

Government Supports in Public–Private Partnership Contracts: Metro Line 4 of the São Paulo Subway System

Luiz E. Brandão¹; Carlos Bastian-Pinto²; Leonardo Lima Gomes³; and Marina Labes⁴

Abstract: In November 2005, the state government of São Paulo, Brazil, announced the intention to bid a 30-year contract to build, operate, and explore passenger services for the Metro Line 4 of the São Paulo Metropolitan Subway System. Given the high risk of the project, to attract private investors the bid documents stipulated that the government would offer risk-mitigation mechanisms such as subsidy payments and a minimum demand guarantee (MDG). Because an MDG has option-like characteristics, the real-options approach is used to analyze the effect of these incentives on the value and the risk of the Metro Line 4 concession project, and their cost and risk to the government. The results indicate that the incentives proposed are effective in reducing the risk, and increase the net value of the project by 36% at a cost to the government of 5% of the total value of the project. Additionally, it is shown that for a given cost, the most effective risk-reduction mechanisms are the ones that include a higher portion of minimum demand guarantees relative to the subsidy payment. The approach developed can assist transportation authorities in designing optimal incentive mechanisms. **DOI:** 10.1061/(ASCE)JS.1943-555X.0000095. © 2012 American Society of Civil Engineers.

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Introduction

In November 2005, the state of São Paulo, Brazil, announced the intention to bid a 30-year contract to build and operate the Metro Line 4 of the São Paulo Metropolitan Subway System, the largest in the country. The line, which was to be built in two phases at a total cost of 3.34 billion Brazilian Reals (R\$1.00 = US\$0.50), would have a length of 12.8 km with 11 stations, and would connect the downtown area to the west side of the capital, adding to the four other existing lines with 55.3 km already in operation.

A study by the government of São Paulo and the International Finance Corporation (IFC) of the World Bank in 1997 indicated that the project would not be attractive to private investors because of the large uncertainties concerning traffic demand (H. Pioner, unpublished manuscript, 2010). Infrastructure projects are subject to a wide variety of risks that should optimally be assigned to the party most capable of bearing them (Marques and Berg 2010, 2011). Although these risks can be technical, construction, operating, demand, financial force majeure, political, or environmental in nature, the demand risk is generally considered to be the most relevant for private infrastructure projects.

In 2004, Congress passed Law 11.079/2004 to regulate joint government and private-sector participation in infrastructure projects

(PPP), which for the first time allowed the government to provide project incentives such as financial subsidies, minimum demand, exchange rate, and return-of-investment guarantees and other benefits to reduce the risk and increase the attractiveness of this class of projects to the private investor. Because of this, the contract offered some unique features in the form of risk-mitigation mechanisms that would have important consequences for the valuation of the project. In the case of the concession of Metro Line 4, the government offered three types of support: a financial subsidy, a partial exchange-rate guarantee, and a staged minimum demand guarantee ranging between 10 and 40% of projected demand, ensuring a division of project risk between the concessionaire and the government of São Paulo. The bidder who agreed to the lowest subsidy payment would be declared the winner.

A minimum demand guarantee (MDG) provides the concessionaire an insurance against traffic levels that are lower than a contractually established threshold. If this limit is breached, the government will cover the difference, which reduces the risk and increases the value of the project. Because of the option like characteristics of the MDG, traditional approaches to the analysis of investment projects, such as the discounted cash-flow method (DCF) cannot be used, because the change in the risk characteristics of the project makes it impossible to know beforehand what the appropriate risk-adjusted discount rate should be. Moreover, because of passenger capacity constraints of the system, the impact of the upward and downward traffic uncertainty is asymmetric. Thus, the valuation of the project under these circumstances requires the use of option-pricing methods such as the real-options approach.

The literature on the use of real options to price government guarantees is extensive. Rose (1998) studied the use of the Melbourne Central Toll Project under the real-options approach and concluded that the embedded options accounted for over half the market value of the project. Cui et al. (2004) questioned the use of warranty contracting in highway construction by state governments in the United States and as an alternative proposed the use of

¹Pontifícia Universidade Católica do Rio de Janeiro, Brazil (corresponding author). E-mail: brandao@iag.puc-rio.br

²Universidade do Grande Rio—Unigranrio, Rio de Janeiro, Brazil. E-mail: bastian@unigranrio.br

³Pontifícia Universidade Católica do Rio de Janeiro, Brazil. E-mail: leonardolima@iag.puc-rio.br

⁴Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, Brazil. E-mail: marina.labes@gmail.com

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warranty options that give the government the right to buy a warranty only if it is deemed necessary toward the end of construction. Rus and Nombela (2003) show that traditional fixed-term minimum-toll concession-contracts are one of the causes of the frequent concession contract renegotiations observed in practice, and propose a flexible-term concession that is adjusted according to the actual observed traffic level, similar to the least present value of revenues (LPVR) method of Engel et al. (1999).

