservation profit, then the NPV in equation (15) should satisfy $NPV\left(I_{\max},\ \lambda_{\min}\right)\geq NPV_{\min}$.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

The participation constraint can also be explained through the risk-adjusted rate of return. First, suppose that the government and concessionaire agree on a specific concession period T^*. Then, the concessionaire's rate of return on the concession project can be calculated from equation (14), which is the corresponding internal rate of return (IRR^*). Obviously, the IRR^* should satisfy the interests of both the government and concessionaire, and be reasonable and fair for both stakeholders. If the concession period is too long, the high return rate will satisfy the concessionaire but damage the government's interests, and vice versa. The concessionaire cannot earn a return in excess of the IRR^* because many competitive tenders may be submitted before the deal is concluded. However, the return should also not be less than the IRR^* else the concessionaire may choose to invest where the government can provide more favorable terms. This competitive internal rate of return can be measured precisely by the Capital Asset Pricing Model or CAPM (Brealey and Myers 2002), which includes the risk premium. That is, if the risk-adjusted rate of return, including the interest rate, inflation rate, and risk premium, is used as the discounted rate in equation (14), the reservation profit should be set as zero, i.e., $NPV_{\min}=0$. Therefore, the following equation is obtained when considering the participation constraint and risk-adjusted discounted rate.

$$\frac{\left(1-\lambda_{\min}\right)R\left(T_C\right)\left(e^{\left(\mu-\bar{r}\right)T-\mu\overline{T_C}}-e^{-\bar{r}\overline{T_C}}\right)}{\mu-\bar{r}}-\frac{I_{\max}\left(1-e^{-\bar{r}\overline{T_C}}\right)}{\bar{r}}\geq NPV_{\min}=0 \qquad \textbf{(15)}$$

Here, $\bar{r}$ is used as the discounted rate, including the risk premium, besides the interest rate and inflation rate. For simplicity, the discount rate on the highway project will not be estimated, instead it will directly set the discount rate $r=15\%$ in the case study below. In fact, there is a large volume of academic literature on how to estimate the discount rate, especially in emerging markets (Estrada 2007). The discount rate used in appraising the highway project in China can be assumed to be an independent subject under continuous study.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Calculation of the Optimal Concession Period

The inequality in equation (15) is further taken into consideration in order to obtain the concession period. At the optimal point, the inequality constraint should be binding. That is, the inequality holds with equality, as is the norm in an optimization problem. Solving the simple equality gives the value of the concession period under the multiple constraints as follows:

$$T = \frac{1}{\mu - \bar{r}} \left(\mu \overline{T_C} + \ln \left(\frac{I_{\max} \left(1 - e^{-\bar{r}\overline{T_C}} \right) \left(\mu - \bar{r} \right)}{\bar{r} \left(1 - \lambda_{\min} \right) R\left(T_C \right)} + e^{-\bar{r}\overline{T_C}} \right) \right) \qquad \textbf{(16)}$$

In this solution, the parameters involved in the operating period include the drift parameter μ, initial tool revenue parameter $R(T_C)$, and the minimum maintenance cost parameter $\lambda_{\min}$. The parameters related to the construction cost and duration include the expected duration $\overline{T_C}$ and the maximum investment rate $I_{\max}$. The discounted rate is the general parameter.

A hypothetical BOT highway project is used to demonstrate the application of the proposed methodology. In this case, the data of the Guang-Shen highway is used. The data is ~~intentionally~~ simplified for the purpose of demonstration. The expected static total cost $E\left[C\left(T_C \right) \right] = I_{\max} \overline{T_C} = 5.823$ billion RMBs, the expected duration $\overline{T_C} = 72$ months, and its standard deviation is set at 5 months. The maximum investment rate is then $I_{\max} = 80.88$ million RMBs. The standard deviation of the monthly cost increment is set at $\sigma_C = 8$ million RMBs. As previously described, the initial toll revenue $R\left(T_C \right) = 42$ million RMBs, $\lambda_{\min} = 10\%$, $\mu = 0.0109$, and the monthly discounted rate $r = 0.0125$, therefore the annual discounted rate equals 15%. Substituting these parameters into equation (16) creates:

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

$$T=\frac{1}{0.0109-0.0125}\left(0.0109\times 72+\ln\left(\frac{80.88\left(1-e^{-0.0125\times 72}\right)\left(0.0109-0.0125\right)}{0.0125\left(1-10\%\right)\times 42}+e^{-0.0125\times 72}\right)\right) \quad \textbf{(17)}$$

The computed result of equation (17) is 391 months, or about 32.5 years. This concession period incorporates the government and the sponsor's interests by considering the two constraints with the goal of continuously improving efficiency, cost effectiveness, and service quality.

Numerical Simulation

When calculating the concession period in the previous section, the risks associated with the toll revenue and the construction cost are not considered. In contemplating a proper concession period, however, the assessment of the toll revenue and construction cost risks is important for both the concerned government and the concessionaire. In order to assess the risks and incorporate the effects of these risks into the model, the degree of risk has to be taken into account using the parameters σ_R and σ_C, which respectively represent the risks associated with the toll revenue and construction cost.

The degree of risk can be solved on both the toll revenue and construction cost. Here, the risk of toll revenue can be solved using two methods. The first method is to directly use the estimated Geometric Brownian motion $dR_t/R_t = 0.0109dt + 0.03974dw_t$ given the initial toll revenue $R(T_C) = 42$ million RMBs. The second method is to calculate the confidence interval of the toll revenue, within which the toll revenue fluctuates at a certain probability. The analytical solution of the variance of the toll revenue, $R(t)$, and its distribution can be obtained as shown in equation (5). The optimistic decision-maker may use $\overline{R}(t)+\sqrt{Var\left[R(t)\right]}$, while the pessimistic decision-maker may use $\overline{R}(t)-\sqrt{Var\left[R(t)\right]}$, as previously discussed.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

On the other hand, the risk of construction cost is more complicated because the distribution of construction cost cannot be directly obtained. The Monte Carlo simulation can replace the analytical solution to allow for a stochastic approach in estimating the optimal concession period. Fig. 3 shows the flow chart for the Monte Carlo simulation and determines the probabilistic distribution of a concession period. This simulation was done based on the established optimal concession period model under incentive and participation constraints.

The flow chart describes the six steps of the Monte Carlo simulation used in this study. First, the simulation steps starts with establishing an equation of total construction cost. By substituting the investment rate, $I(t)=I_{\max}$, into equation (7), and then integrating both sides from the initial time to the completed time T_C, the controlled diffusion process has the following analytical solution:

$$C(T_C)=I_{\max}T_C+\sigma_C\sqrt{T_C}\varepsilon \qquad \textbf{(18)}$$

where $I_{\max}=80.88$ million RMBs and $\sigma_C=8$ million RMBs. Here, ε is the random variable.

Second, the cumulative distributions of the parameters in equation (18) were determined to be used as simulation input. The distribution is obtained using real data of the Guang-Shen highway in the previous section in this study. T_C is normally distributed with the mean 72 months and the standard deviation 5 months, and ε is the standard normal distribution. Third, random numbers were generated to determine the T_C and ε using the cumulative distribution functions. Fourth, the iteration of this numerical simulation generated an empirical cumulative distribution function for total construction cost, $C(T_C)$. The cumulative distribution can be used to show the probability with which the total construction cost might be beyond some set value. Fifth, the output value of the investment rate was then substituted for the equation (16) to generate the individual concession period. Finally, this process has been iterated for 10,000 times to generate an output of empirical normal distribution of concession periods.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Fig. 4 shows the distribution of concession periods that was obtained using the Monte Carlo simulation. In the figure, the 10,000 output values of the concession period are depicted as a histogram. The concession period spans from 292.25 months to 522.99 months with a mean value of 391.68 months. The confidence interval with the probability of 90% ranges from 345 months to 441 months. The *p*-value of the Jarque-Bera statistic, which is often used to test the normality (Hipel and McLeod 1994), is approximately 17.15% and rejects the null hypothesis at any traditionally significant level, such as 1%, 5%, or 10%.

