Entropy Application to Improve Construction Finance Decisions

C. M. Tang[1]; A. Y. T. Leung[2]; and K. C. Lam[3]

Abstract: In financial decision-making processes, the adopted weights of the objective functions have significant impacts on the final decision outcome. However, conventional rating and weighting methods exhibit difficulty in deriving appropriate weights for complex decision-making problems with imprecise information. Entropy is a quantitative measure of uncertainty and has been useful in exploring weights of attributes in decision making. A fuzzy and entropy-based mathematical approach is employed to solve the weighting problem of the objective functions in an overall cash-flow model. The multiproject being undertaken by a medium-size construction firm in Hong Kong was used as a real case study to demonstrate the application of entropy. Its application in multiproject cash flow situations is demonstrated. The results indicate that the overall before-tax profit was HK$ 0.11 millions lower after the introduction of appropriate weights. In addition, the best time to invest in new projects arising from positive cash flow was identified to be two working months earlier than the nonweight system.

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Introduction

Decision-making problem abounds in the real world. Financial decision-making allows contracting firms to optimize their resource allocation, plan for financial need, predict profit, and forecast cash flow liquidity at both the project and the corporate levels. Cash flow is the excess of cash revenues over cash outlays in a given period of time excluding noncash expenses. Cash-flow decision sometimes involves the optimization of conflicting multiobjective functions based on both qualitative and quantitative criteria and uncertain information. Thus the selection of weights for the objective function significantly affects the results of the analysis. Rating is an appraisal of the value of something and weighting is a coefficient assigned to elements of a frequency distribution in order to represent their relative importance. Both methods cannot cope with complex decision-making problems (Guitouni and Martel 1998; Cho 2003). It is not an easy task to determine a reliable and appropriate rating and weighting of attributes, especially when the judgments of decision makers are highly subjective or ill defined. Therefore, raters should be experts who are conscientious and objective. Besides, minimizing the bias of each rater by combining the response of several raters achieves better results. Apart from fuzzy approaches, there are some good methods, like the eigenvector method of Saaty and Hu (1998) and the entropy method of Shannon (1948), to determine weights of the objective functions. Entropy is a quantitative measure of uncertainty and is useful in exploring direct, objective, and internal weights of attributes in decision making. Chan et al. (1999) used fuzzy and entropy methods to reflect the importance structure of the needs of different customers. The vagueness of respondents can be effectively revealed by the use of fuzzy numbers and their biases are effectively minimized by using the entropy method. The rating and weighting obtained using these techniques can only deal with competitive criteria and their applications have been limited to specifying rates and weights to single-objective problems. Hwang and Yoon (1981) noted the deficiency of the entropy method in a multiobjective environment; the method needs a decision matrix as part of its input, which is an impractical requirement in real life.

Financial modeling is necessary to provide information for use by potential investors and the financial management of the company. It is an abstract description of the real world (Rubenstein 1975) and an effort to simplify a world of complexity. Kenley (2003) first had an extensive overview of the financial control models and techniques. Together with the literatures conducted, the nature, strength, and limitation of previous cash-flow modeling were explored as follows. Nazem (1968) first proposed a net cash-flow model that was easy to use based on historic data. Because of the problems in deriving such an average, there were no follow-up actions. Kennedy et al. (1970) and Peer (1982) then used a project schedule to predict cash-flow profiles directly without any model evaluation. Various standard curve models had been proposed (McCaffer 1979; Kenley and Wilson 1986; Kaka and Price 1993). They were normally accepted based on historic data but showed inadequacy in estimating sample sizes of 30 or more. The well-known S curves (Ashley and Teicholz 1977) improved the insufficiencies of the standard curve by providing ac-

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curate and quicker cash-flow forecast in the pre-tender stage but their precisions and flexibilities needed refinement and computerization for long-term use. A net cash-flow model based on cost rather than value was then developed by Kaka and Price (1991), considered variables or constraints, such as inflation and retention, that were ignored previously. There was no effective way in the study of variables that was the main subject domain for the sketch of the cash-flow profiles.

Boussabaine and Elhag (1999) applied fuzzy techniques in outlining the cash-flow profile. However, it showed weakness when fitting curves where extensive information was needed. Lam and Runeson (1999) proposed an overall cash-flow model that optimized three factors including overall profit, risks, and qualitative factors in a fuzzy environment with no provision in determining the weightings in the objective functions. Optimization methods had been improved by using recurrent-neural network (Boussabaine and Kaka 1998; Lam et al. 2001a); integrating the fuzzy reasoning technique and fuzzy optimization method (Lam et al. 2001c); and using adaptive genetic algorithm approach (Lam et al. 2001b). They could not solve the problem of weightings and handle nonlinearity in the objective functions. Financial decisions are thus ill-defined and the optimization of multiproject cash-flow problems involving fuzziness and imprecision have not been modeled and solved mathematically. Besides, the selection of weights for the objective function significantly affects the result and there is an absence of the rating and weighting approach to deal with this.

Based on a previous study, building contractors commonly lack mathematical models or computerized decision support systems for controlling their projects. Less than 30% of respondents (building contractors) claimed that their firms used computer packages or financial modeling tools for managing their projects as they faced difficulties in dealing with the imprecise and dynamic information in the industry (Lam et al. 2002). It led to a growing number of bankruptcies or insolvency in the construction industry after the financial crisis in 1997. The research attempted to provide a solution to the problem. A methodology combining fuzzy and entropy methods is developed to solve the weighting problem of the objective functions in a relatively complex decision-making process of financial modeling with imprecise information. This provides an accurate, objective, reliable, and reasonable basis, that is, a weighting system herein, for decision-making process. The work has the following main objectives: (1) to develop a fuzzy and entropy-based rating and weighting model for deriving weights in a financial decision-making problem with imprecise information and (2) to demonstrate and evaluate the model using a case study.

Conventional Rating and Weighting Methods

Many methods for multiobjective problems need information about the relative importance or weight of each objective. Eckenrode (1965) proposed six techniques for obtaining the opinions of decision makers regarding the relative value of criteria. The methods listed are too simple and inadequate for dealing with large-scale decision-making problems. Zanakis et al. (1998) investigated the performance of eight weighting methods and showed that "elimination et choice translating reality" was not beneficial to the ordinal decision-making problem involving vagueness of information, despite being the most common method of multiattribute decision making. Hajkowicz et al. (2000) applied five weighting techniques to the criteria and showed that ordinal ranking had a high preference due to its ease of use. Poyhonen and Hamalainen (2001) established a novel interactive internet experiment to study the convergence issue of five multiattribute weighting methods for a relatively large-scale decision-making problem. However, different methods have been shown to produce different weights and rates for the same problem. This leads to the question of choosing a suitable rating and weighting method for a specific decision-making problem.

Weber and Borcherding (1993) focused on various behavioral influences or variables including descriptions of attributes, e.g., range and splitting, the effect of hierarchy, and the weight elicitation procedure to study their effects on weighting decisions. Chen and Kane (2001) used two different weightings developed from the same magnitude estimation technique and suggested that the choice of weights by decision makers was indeed the most significant aspect, regardless of the method used. In the wider literature, the following two issues remain unsolved: (1) The conventional rating and weighting methods have deficiencies in decision-making problems, especially with the existence of imprecise information and (2) the rating and weighting judgment of the decision maker has a significant impact on the final outcome.