Cheah and Liu (2006) use the real-options approach to analyze the project to expand the bridge between Malaysia and Singapore, in which the government offered a guarantee if the traffic was less than anticipated and the concessionaire would return excess revenue otherwise. Chiara et al. (2007) suggest that the best guarantee model for build, operate and transfer (BOT) projects is composed of Bermudian options, which can be exercised only once at pre-determined dates, and Australian options, which can be exercised M times at N pre-determined dates where $M \geq N$, and conclude that the least squares method of Longstaff and Schwartz (2001) can be used to price these guarantees.

Bowe and Lee (2004) analyze the Taiwan High-Speed Rail Project based on information from the project and the winning consortium of government and considered the options to defer, expand, contract, abandon, and develop real-estate projects along the right of way of the concession area. The authors conclude that these managerial flexibilities add value and significantly reduce project risk. Analyzing the same project, Huang and Chou (2006) use the real-option approach to value the MDG and the option to abandon before construction commencement, and show that although both MDG and the option to abandon create value, increasing the MDG level decreases the value of the option to abandon. Brandão and Saraiva (2008) also analyze the problem of guarantees in PPP road concessions and focus on their effectiveness in reducing project risk and their cost to the government, and propose measures to limit government liability when granting these guarantees. Galera and Solino (2010) use the approach developed in Brandão and Saraiva (2008) to analyze road concession projects under revenue guarantees, and obtain similar results.

None of the studies available in the literature, however, consider a staged MDG or the impact-capacity limitation on the value and risk of the project. In this article the real-options approach is used to examine the effect of traffic guarantees on the value and the risk of a PPP project, and the cost and the risk of these guarantees to the government. This is applied to the case of the Metro Line 4 concession project. The optimal mix of incentives that minimizes the cost to the government and taxpayers is also determined.

This article is organized as follows: after this introduction and literature review, details of the Metro Line 4 project and results of the DCF valuation are presented. Next the real-options model is developed, the model parameters are determined, and the dynamic analysis (considering the risk-mitigation mechanisms and capacity issues involved) is performed. Following this, alternate incentive mechanisms are analyzed, and finally a conclusion and suggestions are provided.

Metro Line 4 of the São Paulo Subway System

The São Paulo Subway System dates back to 1927, when the Canadian-based São Paulo Tramway, Light and Power Company developed a project to build a network of urban-rail transport in the city. Since then, several projects were proposed and subsequently abandoned until 1966, when the state-owned Companhia do Metropolitano de São Paulo was formed and a plan for a 70-km-long system organized into five color-coded lines

[Blue (1), Green (2), Red (3), Yellow (4), and Lilac (5)] was created. Construction began in 1968, and in 1972, the first line was inaugurated, with additional segments added in the following years. As of 2011, the network had 65.3 km of track and 58 passenger stations carrying an average of 3.6 million passengers daily.

In 1997, the IFC of the World Bank conducted a study on the feasibility of the Metro Line 4 (Yellow Line) project under a BOT model and concluded that without the participation of private capital, significant state resources would be required to implement the project (H. Pioner, unpublished manuscript, 2010). On the other hand, the experience of private investment in infrastructure projects such as toll roads and distribution of natural gas in São Paulo had been very positive, creating a favorable environment for the adoption of this model throughout the state. In 2004 the São Paulo state government signed Law No. 11,688, allowing the Line 4 project to be implemented as a PPP, and in December 2005 an international bid for the operation and exploration of passenger transportation services for a 30-year period (2008–2038) was announced (David 2005).

The bid documents stipulated that the government would pay a financial subsidy of up to R\$120 million in 48 monthly installments, and the winning bid would be the one which demanded the lowest subsidy amount. The bid was won by MetroQuatro consortium with a tender offer of R\$75 million, whereas the only other competitor, the IntegraVias consortium, offered R\$90 million. The first public-private partnership contract in Brazil was signed in November 2006.

The Project

The state government was responsible for the civil works at an estimated cost of R\$2.31 billion, whereas a R\$1.03 billion investment in equipment, rolling stock, and operating system would borne by the concessionaire. The project would be implemented in two phases to speed up the beginning of service in the line. Phase I involved building 12.8 km of tracks, six stations, the structure of three intermediate stations, integration with other metro lines, operating systems, and the maintenance yard, which would allow the line to go into operation and begin to generate revenue for the concessionaire. The completion of the three intermediate stations, construction of two new stations and additional systems would be done in Phase II, which was to begin four years after Phase I.