Overall, the output of the Monte Carlo simulation demonstrated that the standard deviation of the concession period is relatively small at 29.51 months. The coefficient of variation is, therefore, 0.07, which shows that the output is mainly scattered around the mean value of 391.68 months. Since only the construction costs are considered in the simulation, the deviation mainly comes from the uncertainty of the construction cost. Even if the uncertainty on toll revenue is also considered, the standard deviation of the concession period should be under control because the variance of natural logarithm of toll revenue is determined by $\sigma_R \sqrt{(t-T_C)}\varepsilon_t$ in equation (3), where $\sigma_R = 0.03974$ and the variance of ε_t is one.

As Bain and Polakovic (2005) and Bain (2009) discussed, the bias error is usually large when forecasting the traffic volume or toll revenue, even within a short time period. This bias error can make the variances of toll revenue and concession period infinite, and can also lead to unreliable and incredible risk assessment. However, the model diagnosis in this study shows that the model is fitted very well and demonstrates its statistical validity in forecasting the optimal concession period. Based on the reliable distribution, therefore, the concerned government and the concessionaire can effectively negotiate with each other on the concession period according to their own degree of risk tolerance.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013; posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Conclusion

The concession period directly affects the interests of both the concerned government and the concessionaire. Determination of the concession period should be made in a way that satisfies both stakeholders. In this paper, a stochastic, rather than deterministic, approach is presented to model the uncertainties of the toll revenue and construction costs in BOT projects. The stochastic processes of the toll revenue and construction costs were converted into equivalent discrete forms to calibrate the parameters in the process models. Based on the two stochastic processes, the NPV was calculated to show the value of the cash flow in the operating period. This paper then goes a step further to model the determination of the optimal concession period by considering two kinds of constraints, namely, incentive and participation constraints. These constraints were used to integrate the stakeholders' interests in estimating the concession period. A simplified example of a highway project was used to demonstrate the capability of the proposed methodology and its compatibility with the uncertainties of estimating the toll revenue and construction costs.

This paper provides a number of benefits. First, the use of actual data from a BOT project helped to generate the distribution of variables in the model development and to understand the uncertainties inherent in a BOT project. Second, the uncertainties regarding the toll revenue and construction costs are embedded in the concession period determination model. Third, the stochastic process, in relation to the toll revenue and construction costs, can be used to simulate the toll revenue and construction costs according to the special characteristics of a BOT project. The optimal concession period can then be determined by using the simulated parameters, and the concession period risk can be assessed. Fourth, the presented method has the ability to balance interests between the government and the concessionaire. The contractual parties can, therefore, take advantage of this model by using the probability distribution when negotiating a concession period.

In the actual application of the stochastic revenue and cost process, the estimation of the model parameters requires rigorous work. The estimation reliability depends heavily on historical data and the operating information of similar existing projects. Without this information, it is very difficult to determine parameters in the stochastic model. In addition, the discount rate, including the risk premium, plays a crucial role in appraising a BOT project. However, it is very difficult to estimate the discount rate precisely at a practical level in developing countries due to both the unavailability of historical data and the lack of well-functioning risk markets. Therefore, future research should try to overcome these limitations and focus on the estimation of discount rates, particularly in emerging markets.

Acknowledgement

The first author appreciates financial support from the China Scholarship Council Grant #2007U23067. This work was also supported by the Second Stage of Brain Korea 21 Project in Division for Ubiquitous-Applied Construction of Port Logistics Infrastructure.

REFERENCES

Bain, R. (2009). “Error and optimism bias in toll road traffic forecasts.” *Transportation,* 36(5), 469-482.

Brandao, L. E. T., and Saraiva, E. (2008). “The option value of government guarantees in infrastructure projects.” *Construction Management and Economics,* 26(11), 1171-1180.

Brealey, R. A., and Myers, S. C. (2002). *Principles of corporate finance*, 7th edition, McGraw-Hill, New York.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Cheung, E. and Chan, A. (2011). ”Evaluation Model for Assessing the Suitability of Public-Private Partnership Projects.” *J. Manage. Eng.*, 27(2), 80–89.