Fuzzy sets were first proposed in the 1920s (Rescher 1969) and then a fuzzy reasoning technique, which was able to represent a level of truth values, was developed. Zadeh (1965) then put further work into this possibility theory to create a formal mathematical representation of fuzzy terms called fuzzy logic. The employment of triangular fuzzy numbers (Laarhoven and Pedryce 1983; Lai and Hwang 1994) provides a well-established mechanism for imprecision. Being mainly used for fuzzy numbers, fuzzy arithmetic is treated as a direct application of extension theory (Zadeh 1975). The fuzzy arithmetic technique of Chen and Hwang (1992) was well enough to determine the ratings of criteria in the multicriteria decision aid (MCDA) methods. Another kind of information measure called "entropy." Its concept is used for quantitative measurement of the uncertainty that presents in every probability distribution and has been adopted in a variety of disciplines. Elimination of uncertainty is a way to provide information. Shannon (1948) was the first one to build a measure of uncertainty for a probability distribution $p=(p1,p2,\ldots,pn)$ and found the key measure as shown as

$$H(p_1,p_2,\ldots,p_n) = -\phi_k \sum_{i=1}^{k} p_i \ln(p_i) \tag{1}$$

where H=entropy; $\phi k=1/\ln(k)$ is a positive constant which guarantees $0 \leqslant H(p_1,p_2,\ldots,p_n) \leqslant 1$; i=constant from 1 to k; and k=number of scales. In fact, this measure of information was given by Shannon and Weaver (1947). The rules for measuring this uncertainty followed the rules by Kapur (1989). A decision matrix was a must, enabling the use of the entropy method in a multiobjective environment. Besides, maximum entropy can always be achieved if all the respondents have a similar bias, either toward least or most, on the significance of the criteria.

The Proposed Rating and Weighting Model

As mentioned, the rating and weighting method of Chan et al. (1999) can only deal with competitive criteria and their applications have been limited to specifying rates and weights to single-objective problems. The method is not comprehensive and needs

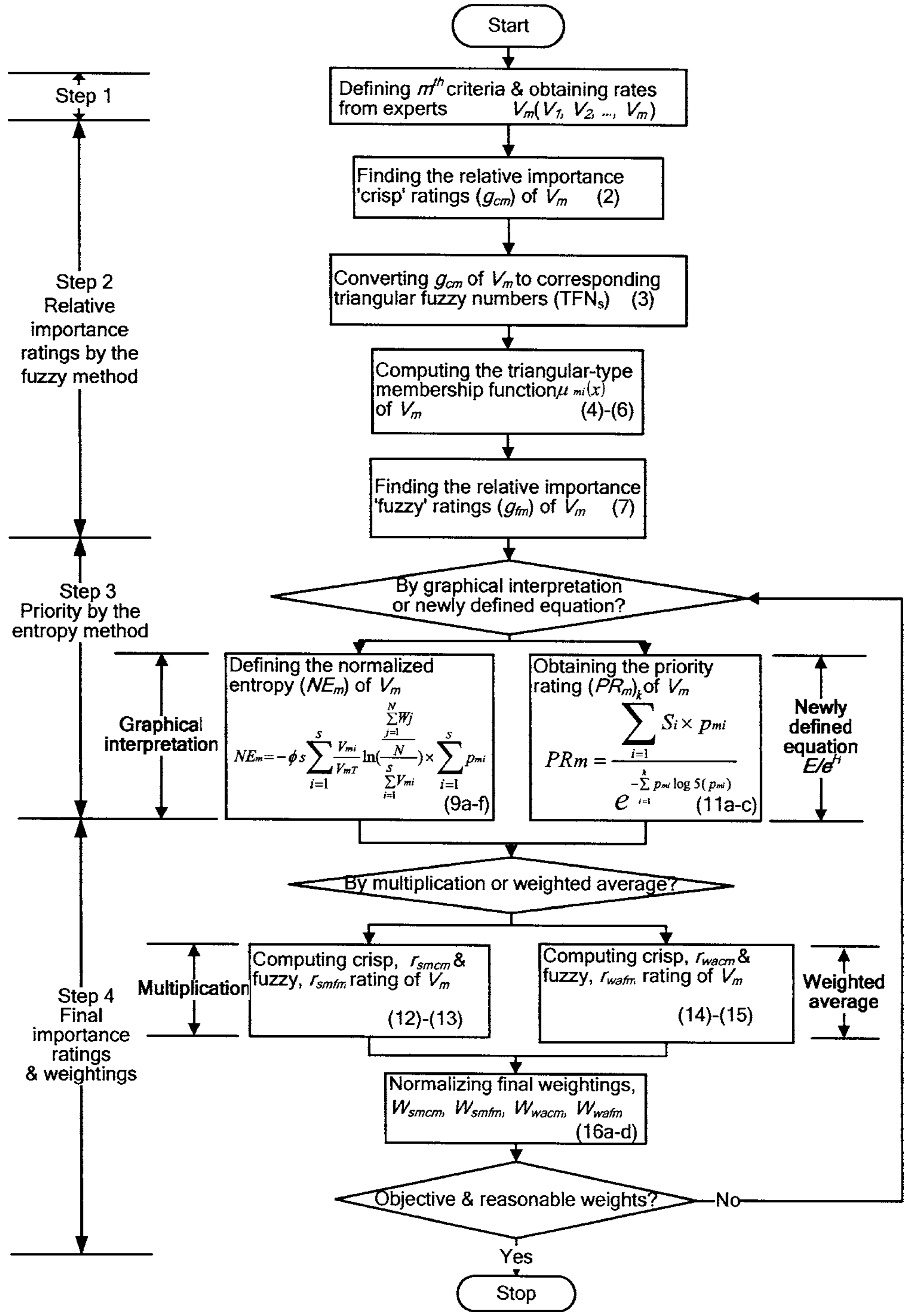

Fig. 1. Procedures for developing the weights of multiobjective functions

some modifications. The proposed rating and weighting model is fuzzy and entropy based. Fig. 1 shows the four main steps of the model to determine the rating and weighting of coefficients in the objective functions.

Relative Importance Rating Using the Fuzzy Method

The mathematical algorithm of the fuzzy method is shown in Eqs. (1)–(6) in Fig. 1, where g_{cm}="crisp" rating of V_m; i=constant from 1 to N_t; N_t=total number of respondents; W_{cmi}=rating provided by the respondent; M_{mi}=triangular fuzzy numbers (TFNs) of V_m; W_{f1mi}, W_{f2mi}, and W_{f3mi}=special fuzzy numbers of V_m; $\mu_{mi}(x)$=membership value that V_m is assigned a rating of x; and g_{fm}="fuzzy" rating of V_m.

First, find the relative importance crisp ratings of V_m by averaging the ratings provided by the respondents

$$g_{cm} = \sum_{i=1}^{N_t} W_{cmi}/N_t \qquad (2)$$

For example, suppose there are 26 respondents providing opinions on 17 criteria based on a five-point scale, thus, for $m=1$,

$$g_{c1}=\sum_{i=1}^{26} W_{c1i}/26=\frac{2+4+\cdots+4+4}{26}=\frac{95}{26}=3.6538$$

Second, convert the crisp rating on the relative importance of V_m to corresponding TFNs

$$M_{mi}=\text{“approximately } W_{cmi}\text{”}=\text{“approximately } W_{f2mi}\text{”}$$
$$=(W_{f1mi},W_{f2mi},W_{f3mi}) \tag{3}$$

For $W_{cmi}=W_{f2mi}$, $W_{f1mi}=W_{f2mi-1}$, and $W_{f3mi}=W_{f2mi+1}$; if W_{cmi}=minimum value of the scale, then $W_{f1mi}=W_{f2mi}$; if W_{cmi}=maximum value of the scale, then $W_{f3mi}=W_{f2mi}$.