The concessionaire would be reimbursed for the capital investment, maintenance, and operation of the line by collecting passenger fares of R\$2.08 per passenger, referenced to February 2005. The concessionaire would receive 100% of the fare of passengers who relied exclusively on the Metro Line 4 and 50% of the fare of passengers who also used other connecting subway lines. The fare rate would be adjusted for inflation throughout the life of the concession by the consumer price index, and was conditional on the compliance of the concessionaire to certain standards of quality of services and maintenance. The concessionaire would also have the right to collect nontariff revenues from the exploration of commercial activities and advertising in the stations, provided they did not interfere with the quality of the services. For purposes of the project, these revenues were estimated at 5% of total revenues.

The government would be responsible for the construction risks, and was required to compensate the concessionaire for any delays in the civil works that could delay the start of the operation of the line. The concessionaire, on the other hand, would be responsible for all risks relative to the implementation, operation, and maintenance of the systems under its responsibility, including delays in delivery of equipment and in securing capital for the investment. Given that a significant portion of the capital equipment would be imported, adverse variations in the exchange rate

could negatively affect the concessionaire's ability to pay its foreign currency commitments, because all the project revenues were denominated in Brazilian Reais. To mitigate this risk, the government would bear 50% of the cost of any adverse exchange-rate variations.

The traffic-demand risk is associated with the possibility that the projected demand may not occur, and traditionally is the main risk in this class of projects. The mitigation mechanism established in the bid documents stated that there would be staged demand guarantees that would start six months after the beginning of commercial operations of Phase I and would remain in effect for ten years (2008–2018). The contract stated that the concessionaire would receive compensation (M) whenever the actual demand (D_R) fell below 90% of the projected demand for the period (D_P). Likewise, if $D_R > 110\% D_P$, the concessionaire would have to hand over part of this gain to the government. The value of the compensation M would depend on the observed traffic levels, as show in Table 1.

Static Analysis

The project was initially analyzed using the traditional DCF method. The projected demand was based on information listed in Appendix IV of *International Bid Documents No. 42325.212* (David 2005) and is illustrated in Table 2, which assumes that there will be an accelerated growth in the first four years, followed by a 1.3% growth from 2012 to 2020, after which demand will remain constant until the end of the concession in 2038.

Revenue was determined taking into account passenger demand, fare, and passenger type (exclusive or integrated), quality of services and maintenance-compliance factor (estimated at 98% of revenue), and nonfare revenues (assumed to be 5% of total revenues).

Costs were estimated on the basis of the criteria defined in the bid documents, and involve direct and indirect operating costs, taxes, marketing expenditures, insurance, warranties, and legal, administrative and other costs and investment required for implementation of Phases I and II. The total investment made by the concessionaire is R\$1.027 billion, representing 31% of the total project cost. R\$2.315 billion will be invested by the state

government of São Paulo in civil works, stations, and command and control systems, as illustrated in Table 3.

To determine the weighted average cost of capital (WACC), the equity cost of capital is first adjusted to the country risk. The return of the 30 year treasury bonds of the U.S. Government at the time of the release of the bidding documents [4.63% per year (Federal Reserve Board 2005)] is then considered, less the projected U.S. inflation of 1% per year, as the real risk-free rate ($R_F = 3.63\%$). The historical risk premium of the U.S. market (λ) was estimated at 8% per year, the Brazil country risk (R_B) at 4% per year, and the unlevered beta (β_U) at 0.75, where β_U represents the sensitivity of the returns of a firm to the market returns, assuming the firm carries no debt. The value of β_U was determined from the industry Beta of North American Railroad firms (0.58) obtained from Damodaran (2005), adjusted high to reflect the higher risk of urban transport. The transformation shown in Eq. (1) was used to calculate the levered beta (β_L), considering an effective tax rate of 30%, a Debt (D)/Equity (E) relationship of 40/60, resulting in the value of $\beta_L = 1.1$

$$\beta_L = \beta_U x \left[1 + (1 - IR) x \left(\frac{D}{E} \right) \right] \quad (1)$$

Using the capital asset pricing model (CAPM) (Sharpe 1964) $K_e = R_F + \beta_L \lambda + R_B$ then produces $K_e = 16.43\%$ per year, which represents the expectation of the investor at the time of bid and decision making (December 2005). Considering a debt level of 40% and assuming a gross cost of debt of 9%, the real WACC is 12.38%.

The DCF analysis provides a net present value (NPV) of R \$151.8 million and a modified internal rate of return (MIRR) of 13.38%. Without the government subsidy of R\$75 million, the NPV decreases to R\$108.2 million. However, the static analysis does not allow for an accurate indication of the risks involved, nor does it capture the value of risk-mitigation mechanisms available for this project or the impact of the capacity limitation. This will be done in the next section.