Dagum, E. B. (1988). *The X11ARIMA/88 seasonal adjustment method: Foundations and user's manual.* Statistics Canada.

Damnjanovic, I., Waller, S. T., Vajdic, N., and Suescún, D. R. (2010). “*Quantifying the effects of network improvement actions on the value of new and existing toll road projects*.” Technical Report, National Technical Information Service, Springfield, Virginia. 〈http://tti.tamu.edu/documents/0-5881-1.pdf〉.

Dixit, A. K., and Pindyck, R. S. (1994). *Investment under Uncertainty*, Princeton University Press, New Jersey.

Estrada, J. (2007). “Discount rates in emerging markets: four models and an application.” *J. Appl. Corp. Fin.,* 19(20), 72-77.

Flyvbjerg, B., Holm, M. K. S. and Buhl, S. L. (2006). “Inaccuracy in traffic forecasts.” *Transport Reviews,* 26(1), 1-24.

Hopewell Highway Infrastructure Ltd. (HHI) (1994-2010). “Average daily traffic data and average daily revenue data from May 2004 to March 2010.” 〈http://www.hopewellhighway.com/WebSite_en/bo/gs_adtd_1994.htm〉.

Hopewell Highway Infrastructure Ltd. (HHI) (2001-2008). *Annual and interim reports from 2003 to 2008*, Hong Kong.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Hipel, K. W., and McLeod, A. L. (1994). *Time series modeling of water resources and environmental systems,* Elsevier Science, Netherlands.

Iyer, K. and Sagheer, M. (2012). ”Optimization of Bid-Winning Potential and Capital Structure for Build-Operate-Transfer Road Projects in India.” *J. Manage. Eng.*, 28(2), 104–113.

Kumaraswamy, M. M., and Zhang, X. Q. (2001). “Government role in BOT-led infrastructure development.” *Int. J. Proj. Mgmt.*, 19(4), 195-205.

Laffont, J. J., and Martimort, D. (2001). *The theory of incentives: The principal-agent model*, Princeton University Press, Princeton, N.J.

Levy, S. M. (1996). *Build, Operate, Transfer*, John Wiley and Sons, New York, N.Y.

Øksendal, B. (2007). *Stochastic differential equations: An introduction with applications,* 6th edition, Springer, New York, N.Y.

Shen, L. Y., Li, H., and Li, Q. M. (2002). “Alternative concession model for build operate transfer contract projects.” *J. Constr. Eng. Manage.*, 128(5), 326-330.

Shen, L. Y., and Wu, Y. Z. (2005). “Risk Concession Model for Build/Operate/Transfer Contract Projects.” *J. Constr. Eng. Manage.*, 131(2), 211-220.

Ross, S. A., Westerfield, R. W., and Jaffe, J. F. (2008). *Corporate Finance*, 8th Edition, McGraw-Hill Irwin.

Tiong, R. L. K. (1990). “Comparative study of BOT projects.” *J. Manage. Eng.,* 6(1), 107-122.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Watsham, T. J., and Parramore, K. (1997). *Quantitative Methods in Finance*. International Thomson Business Press, London, U.K.

Zhang, X. (2009). "Win-Win Concession Period Determination Methodology." *J. Constr. Eng. Manage.*, 135(6), 550-558.

Zhang, X., and AbouRizk, S. M. (2006). "Determining a reasonable concession period for private sector provision of public works and services." *Can. J. Civ. Eng.*, 33, 622-631.