Hence, there are totally 26 TFNs and are classified into five types as a five-point scale is used. For $m=1$ and $i=1$, M_{11}=approximately “W_{c11},” for $W_{c11}=W_{f211}$; $W_{f111}=W_{f211}-1$; $W_{f311}=W_{f211}+1$, M_{11}=approximately “W_{f211}”$=(W_{f111},W_{f211},W_{f311})$=approximately “2”=(1,2,3).

Third, compute the triangular-type membership function of V_m,

$$\mu_{mi}(x)=\begin{cases}0, & x\leqslant W_{f1mi}\ or\ x\geqslant W_{f3mi} \quad (4)\\ \dfrac{(x-W_{f1mi})}{(W_{f2mi}-W_{f1mi})}, & W_{f1mi}\leqslant x\leqslant W_{f2mi} \quad (5)\\ \dfrac{(W_{f3mi}-x)}{(W_{f3mi}-W_{f2mi})}, & W_{f2mi}\leqslant x\leqslant W_{f3mi} \quad (6)\end{cases}$$

The 26 triangular-type membership functions of V_1. For example, for $m=1$ and $i=3$

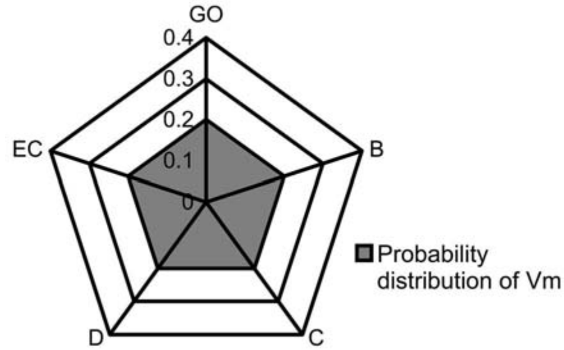

Fig. 2. Probability distribution between different respondents of V_m (GO=government officials; B=banker; C=contractors; D=developers; and EC=engineering consultants)

$$\mu_{13}(x)=\begin{cases}0, & x\leqslant W_{f113} \text{ or } x\geqslant W_{f313}\\ \dfrac{(x-W_{f113})}{(W_{f213}-W_{f113})}, & W_{f113}\leqslant x\leqslant W_{f213}\\ \dfrac{(W_{f313}-x)}{(W_{f313}-W_{f213})}, & W_{f213}\leqslant x\leqslant W_{f3131}\end{cases}$$

So

$$\mu_{13}(x)=\begin{cases}0, & x\leqslant 2 \text{ or } x\geqslant 4\\ x-2, & 2\leqslant x\leqslant 3\\ 4-x, & 3\leqslant x\leqslant 4\end{cases}$$

For $x=2.0$, $\mu_{13}(x)=\mu_{13}(2.0)=0.0$; for $x=2.5$, $\mu_{13}(x)=\mu_{13}(2.5)=0.5$; for $x=3.0$, $\mu_{13}(x)=\mu_{13}(3.0)=1.0$; for $x=3.5$, $\mu_{13}(x)=\mu_{13}(3.5)=0.5$; for $x=4.0$, $\mu_{13}(x)=\mu_{13}(4.0)=0.0$.

Finally, find the relative importance fuzzy ratings of V_m by averaging the ratings of TFNs

$$g_{fm}=\sum_{i=1}^{N_t}(W_{f1mi},W_{f2mi},W_{f3mi})/N_t \tag{7}$$

$$g_{f1}=\frac{\sum_{i=1}^{26_t}(W_{f11i},W_{f21i},W_{f31i})}{26}=\frac{\sum_{i=1}^{26}((W_{f111},W_{f211},W_{f311})+(W_{f112},W_{f212},W_{f312})+\cdots(W_{f11i},W_{f21i},W_{f31i}))}{26}$$

For V_1

$$=[(1,2,3)+(3,4,5)+\cdots+(3,4,5)+(3,4,5)]/26$$
$$=(70,93,119)/26=(2.6923,3.5769,4.5769)$$

Priority Rating Using the Entropy Method

For the Competitive Criteria

An objective entropy method was used to prioritize the mth criteria, V_m, in the mth row of the comparison matrix $[V]$, through graphical interpretation

$$\begin{array}{c} \mathbf{V_1} \\ \vdots \\ \mathbf{V}_m \end{array}\begin{bmatrix}\mathbf{p}_{11} & \mathbf{p}_{12} & \cdots & \mathbf{p}_{1k}\\ \vdots & \vdots & \cdots & \vdots\\ \mathbf{p}_{m1} & \mathbf{p}_{m2} & \cdots & \mathbf{p}_{mk}\end{bmatrix} \tag{8}$$

where ϕ_s=positive constant, $1/\ln(s)$; i=constant from 1 to s; s=number of categories of respondents; V_{mi}=average ratings for a category of respondents; V_{mT}=total score of V_m; j=constant from 1 to N; N=number of respondents of a category, s; W_j=rating provided by the respondent in a particular category; and p_{mi}=probability distribution of V_m The entropy value actually represents the probability distribution of the significance of the criteria being studied. The method is used when the criterion is competitive. For example, in Fig. 2, there is no bias shown over V_m between five kinds of construction respondents. They will have $1/5=0.2$ rating on a criterion and a perfect trapezium will result in maximum entropy.

First, for $i=1$ to s, average the ratings provided by the respondents

$$V_{mi}=\sum_{j=1}^{N} W_j/N \tag{9a}$$

Thus, for $i=1-5$ and V_1, for GO, $N=4$, $V_{11}=(2+4+3+4)/4=3.25$; for B, $N=4$, $V_{12}=(1+3+3+4)/4=2.75$; for C, $N=5$, $V_{13}=(4+4+5+4+4)/5=4.20$; for D, $N=9$, $V_{14}=(2+4+4+4+4+3+4+4+4)/9=3.67$; for EC, $N=4$, $V_{15}=(4+5+4+4)/4=4.25$.

Second, compute the total score of V_m

$$V_{mT}=\sum_{i=1}^{s} V_{mi} \tag{9b}$$

For V_1, $V_{1T}=\Sigma_{i=1}^{5}V_{1i}$, so, $V_{1T}=V_{11}+V_{12}+V_{13}+V_{14}+V_{15}=3.25+2.75+4.20+3.67+4.25=18.12$.

Third, obtain the "probability distribution" of V_m

$$p_{mi}=V_{mi}/V_{mT} \tag{9c}$$

For V_1, $p_{1i}=V_{1i}/V_{1T}$, so, $p_{11}=V_{11}/V_{1T}=3.25/18.12=0.1794$; $p_{12}=V_{12}/V_{1T}=2.75/18.12=0.1518$; $p_{13}=V_{13}/V_{1T}=4.20/18.12=0.2318$; $p_{14}=V_{14}/V_{1T}=3.67/18.12=0.2025$; $p_{15}=V_{15}/V_{1T}=4.25/18.12=0.2345$.