Table 1. MDG Compensation Levels

Actual demand (D_R)	Compensation (M)	Received by
$90\%D_P < D_R \leq 110\%D_P$	$M = 0$	—
$80\%D_P < D_R \leq 90\%D_P$	$M = 0.6(0.90 \times D_P - D_R)$	Concessionaire
$60\%D_P < D_R \leq 80\%D_P$	$M = 0.06 \times D_P + 0.9(0.80 \times D_P - D_R)$	Concessionaire
$D_R < 60\%D_P$	Contract renegotiation	Concessionaire
$110\%D_P < D_R \leq 120\%D_P$	$M = 0.6(D_R - 1.1 \times D_P)$	Government
$120\%D_P < D_R \leq 140\%D_P$	$M = 0.06 \times D_P + 0.9(D_R - 1.2 \times D_P)$	Government
$D_R > 140\%D_P$	Contract renegotiation	Government

Table 2. Demand Projections ($\times 1,000$ Passengers)

Phase	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
I	308	704	730	739							
II					970	981	992	1,003	1,015	1,026	1,038
Year		2019	2020	2021	2022	2023	2024	2025	2026	2027	2028
II		1,049	1,061	1,061	1,061	1,061	1,061	1,061	1,061	1,061	1,061
Year		2029	2030	2031	2032	2033	2034	2035	2036	2037	2038
II		1,061	1,061	1,061	1,061	1,061	1,061	1,061	1,061	1,061	1,061

Table 3. Projected Investments (R\$ millions)

Party	Phase		Total	Proportion (%)
	I	II		
State government	1,845	470	2,315	69
Civil works	1,700	98	1,797	
Stations	50	207	258	
Systems	95	165	260	
Concessionaire	520	507	1,027	31
Trains	362	467	829	
Systems	158	40	198	
Total	2,365	977	3,342	100

Note: Source: Bid document No. 42325.212 (David 2005).

Dynamic Analysis

The primary uncertainty in the Metro Line 4 is related to the number of passengers that will demand service. As is standard in the literature, traffic levels are modeled as a Geometric Brownian Motion diffusion process as shown in Eq. (2) (Dixit and Pindyck 1994)

$$dS = \mu S dt + \sigma S dz \quad (2)$$

where dS = incremental variation in traffic levels in the time interval dt ; μ = instantaneous traffic growth rate; σ = volatility; and $dz = \epsilon \sqrt{dt}$ = standard Wiener process.

The discretization of the stochastic traffic that is necessary for the simulation model is represented by Eq. (3)

$$S_{t+1} = S_t e^{(\mu - \sigma^2/2)\Delta t + \sigma \epsilon \sqrt{\Delta t}} \quad (3)$$

It is also assumed that there is uncertainty concerning the initial level of traffic, which was modeled as a triangular distribution with an expected value of 308,000 passengers and a minimum and maximum of 215,000 and 400,000, respectively. The expected rate of traffic growth (μ) over the life of the project is known, as it was used to determine the static NPV.

The usual proxy for the volatility parameter (σ) is the standard deviation of the log returns of regional gross domestic product (GDP) (Irwin 2007), because there is a strong correlation between traffic levels and economic growth, as greater demand for services and goods will increase traffic volume and loads. The GDP series for the city of São Paulo from 1999 to 2008 from Instituto

Brasileiro de Geografia e Estatística (IBGE 2011) indicates a volatility of 3.9%. On the other hand, metropolitan subway demand involves only passenger traffic, so the correlation may be less than for highways and interstate rail service, which also transport cargo loads. A historical series of subway traffic in São Paulo between 1977 and 1987 presents a significantly higher volatility of 15.74% (Piovezan 1991). For the purposes of this analysis, a volatility of 8% is assumed, and a sensitivity analysis is performed on this parameter.

The dynamic analysis of the base case with no government supports shows that there is a 34.8% probability that the NPV will be negative, which indicates that the initiative to grant government supports and risk-mitigating mechanisms is appropriate. Considering the financial subsidy of R\$75 million, which was the amount specified in the winning bid, the NPV increases to R\$151.5 million and the risk of a negative NPV reduces to 27.2%, as illustrated in Fig. 1.

Risk Mitigation

The risk-mitigation mechanisms established in the bidding documents of the Line 4 project were designed to increase the attractiveness of the project to private investors. This was achieved by means of an MDG that is equivalent to a government-backed insurance policy, which has option-like characteristics. Thus, analyzing the effect of the MDG requires option-pricing methods, such as the real-options approach.