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

List of Figure Captions

Fig. 1. Seasonally adjusted monthly toll revenues from July 1994 to December 2010

Fig. 2. The changes of natural logarithm of seasonally adjusted monthly toll revenues

Fig. 3. Flow chart of Monte Carlo simulation

Fig. 4. Distribution of Concession Periods

Figure 1

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

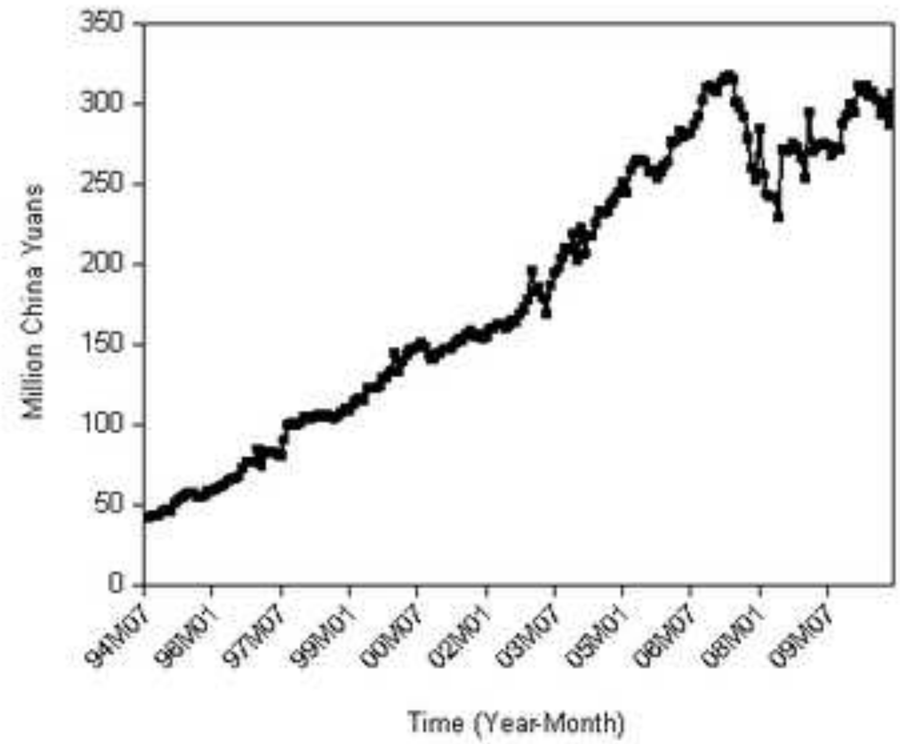

Fig. 1. Seasonally adjusted monthly toll revenues from July 1994 to December 2010

Figure 2

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

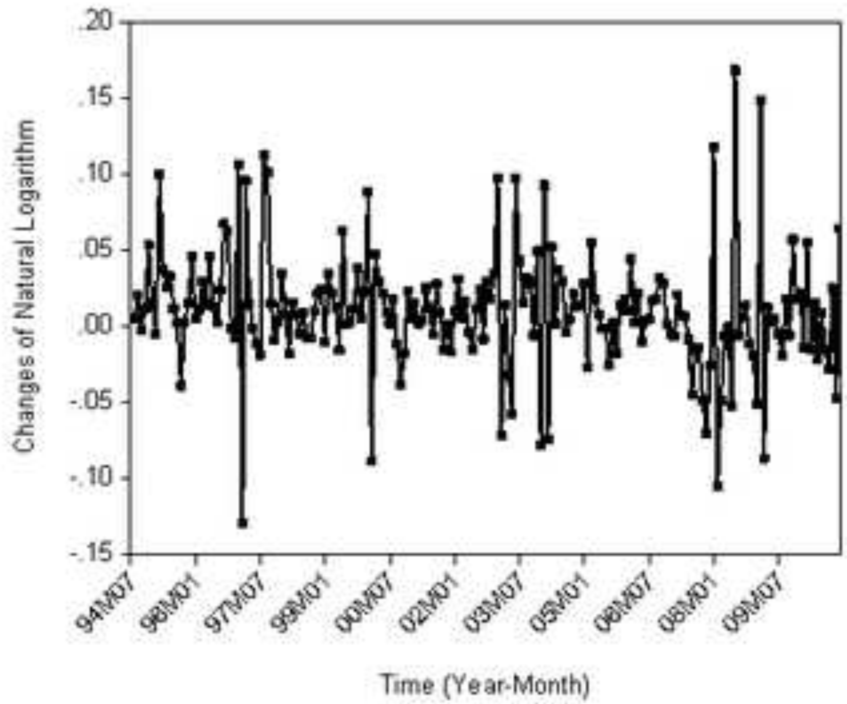

Fig. 2. The changes of natural logarithm of seasonally adjusted monthly toll revenues