Fourth, compute the normalization factor (p_{mT})

$$p_{mT}=\sum_{i=1}^{s} p_{mi} \tag{9d}$$

Therefore, $p_{1T}=\Sigma_{i=1}^{s}p_{1i}=p_{11}+p_{12}+p_{13}+p_{14}+p_{15}=0.1794+0.1518+\cdots+0.2345=1.00$. Fifth, define the entropy of V_m

$$E_m=-\varphi_s\sum_{i=1}^{s} p_{mi}\ln(p_{mi}) \quad \text{or} \quad -\phi_s\sum_{i=1}^{s}\frac{V_{mi}}{V_{mT}}\ln\left(\frac{V_{mi}}{V_{mT}}\right) \tag{9e}$$

Therefore, for V_1

$$\begin{aligned}E(V_1)&=-\phi 5\sum_{i=1}^{5} p_{1i}\ln(p_{1i})=-[0.1794\ln(0.1794)\\&\quad+0.1518\ln(0.1518)+0.2318\ln(0.2318)\\&\quad+0.2025\ln(0.2025)+0.2345\ln(0.2345)]/\ln(5)\\&=-(-1.5968)/1.6094=0.9921\end{aligned}$$

Finally, multiply by p_{mT} for each entropy of V_m

$$e_m=E_m\times p_{mT} \tag{9f}$$

Therefore, for V_1, $e1=E(V_1)\times p_{1T}=0.9921\times 1.000=0.9921$.

For the Noncompetitive Criteria

The newly defined equation

$$\frac{\textit{Expected value of the five scales of significance level, } E}{e^{\textit{Entropy of those criteria, } H}} \tag{10}$$

First, the expected value of V_m is computed as

$$E_m=\sum_{i=1}^{k} S_i\times p_{mi} \quad \text{or} \quad \left(\sum_{i=1}^{k} S_i\times f_{mi}\right)\Big/ N_t \tag{11a}$$

For V_1, $E_1=\Sigma_{i=1}^{5}S_i\times p_{1i}$, for example, $S_1=1$, $p_{11}=1/26$; $S_2=2$, $p_{12}=2/26$; $S_3=3$, $p_{13}=4/26$; $S_4=4$, $p_{14}=17/26$; $S_5=5$, $p_{15}=2/26$; so, $E_1=1\times 1/26+2\times 2/26+3\times 4/26+4\times 17/26+5\times 2/26=(1+4+12+68+10)/26=3.65$

Second, apply Shannon's entropy (1984) H_m of V_m is

$$H_m=-\sum_{i=1}^{k} p_{mi}\log_5(p_{mi}) \tag{11b}$$

For V_1, $H_1=-\Sigma_{i=1}^{5}p_{1i}\log_5(p_{1i})$

$$H_1=-\left[\tfrac{1}{26}\times\log_5\left(\tfrac{1}{26}\right)+\tfrac{2}{26}\times\log_5\left(\tfrac{2}{26}\right)+\tfrac{4}{26}\times\log_5\left(\tfrac{4}{26}\right)+\tfrac{17}{26}\times\log_5\left(\tfrac{17}{26}\right)+\tfrac{2}{26}\times\log_5\left(\tfrac{2}{26}\right)\right]$$

So

$$\begin{aligned}&=-\left\{\frac{1}{26}\times\left(\frac{\log\frac{1}{26}}{\log 5}\right)+\frac{2}{26}\times\left(\frac{\log\frac{2}{26}}{\log 5}\right)+\frac{4}{26}\times\left(\frac{\log\frac{4}{26}}{\log 5}\right)\right.\\&\quad\left.+\frac{17}{26}\times\left(\frac{\log\frac{17}{26}}{\log 5}\right)+\frac{2}{26}\times\left(\frac{\log\frac{2}{26}}{\log 5}\right)\right\}\\&=0.6746\end{aligned}$$

Finally, the priority rating of V_m is

$$\mathrm{PR}_m=\frac{E_m}{e^{H_m}} \tag{11c}$$

For V_1, $\mathrm{PR}_1=E_1/e^{H1}=3.6538/e^{0.6746}=1.8611$, where PR_m=priority rating of V_m; H_m=entropy of V_m; E_m=expected value of V_m; i= constant from 1 to k; k=number of scales; S_i=scale of a degree of significance; and p_{mi}=probability of a scale.

Methods for Computing the Final Weightings

By combining the results of Steps 2 and 3, the final importance rating and weighting for each criterion can be obtained by multiplication (Hwang and Yoon 1981) and weighted average. The weighted ranking of the relative importance ratings, either crisp or fuzzy, and the priority ratings are the weightings adopted for each criterion in this step, where r_{smcm}=final importance rating by multiplication for crisp rating of V_m; r_{smfm}=final importance rating by multiplication for fuzzy rating of V_m; r_{wacm}=final importance rating by weighted average for crisp rating of V_m; r_{wafm}=final importance rating by weighted average for fuzzy rating of V_m; w_{cm}=weighting of the relative importance rating (crisp) of V_m; w_{fm}=weighting of the relative importance rating (fuzzy) of V_m; w_{cprm}=final of the priority rating (crisp) of V_m; w_{fprm}=weighting of the priority rating (fuzzy) of V_m; w_{smcm}=final weighting for the multiplicated crisp rating of V_m; w_{smfm}=final weighting for the multiplicated fuzzy rating of V_m; and w_{wacm}=final weighting for the weighted-averaged crisp rating of V_m;w_{wafm}=final weighting for the weighted-averaged fuzzy rating of V_m. For $m=1$ to number of variables, computing r_{smcm} of V_m by multiplication

$$r_{smcm}=g_{cm}\times\mathrm{PR}_m \tag{12}$$

computing r_{smfm} of V_m by multiplicaion

$$r_{smfm}=g_{fm}\times\mathrm{PR}_m \tag{13}$$

computing r_{wacm} of V_m by weighted average

$$r_{wacm} = w_{cm} \times g_{cm} + w_{cprm} \times \mathrm{PR}_m \quad (14)$$

computing r_{wafm} of V_m by weighted average

$$r_{wafm} = w_{fm} \times g_{fm} + w_{fprm} \times \mathrm{PR}_m \quad (15)$$

computing r_{smcm} of V_m for multiplicated crisp rating of V_m

$$w_{smcm} = \left(\frac{1}{r_{smcm}}\right) \Bigg/ \left(\sum_{i=1}^{m} \frac{1}{r_{smcm}}\right) \quad (16a)$$

For w_{smc1} of V_1

$$w_{smc1} = \frac{\dfrac{1}{r_{smc1}}}{\sum_{i=1}^{m} \dfrac{1}{r_{smcm}}} = \frac{\dfrac{1}{6.800}}{4.5902} = 0.0320$$

computing r_{smfm} of V_m for multiplicated fuzzy rating of V_m

$$w_{smfm} = \left(\frac{1}{r_{smfm}}\right) \Bigg/ \left(\sum_{i=1}^{m} \frac{1}{r_{smfm}}\right) \quad (16b)$$