A few simplifying assumptions were adopted. Although the bidding documents establish that risk mitigation will be done quarterly, this paper adopts an annual basis in its calculations. This assumption is conservative because it considers that any payments received by the concessionaire will occur only at the end of each year, rather than quarterly. The same reasoning applies to payments by the concessionaire to the government, but it will be shown to be significantly lower. The portion (50%) of foreign currency risk borne by the private investor is also disregarded, because the R\$/US\$ exchange rate has appreciated considerably since 2005, and the concessionaire was actually able to effect all foreign purchases at a more favorable rate than expected. The bid documents also state that if the actual demand is less than 60% or greater than 140% of projected demand, the contract terms will be renegotiated. It was assumed that an eventual renegotiation will necessarily result in an additional burden to the government, because the services cannot be interrupted and the passenger-capacity limitation ensures

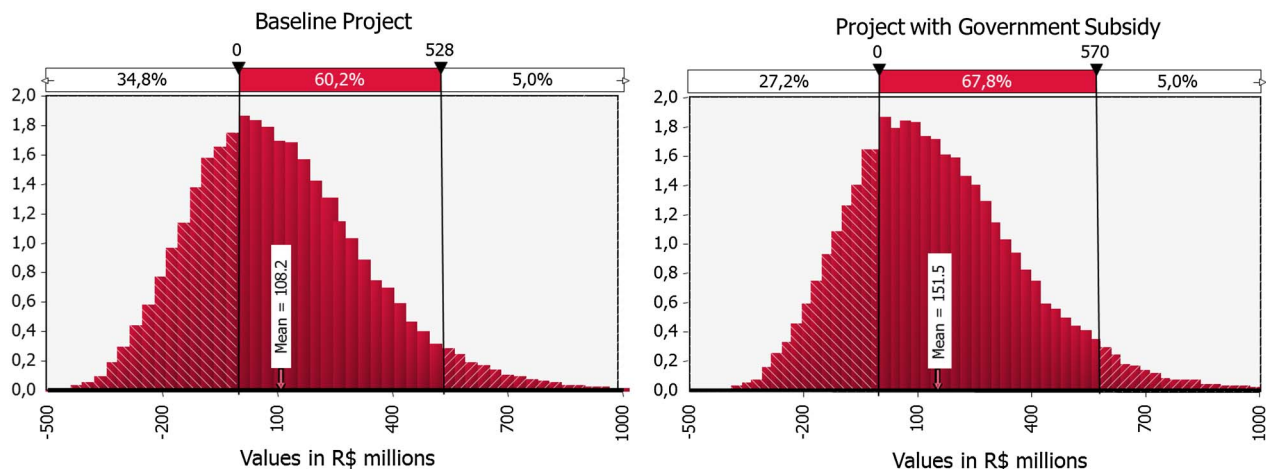


Fig. 1. Risk analysis of baseline case and financial subsidy

that levels above 140% of expected demand will seldom be achieved. This implies that the probability of favorable renegotiation to the government is almost nonexistent. Thus, this limit was ignored and the mitigation rules applied to all possible ranges of traffic demand. The risk-mitigation mechanism is shown in Fig. 2.

The risk-mitigation mechanisms were analyzed as a bundle of European options with maturities between 1 and 30 years (Charoenpornpattana et al. 2003). The MDG was modeled as a series of Put options, while concessionaire obligations to turn over excess revenues were modeled as Call options in favor of the government. The valuation process assumes that at each time $t = \tau$ the optimal decision will be made on whether to consider the actual traffic volume S_τ^A or the minimum guaranteed traffic-demand level S_τ^F , as long as the traffic levels are below the maximum threshold level S_τ^C

$$S_\tau = \min\{\max\{S_\tau^A; S_\tau^F\}; S_\tau^C\}$$

Once the optimal decision is exercised in $t = \tau$, the corresponding revenue $\pi_\tau = S_\tau \cdot \phi$, where ϕ = passenger fare, and project cash flows $f(\pi_\tau, \tau)$ can be computed. The value of the project is shown in Eq. (4)

$$NPV = \int_{\tau=0}^{30} f(\pi_\tau^*, \tau) e^{-r\tau} d\tau \quad (4)$$

where π_τ^* = project revenues under the risk-neutral measure of traffic $dS_R = (\mu - \lambda)S_R dt + \sigma S_R dz$, where λ = risk premium of the passenger traffic.

For marketed assets, the risk premium can be simply observed from market data. However, for assets for which the market is incomplete, such as traffic demand, it is necessary to use indirect methods to determine the risk premium. Hull (2006) proposes an approximation based on the market rate of return of the project. A similar but more intuitive way is to take advantage of the fact that the expected project cash flows discounted at the opportunity cost of the firm (WACC) must be identical to the risk-neutral cash flows discounted at the risk-free rate. This condition can be expressed algebraically by Eq. (5) (Freitas and Brandão 2010)

$$\int_{t=1}^n E(f[S(t)]) e^{-\mu t} dt = \int_{t=1}^n E(f[S_R(t)]) e^{-r t} dt \quad (5)$$