Figure 3

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

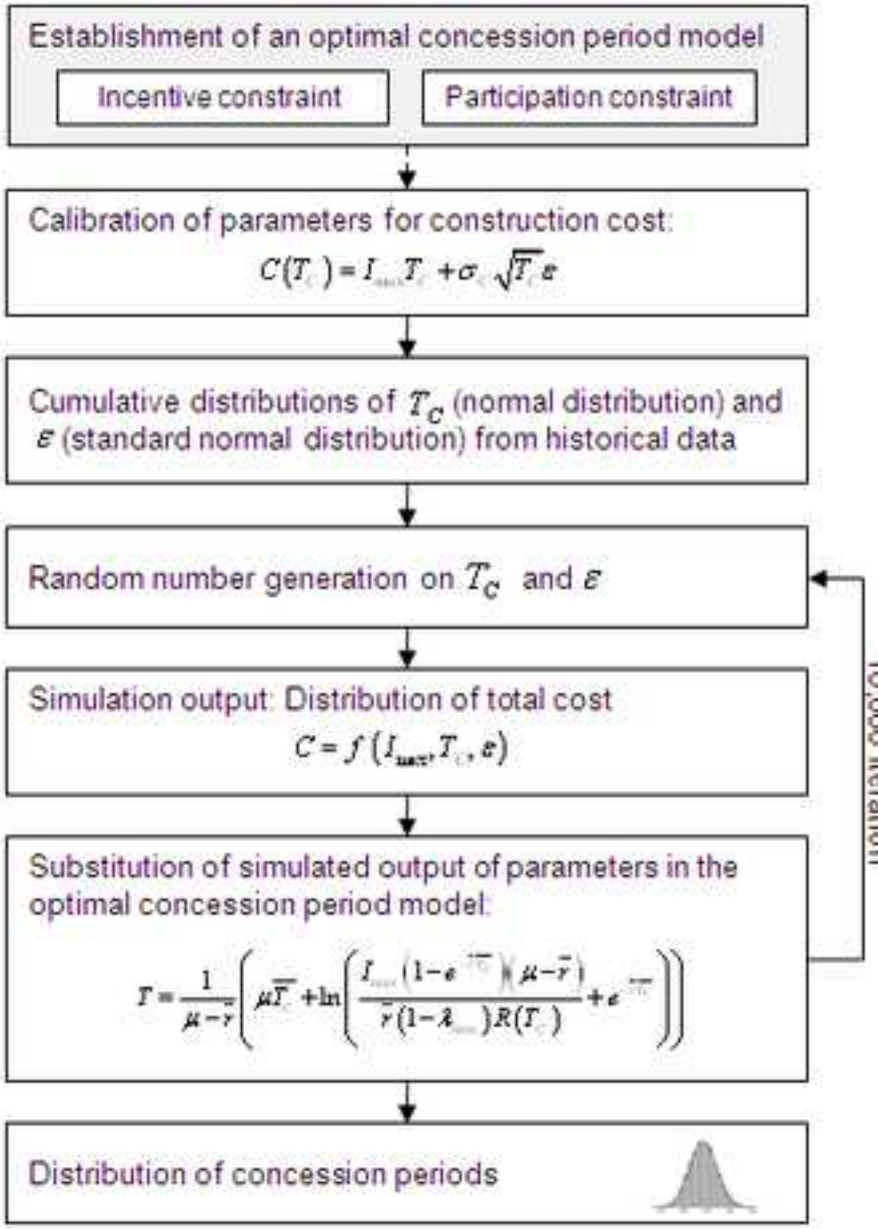

Fig. 3. Flow chart of Monte Carlo simulation

Figure 4

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

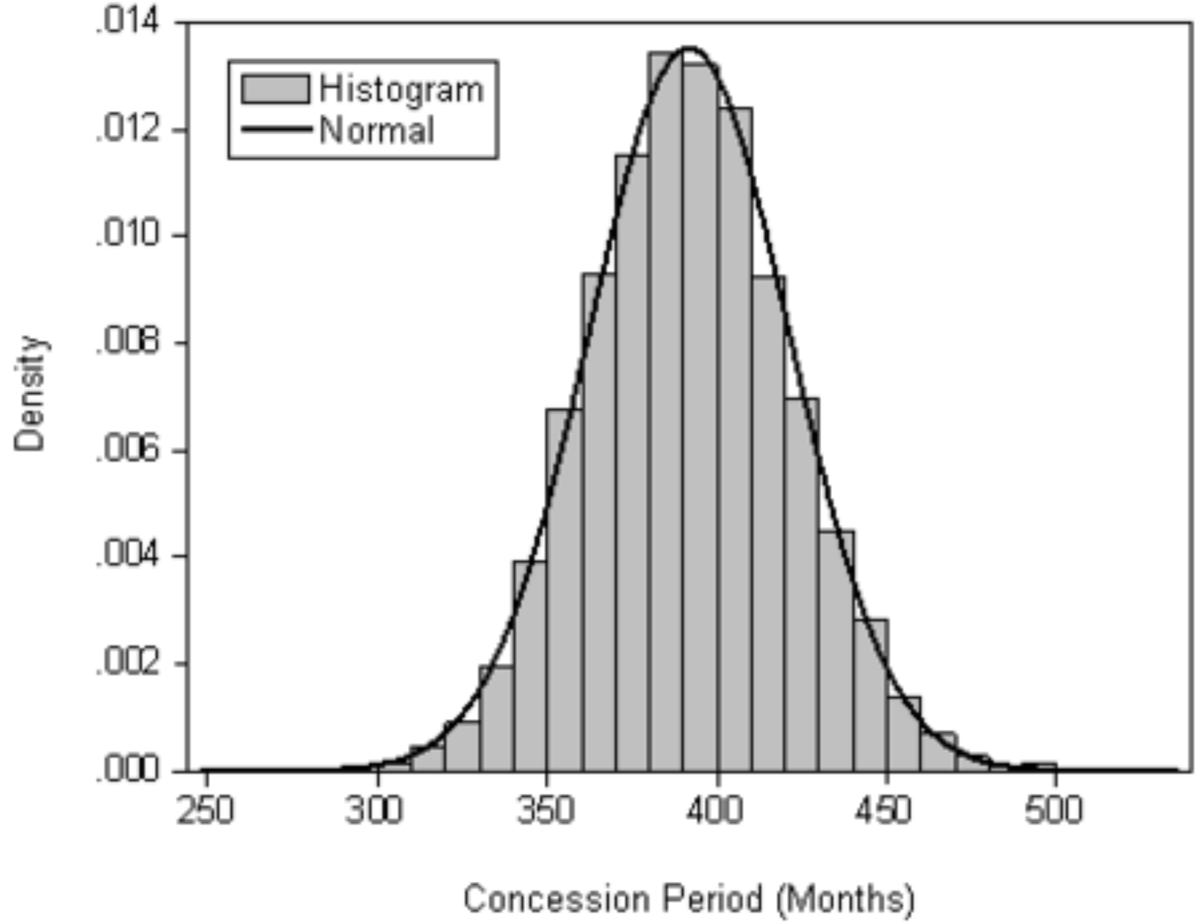

Fig. 4. Distribution of Concession Period

Journal of Management in Engineering. Submitted July 17, 2012; accepted January 23, 2013;
posted ahead of print January 25, 2013. doi:10.1061/(ASCE)ME.1943-5479.0000182

Table 1. Calculation of the maintenance cost proportional to the toll revenue

Financial Year	2001.7 - 2002.6	2002.7 - 2003.6	2003.7 - 2004.6	2004.7 - 2005.6	2005.7 - 2006.6	2006.7 - 2007.6
λ_1	0.1194	0.1299	0.1363	0.1097	0.0989	0.0922
λ_2	0.1337	0.2297	0.1695	0.1593	0.1667	0.1715