For w_{smf1} of V_1

$$w_{smf1} = \frac{\dfrac{1}{r_{smf1}}}{\sum_{i=1}^{m} \dfrac{1}{r_{smfm}}} = \frac{\dfrac{1}{(5.0106, 6.6570, 8.5181)}}{(6.4033, 4.5313, 3.4188)}$$

$$= (0.0312, 0.0332, 0.0343)$$

computing w_{wacm} for weight-averaged crisp rating of V_m

$$w_{wacm} = \frac{w_{cm}\left(\dfrac{1}{g_{cm}}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{g_{cm}} + w_{cprm}\left(\dfrac{1}{\mathrm{PR}_m}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{\mathrm{PR}_m}}{\sum_{i=1}^{m} \left[w_{cm}\left(\dfrac{1}{g_{cm}}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{g_{cm}} + w_{cprm}\left(\dfrac{1}{\mathrm{PR}_m}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{\mathrm{PR}_m} \right]} \quad (16c)$$

For w_{wac1} of V_1 (total ratings were not shown in the calculations),

$$w_{wac1} = \frac{w_{c1}\left(\dfrac{1}{g_{c1}}\right) \Bigg/ \left(\sum_{i=1}^{m} \dfrac{1}{g_{cm}}\right) + w_{cpr1}\left(\dfrac{1}{\mathrm{PR}_1}\right) \Bigg/ \left(\sum_{i=1}^{m} \dfrac{1}{\mathrm{PR}_m}\right)}{\sum_{i=1}^{m} \left[w_{cm}\left(\dfrac{1}{g_{cm}}\right) \Bigg/ \left(\sum_{i=1}^{m} \dfrac{1}{g_{cm}}\right) + w_{cprm}\left(\dfrac{1}{\mathrm{PR}_m}\right) \Bigg/ \left(\sum_{i=1}^{m} \dfrac{1}{\mathrm{PR}_m}\right) \right]}$$

$$= \frac{\dfrac{12}{29}\left(\dfrac{1}{3.6538}\right) \Big/ 5.6935 + \dfrac{17}{29}\left(\dfrac{1}{1.8611}\right) \Big/ 13.1288}{0.9957}$$

$$= 0.0441$$

computing the final weighting (w_{wafm}) for weight-averaged fuzzy rating of V_m, one has

$$w_{wafm} = \frac{w_{fm}\left(\dfrac{1}{gf_m}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{g_{fm}} + w_{fprm}\left(\dfrac{1}{\mathrm{PR}_m}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{\mathrm{PR}_m}}{\sum_{i=1}^{m} \left[w_{fm}\left(\dfrac{1}{g_{fm}}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{g_{fm}} + w_{fprm}\left(\dfrac{1}{\mathrm{PR}_m}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{\mathrm{PR}_m} \right]} \quad (16d)$$

For w_{waf1} of V_1

$$w_{waf1} = \frac{w_{f1}\left(\dfrac{1}{g_{f1}}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{g_{fm}} + w_{fpr1} \times \left(\dfrac{1}{\mathrm{PR}_1}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{\mathrm{PR}_m}}{\sum_{i=1}^{m} \left[w_{fm}\left(\dfrac{1}{g_{fm}}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{g_{fm}} + w_{fprm} \times \left(\dfrac{1}{\mathrm{PR}_m}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{\mathrm{PR}_m} \right]}$$

$$= \frac{\dfrac{12}{29}\left(\dfrac{1}{2.6923, 3.5769, 4.5769}\right) \Bigg/ (7.9594, 5.6489, 4.3154) + \dfrac{17}{29}\left(\dfrac{1}{1.8611}\right) \Bigg/ \sum_{i=1}^{m} \dfrac{1}{13.1288}}{(0.4842, 1.4861, 0.4876) + 0.5068}$$

$$= (0.0399, 0.0421, 0.0430) + 0.0473$$

Case Study

The rating and weighting model was evaluated using a real case study to determine the horizontal and vertical rating and weighting of criteria in an overall cash-flow model. The overall cash-flow model was developed by Lam and Runeson (1999). The basic objectives of the model are to maximize the before-tax profit and to minimize the total risk cost for all projects. With the inclusion of the rating and weighting between projects, activities and risk criteria (having heavy rainfall, shortages of labor supply, time constraints, and delays in payment) relative to each activity, the trade-off between corporate profit and risk can be weighted. The mathematical algorithm of the objective functions for the overall scalar cash flow model is as follows:

$$\mathrm{Max}\, f_1(X) = \sum_{i=1}^{\mathfrak{I}} \sum_{j=1}^{\mathfrak{R}} \sum_{k=1}^{\aleph} w_{pijk} \Psi_{ik} \pi_{ik} X_{ijk} e^{-m} \quad \text{(before-tax profit)} \quad (17a)$$

Table 1. Details of the Case Study Projects

Project type	Building (public)	Building (public)	Building (public)	Building+civil engineering
Contract sum (HK$ million)	6.85	18.40	10.19	14.69
Contract duration (months)	9	21	15	13
Contract start to finish (month)	10–19	3–22	5–20	14–27
Profit margin (%)	17.5	20.0	18.0	25.0
Resources cost				
Labor (%)	43.5	38.0	34.5	18.5
Material (%)	42.0	45.0	43.0	38.5
Plant (%)	6.5	7.0	12.0	34.0
Management and overhead (%)	10.0	10.0	10.5	9.0
Payment term				
Certificate period	21 days	21 days	21 days	21 days
Payment period	Further 21 days	Further 21 days	Further 21 days	Further 21 days
Retention	1.5% or max. 0.1 million	1.5% or max. 0.22 million	1.5% or max. 0.158 million	1.5% or max. 0.2 million
Release retention	Release after 3-month defects liability periods (DLP)	Release after 3 month DLP	Release after 3 month DLP	Release after 3 month DLP

$$\text{Min}\, f_2(x) = f_2(X_{ijk}, R_j, SL_i, TC_{ik}, DP_{j,})$$
$$= \sum_{i=1}^{\mathfrak{I}} \sum_{j=1}^{\mathfrak{R}} \sum_{k=1}^{\aleph} w_{rijk} X_{ijk} \quad \text{(risk factor)} \qquad (17b)$$

where $f_1(X)$=present value of the sum of future before-tax profits; $f_2(X)$=risk cost; $\mathfrak{I}$=the total and last number of projects; $\mathfrak{R}$=the total and last number of working months; $\aleph$=the total and last number of activities in each project; w_{pijk}=overall weight of profit of the kth activity in the jth month in the ith project relative to the risk; Ψ_{ik}=profit margin of the kth activity in the ith project; π_{ik}=contract sum of the kth activity in the ith project; X_{ijk}=the completion percentage of the kth activity in the jth month of the ith project; e^{-m}=discount factor; R_j=rainfall of the jth month; SL_I=shortage of labour in the ith project; TC_{ik}=time constraint of the kth operation in the ith project; DP_j=payments delays in the jth month; and w_{rijk}=overall weight of risk of the kth activity in the jth month in the ith project relative to the profit.

Problem Background

A medium-size building contracting firm that had three on-going projects and a fourth project that was recently obtained, was examined to evaluate the applicability of the weightings obtained from the rating and weighting model. Table 1 shows the details of the four projects. The construction activities of the projects were broken down into five operations including substructure, superstructure, wet and dry trades, building services, and external works. A 27 month period was used as the examination period for the four projects. Payment terms were based on the measurement of work done, simply with two months delay including time for issuance of payment certificate from client to the contractor after measurement and for submission of the certificate for payment from the contractor.