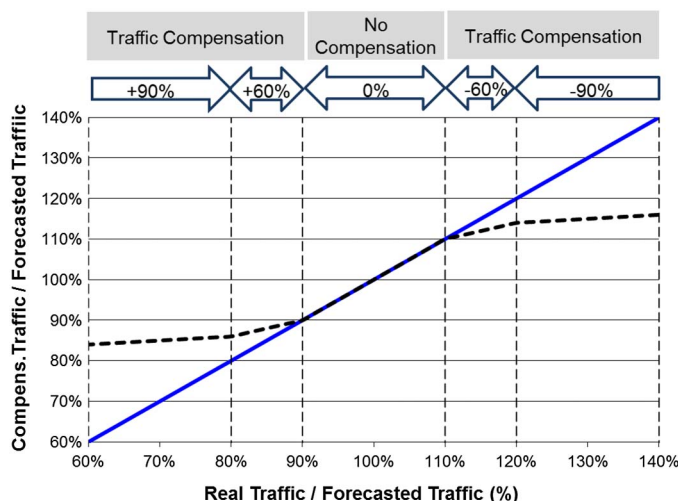


Fig. 2. MDG risk-mitigation mechanism

where $dS = \mu S dt + \sigma S dz$ is the true diffusion process of traffic demand; $dS_R = (\mu - \lambda) S_R dt + \sigma S_R dz$ is the risk-neutral diffusion process; λ = market risk premium; μ and r = risk-adjusted rate $E[f(\cdot)]$ and the risk-free rate, respectively, and are the expected project cash flows.

A value of $\lambda = 2.308\%$ is obtained, and the project was then analyzed under the risk-neutral measure with a Monte Carlo simulation, considering the financial subsidy of R\$75 million and the ten-year MDG represented by the risk-mitigation mechanisms described in Table 2 and illustrated in Fig. 2. The results indicate that the risk-mitigation mechanisms increase the project NPV by 35.6% to \$206.6 million, while at the same time reducing the risk that the project may a negative return from 27.2 to 16.3%. If, by hypothesis, the duration of the MDG is extended to 20 and 30 years, the NPV increases to R\$303.7 million and R\$402.1 million, respectively, indicating that the project is very sensitive to the duration of the MDG. This significant increase is caused by the fact that demand uncertainty increases with time, which makes the government guarantees more valuable. This indicates that the government of the state of São Paulo was prudent in limiting the duration of the MDG to only ten years. Table 4 presents the results of sensitivity to the volatility parameter considering the base case, where it can be observed that as expected, the value of MDG options increase with volatility. It can be noticed, though, that the sensitivity is low: a 100% increase in volatility increases the NPV by only 7.2%, because of the fact that the MDG has limited time duration.

On the other hand, the MDG are a contingent liability to the government, which on average is equal to the option value added to the project. Fig. 3 shows that the expected cost to the government of the MDG is R\$88.7 million, and that there is a 5% probability that this amount may exceed R\$281.8 million. On the other hand, the expected value of receipts by government because of excess traffic are only R\$12.3 million, which shows that although the mitigation mechanism is symmetric, the results are not. This occurs because as one assumes that traffic demand follows a geometric Brownian diffusion process, the probability distribution is lognormal, which is skewed relative to the average because it is bounded below but not above.

Impact of Capacity Limitation

The ability to transport passengers in Line 4 is limited by the investment in rolling stock, infrastructure, and design of the stations. It is reasonable to assume that overcapacity is unlikely to occur in the first years of operation as traffic ramps up, and that the manufacture and delivery of additional rolling stock, should it become necessary, may take several years. Thus, if the actual demand is much higher than the projected demand, the system will not be able to absorb this excess capacity. Given that the risk mechanism has a duration of only ten years, this implies that it is unlikely that the concessionaire will make significant overcapacity payments to the government. To verify this, capacity constraints were modeled as barriers 30, 20, and 10% higher than originally projected traffic levels. The introduction of these barriers affects the value