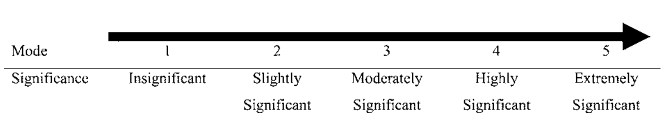

Fig. 3. Definition of mode with respect to significance

Data Collection Process

A survey was conducted between December 2001 and January 2002 to investigate the opinions of experts on various aspects of significance stated in the following. Totally, there were 26 respondents. Among them, 4 were government officials, 4 were bankers, 5 were contractors, 9 were developers, and 4 were engineering consultants. A five-point scale was employed in the questionnaire and was shown in Fig. 3 to investigate the following three aspects:

1. The degree of relative significance of Projects 1–4 as shown in Fig. 4;
2. The degree of significance of before-tax profit and risk (having heavy rainfall, shortages of labor supply, time constraints and delays in payment) arising for various projects as shown in Fig. 5; and
3. The degree of significance of various types of risk (having heavy rainfall, shortages of labor supply, time constraints and delays in payment) for each activity (substructure, superstructure, wet and dry trades, building services and external works) arising for various projects as shown in Fig. 6.

As shown in Fig. 1, a total of four kinds of weightings were determined for the previous three aspects. As the criteria are noncompetitive, the newly defined equation is adopted in this case.

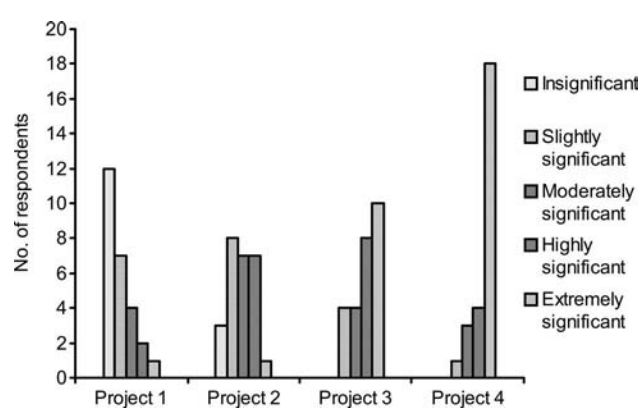

Fig. 4. Distributions of relative significance for Projects 1–4

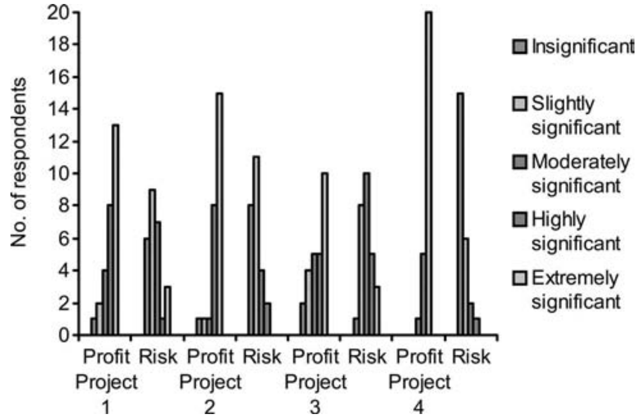

Fig. 5. Distributions of significance of profit and risk for Projects 1–4

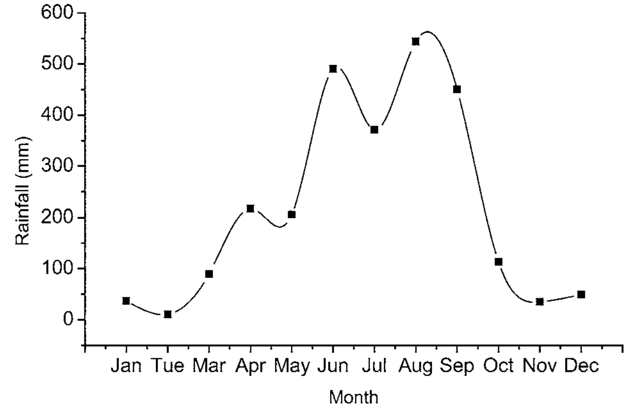

Fig. 7. Monthly rainfall diagram (data adapted from monthly weather summary, January–December 1998–2002, Royal Observatory, Hong Kong SAR 2002)

The final weighted averages of the crisp ratings which were more objective and reasonable were adopted for application in the overall cash flow model.

Fuzzification of the Ingredients

Before the fuzzy reasoning technique could be applied to calculating the weighted and non-weighted risk parameters, linguistic or fuzzy subsets (Lam and Runeson 1999) had to be assigned to the four risk factors. In the study, the risk factors were assessed monthly. For example, a measurement of rainfall in mm per month was used to quantify the magnitude of the weather each month. The norm of the rainfall was determined using historical data from the Royal Observatory, Hong Kong covering the past five years. The average monthly rainfall was plotted in Fig. 7. Similarly, the average unemployment rate, which is an indicator of labor supply, is illustrated in Fig. 8. The popular functions used for justification are either the S function or the π function that are, in fact, combinations of two S functions (Zadeh 1975), and the fuzzification of having heavy rainfall is shown in Fig. 9.

Determination of the Overall Weights

The overall weights on profit and risk for activity k in project i from January to December can be explained by the procedures shown in Fig. 10. Normalization of the criteria values is essential to the maximum operation (Hwang and Yoon 1981). Before normalization, the overall weights on profit and risk were, in turn, composed of various kinds of weights as shown in Fig. 10.

The three kinds of weights including weights on each project, weights on profit and risk in each project, and weights on each activity in each project relative to the risk were calculated from the rating and weighting model. Taking substructure of Project 1 in January as the example, the profit and risk tradeoff was calculated as follows by the final weighted averages of the crisp ratings:

1. The degree of relative significance of Project 1: 0.15;
2. The degree of significance of before-tax profit and risk arising for Project 1: 0.64 and 0.36 respectively; and
3. The degree of significance of various types of risk (having heavy rainfall, shortages of labor supply, time constraints and delays in payment) for substructure arising for project 1: 0.19, 0.25, 0.36, 0.20, respectively.