Table 4. Sensitivity to Volatility

Volatility (%)	NPV (R\$,1,000)
4	190,885
8	206,611
12	213,629
16	221,606

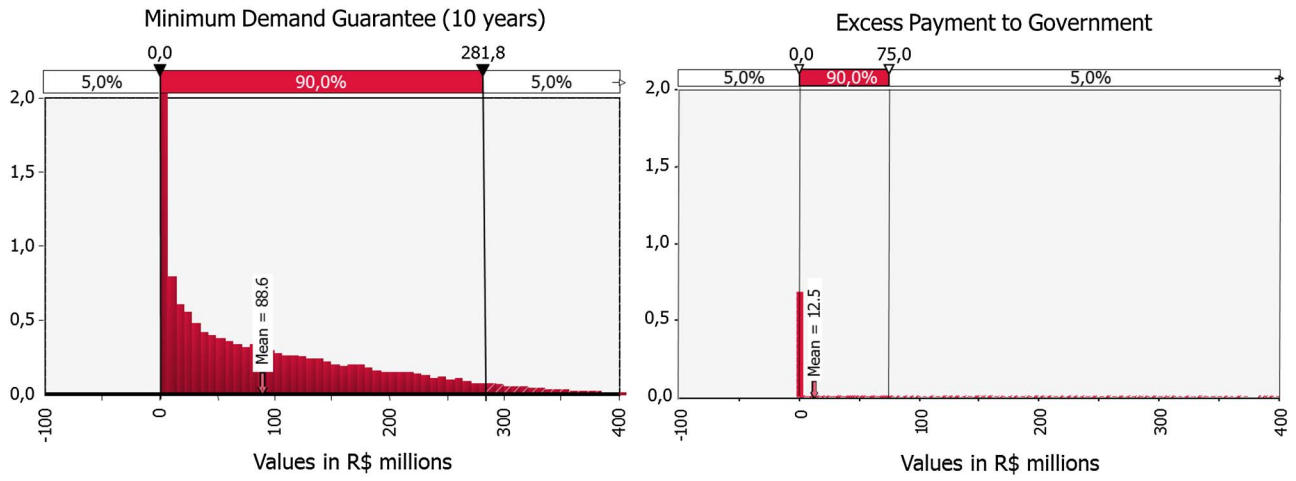


Fig. 3. Probability distribution

Table 5. Impact of Capacity Constraints

Situation	NPV (R\$1,000)	
	Without MDG	With MDG
Baseline case, no barrier	151,789	206,611
30% Barrier	147,220	203,891
20% Barrier	142,856	200,851
10% Barrier	130,171	194,377

of the project, as they have asymmetric effect on the probability distribution of the project NPV by limiting the highest values above the barrier without affecting the lower values. This causes the expected future demand to decrease relative to the previously expected values. The project NPV with and without the MDG is illustrated in Table 5.

In all cases the subsidy payment of R\$75 million was considered. These capacity constraints also negatively affect the expected