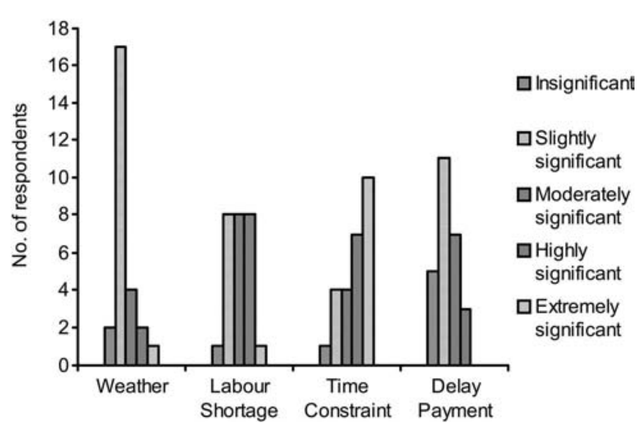

Fig. 6. Distributions of risk significance for the sub-structure of Project 1

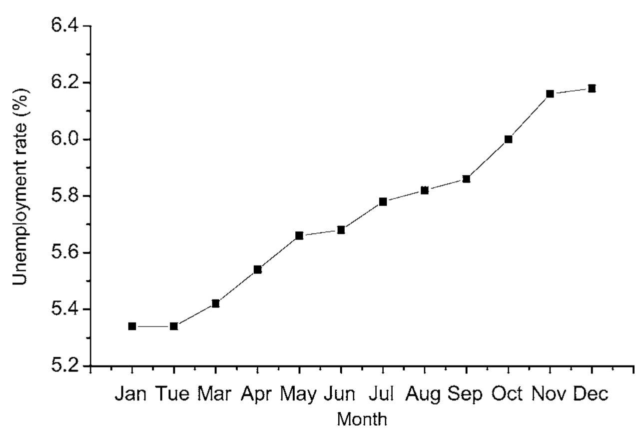

Fig. 8. Monthly unemployment rate (data adapted from Statistics on the labour force and unemployment, January–December 1998–2002, Census and Statistics Department, Hong Kong SAR 2002)

After the procedure of justification, the data was passed into the inference engine where a rule base was established. First, select the following rules and read their corresponding membership functions from the fuzzy charts:

Months	Rules	If R_j	and SL_j	and TC_{ik}	and DP_j	Then the risk is	Result	Risk factor
Jan	First	Very low rainfall $0.48{:}R_{15}$	Adequate labor supply $0.03{:}SL_{23}$	Normal schedule $0.11{:}TC_{114}$	Low payment delay $0.29{:}DP_{243}$	Low	3%	5
Jan	Second	Low rainfall $0.53{:}R_{14}$	Shortage labor supply $0.97{:}SL_{22}$	Tight schedule $0.89{:}TC_{113}$	Medium payment delay $0.72{:}DP_{242}$	Very high	18%	2

Second, calculate the weighted base rules of risk, taking the first rule as the example,

Months	Weighted rules	If R_{15}	and SL_{23}	and TC_{114}	and DP_{243}
Jan	First	$0.48\times0.19=0.09$	$0.03\times0.25=0.01$	$0.11\times0.36=0.04$	$0.29\times0.20=0.06$

Third, use popular max-min operation to identify the membership function of risk for the substructure in Project 1, $\mu=\min[0.09,0.01,0.04,0.06]=0.01$.

Repeat the previous steps for the other projects. Fourth, use the max-min operation to identify the membership function of risk for the substructure of all projects in all rules. The overall risk factor of the substructure was given by the center-of-gravity method, that is, the sum of scalar rules was divided by the sum of rules. For example, if the identified membership functions of Rules 1 and 2 is 0.063 and 0.09975, respectively, for the substructure of Project 1, then the overall risk factor was, $(0.0063\times5+0.0998\times2)/(0.0063+0.0998)=2.1782$.

Finally, compute the overall weights of risk and profit tradeoff. The weights on the activities of each project were actually calculated by normalizing the weighted membership functions of the risk in each activity.

The overall profit weight=(weight of project 1)×(weight of profit in project 1)×(weight in each rule on substructure in Project 1 relative to the risk)=0.15×0.64×[(0.23+0.15)/2]=0.0182 and the overall risk weight=(weight of Project 1)×(weight of profit in Project 1)×(risk factor)×(weight in each rule on substructure in Project 1 relative to the risk)=0.15×0.36×2.1782 ×[(0.23+0.15/2]=0.0223.

By normalization, the overall profit and risk tradeoff is: profit=0.0182/(0.0182+0.0223)=0.45; and risk=(1−0.45)=0.55. Finally, the decisions on choosing weights for new and ongoing projects were made as an input to the overall cash flow model.

Results and Analysis

Contractors can use the overall cash-flow model as a reference for cash-flow forecasting. There are plenty of weight combinations that may be applied to profit and risk, which are in turn dependent on the rainfall, labor shortages, time constraints, payment delays, activities, and contractor attitudes. Through the rating and weighting system shown in Fig. 1, the relative weights on profit and risk were determined and introduced in the overall cash flow model. The model was then solved using LINDO which is the most traditional packages for solving linear, integer, and quadratic optimization models (Lindo System Inc. 1996). The resulting differences between the weighted and nonweighted systems for multi-project cash flow optimization are then analyzed.

Overall Weights on Profit and Risk

Survey data regarding the degree of significance of the three main study aspects were input into the model to explore the rating and weighting of the objective function in the overall cash-flow model. In total, 12 sets of final importance rating and weighting were obtained. The monthly overall weights on profit and risk, determined from the crisp final weightings by the weighted average method, are shown in Table 2. In effect, these results describe a cycle showing the relative weights on profit and risk throughout a year. The introduction of weights on projects and activities showed a consistent result for each month. For Project 4, which had a high profit margin, the profit weight was relatively higher than the other three projects. All sets of weightings in Table 2 showed that the contractor was mostly risk-averse throughout the year. This was obviously due to the adverse economic situation nowadays.

During the period between November and December, the contractor had a lower risk weight. As Project 4 involved external works, a sharp drop in rainfall in these two months reduced the risk, as shown in Fig. 8. In January, March, April, May, July, and August, the tradeoff between profit and risk was nearly balanced. In July and August, the tradeoff was exactly the same. This was due to the fact that the amount of increase and decrease were nearly or exactly the same for different risk factors over the same period, i.e., the weather and the shortage of labor supply (see Figs. 7 and 8) over the six months. Similarly, in February and December and in June and September, the overall tradeoffs between profit and risk were also exactly the same because the historic data describing these two risks were the same over these two sets of month.

In the literature, the MCDA methods for handling weights required a human at the time of analyzing the decision. In construc-

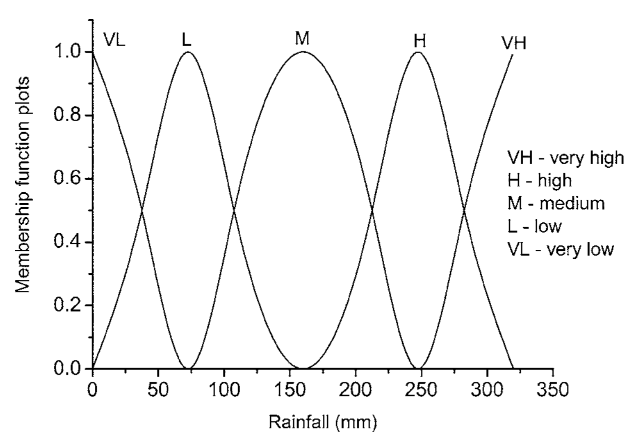

Fig. 9. Fuzzification function of "having heavy rainfall"