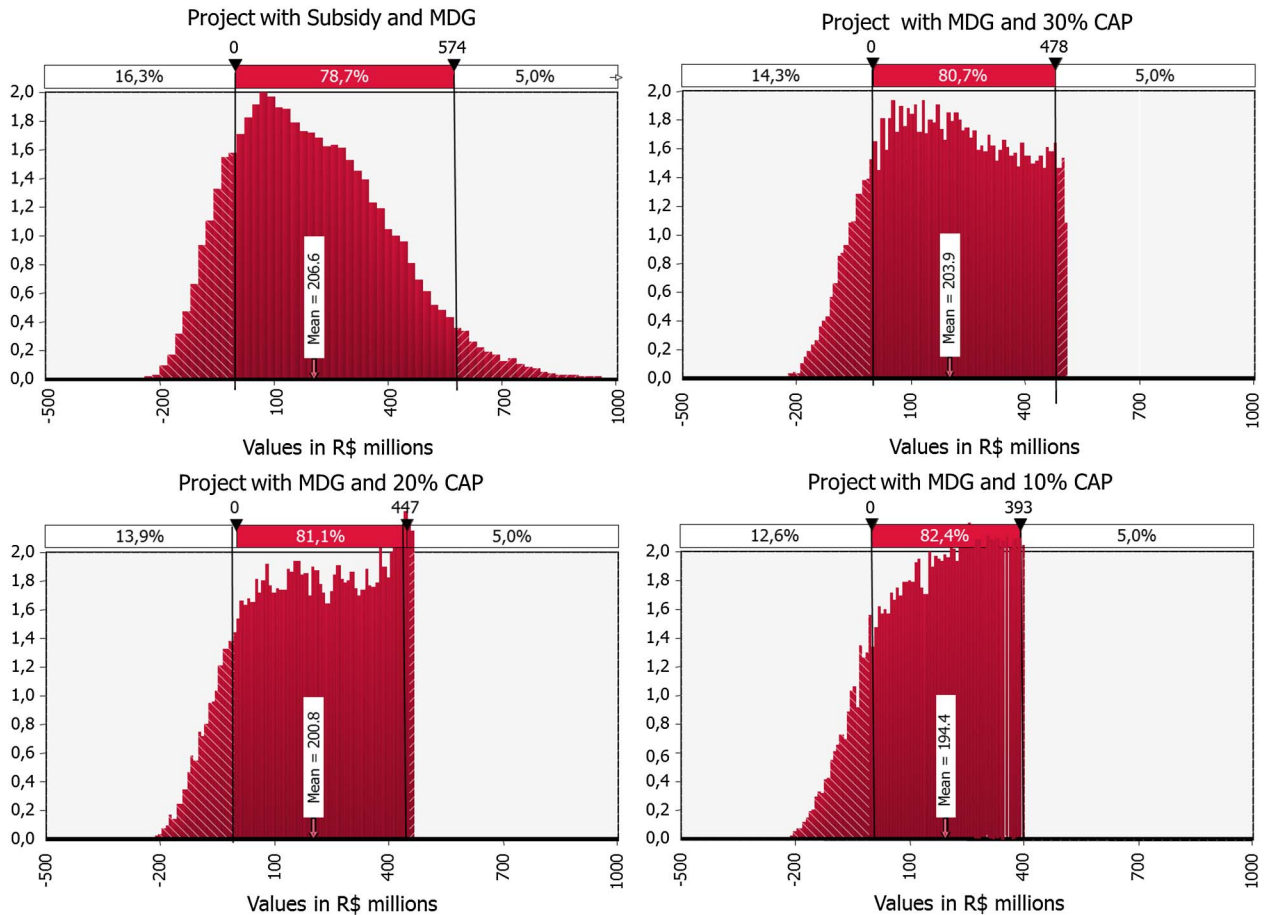


Fig. 4. Probability distribution of baseline case and with barriers

Table 6. Alternate Model Results

Model	MDG years	MDG Cost	Subsidy Cost	Total Cost	NPV	Probability NPV < 0 (%)
A	0	0	163,688	163,688	203,074	19.40
B	7	47,171	116,517	163,688	208,034	17.10
Base case	10	88,688	75,000	163,688	206,611	16.40
C	12	119,606	44,082	163,688	205,706	15.70
D	15	167,121	-3,433	163,688	205,707	14.60

values received by the government, because even if passenger demand is higher than expected it will not be absorbed by the system because of capacity constraints. Probability distributions for each case are shown in Fig. 4.

Optimal Design of Risk-Mitigating Mechanism

The risk-mitigation mechanism adopted in the bidding documents resulted in a cost to the government of R\$75 million relative to the subsidy and an MDG with an expected present value over its ten-year period of R\$88.7 million, for a total cost of R\$163.7 million. This corresponds to 5% of the total project cost or 16% of the total capital invested by the private partner. These supports reduced the risk of a negative NPV occurring from 34.8 to 16.4%.

Assuming the government is risk-neutral, it would be indifferent between paying a subsidy and granting an MDG of same value. Using the model developed in this study, other possible combinations of subsidy and MDG were analyzed to verify if there would be other, more efficient incentive schemes from the standpoint of risk reduction. For this, four different models with the same total cost were analyzed, with different combinations of subsidy payments and MDG ranging from 0 to 100%.

The cost of the MDG was calculated for periods of 7, 12 and 15 years (models B, C, and D), whereas the subsidy payment was set so that the total cost to the government remained constant at R \$163.7 million. For model D, as the guarantee amount exceeds this limit, a small negative subsidy was assumed as compensation. The value of the project for each combination of subsidy and MDG was calculated using the risk-neutral measure, whereas the probability distribution for each model was determined using the risk-adjusted rate for each case.

The results in Table 6 show that although the project NPV remains approximately constant across all models, the project risk to the concessionaire decreases as the proportion of MDG increases.

Conclusions

The value and cost of government supports for risk mitigation established in the bid documents for the construction, operation, and exploration of Metro Line 4 of the São Paulo Subway System were analyzed to determine the effect of these incentives on the value and risk of the project. One component of the risk-mitigating mechanism was an MDG, which transfers to the government the risk that future passenger traffic is less than expected. This type of guarantee has option-like characteristics whose value cannot be determined by traditional evaluation methods. The real-options approach is used to value these incentives, show how such valuation models can be constructed and how an optimal incentive mechanism can be determined.

The results indicate that the government supports proposed are effective at reducing the risk of a project that otherwise would

probably not have been undertaken by private investors. The subsidy payment of R\$75 million reduces the probability of a negative NPV from 34.8 to 27.2%, and the MDG reduces this even further to 16.3%. On the other hand, the project value almost doubles from R\$108.2 million to R\$206.6 million. The MDG has an expected cost to the government of R\$88.7 million, with a 5% probability of being greater than R\$281.8 million. The results also show that the effect of the MDG and of the capacity limitation of the line is asymmetric, which makes the probability that the government will receive payments for excess traffic very small.

The valuation model was also applied to different combinations of risk-mitigation mechanisms, while keeping the total cost of these supports constant. It is verified that while the project value remains constant, the effect on risk reduction increases with the proportion of MDG. This result suggests that for a given level of risk reduction required for the project, the lower-cost alternative for the government is to increase the MDG and reduce the subsidy amount.

The approach used in this work can be applied to other problems of valuation of PPP projects where there is a need to design optimal risk-mitigating mechanism, and determine the cost and risk of these mechanisms to the government. The use of options-pricing models can provide the public authority with the tools required to achieve a better understanding of the risks and costs involved in granting government supports when they are deemed necessary to attract private investment in infrastructure projects of interest to society.

The main limitations of this work concern the volatility parameter, because its real value can only be determined after the project implemented, and data on the actual capacity constraints of the project. Extensions of this work could include studying other forms of supports and the optimal design of incentives considering that the government is risk averse.

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