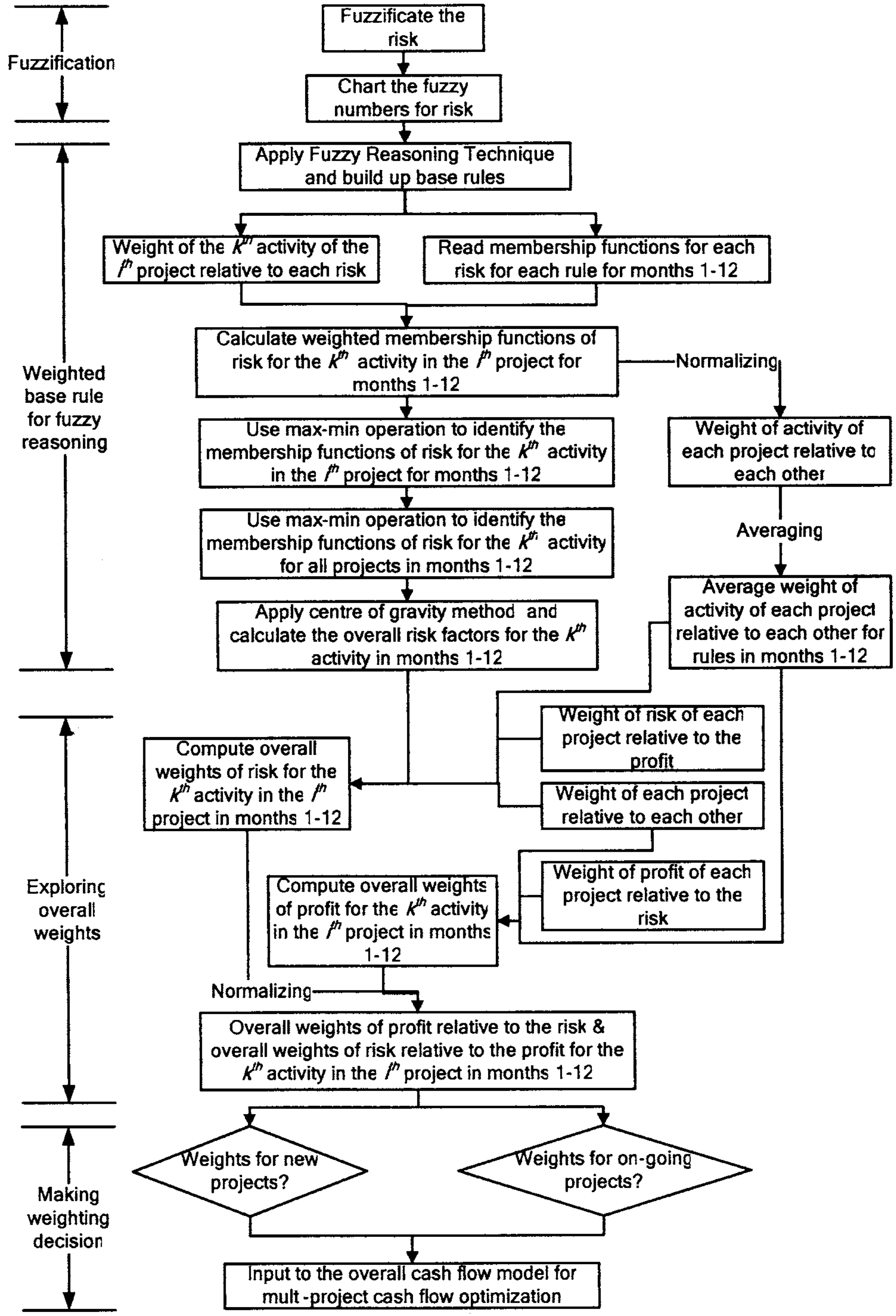

Fig. 10. Procedures for exploring the overall weights on profit and risk

tion industry, an early profit and risk tradeoff warning system could be beneficial to the survival of the contractor. Besides, the conventional MCDM ignored the complex interaction or the uncertainties of decision-makers. As shown in Figs. 4–6, the survey results could not provide the actual importance of the studied aspect and more importantly the profile from different respondents. For example, in Fig. 4, Project 4 seemed to be more significant than Project 3 but their actual weights from different respondents were 0.35 and 0.30, respectively, with 0.05 differences only. In Fig. 6, the survey results of weather and delay payment seemed to be different but their actual weights from different respondents were 0.19 and 0.20, respectively, with 0.1 differences. The weighting system could be input practically into the optimization model. In this study, the sets of weight throughout a year would be input to a multiproject cash-flow optimization model to facilitate the decision-making process by knowing the tradeoff between the profit and risk in dynamic and uncertain projects well in advance.

Implications of Using the Entropy Method

The implications of using the entropy methods are both positive and negative. For the positive sides:

1. There is imprecision or fuzziness about the possibility of a

Table 2. Overall Weights on Profit and Risk for Each Activity in Each Project from January to December

		Jan		Feb		Mar		Apr		May		Jun		Jul		Aug		Sep		Oct		Nov		Dec	
	Project	P	R	P	R	P	R	P	R	P	R	P	R	P	R	P	R	P	R	P	R	P	R	P	R
1	Substructure	0.45	0.55	0.37	0.63	0.47	0.53	0.45	0.55	0.46	0.54	0.31	0.69	0.47	0.53	0.47	0.53	0.31	0.69	0.55	0.45	0.59	0.41	0.37	0.63
	Superstructure	—	—	—	—	—	—	0.44	0.56	—	—	—	—	—	—	—	—	—	—	0.52	0.48	0.56	0.44	—	—
	Wet and dry trades	0.46	0.54	—	—	—	—	0.45	0.55	—	—	—	—	—	—	—	—	—	—	0.54	0.46	—	—	—	—
	Services	—	—	—	—	—	—	0.44	0.56	—	—	—	—	—	—	—	—	—	—	—	—	0.57	0.43	—	—
	External work	—	—	—	—	—	—	0.45	0.55	—	—	—	—	—	—	—	—	—	—	—	—	0.56	0.44	—	—
2	Substructure	0.52	0.48	0.44	0.56	0.54	0.46	0.51	0.49	0.53	0.47	0.37	0.63	0.54	0.46	0.54	0.46	0.37	0.63	0.62	0.38	0.65	0.35	0.44	0.56
	Superstructure	0.51	0.49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.59	0.41	0.62	0.38	—	—
	Wet and dry trades	0.53	0.47	—	—	—	—	0.52	0.48	—	—	—	—	—	—	—	—	—	—	0.60	0.40	0.63	0.37	—	—
	Services	—	—	—	—	—	—	0.51	0.49	—	—	—	—	—	—	—	—	—	—	0.61	0.39	0.64	0.36	—	—
	External work	—	—	—	—	—	—	0.52	0.48	—	—	—	—	—	—	—	—	—	—	0.60	0.40	0.63	0.37	—	—
3	Substructure	0.38	0.62	0.31	0.69	0.40	0.60	0.37	0.63	0.39	0.51	0.25	0.75	0.40	0.60	0.40	0.60	0.25	0.75	0.48	0.52	0.51	0.49	0.31	0.69
	Superstructure	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.45	0.55	0.48	0.52	—	—
	Wet and dry trades	0.39	0.61	—	—	—	—	0.38	0.62	—	—	—	—	—	—	—	—	—	—	0.46	0.54	0.49	0.51	—	—
	Services	—	—	—	—	—	—	0.37	0.63	—	—	—	—	—	—	—	—	—	—	0.47	0.53	0.50	0.50	—	—
	External work	—	—	—	—	—	—	0.38	0.62	—	—	—	—	—	—	—	—	—	—	0.46	0.54	0.49	0.51	—	—
4	Substructure	0.62	0.38	0.54	0.46	0.64	0.36	0.62	0.38	0.63	0.37	0.47	0.53	0.64	0.36	0.64	0.36	0.47	0.53	0.71	0.29	0.74	0.26	0.54	0.46
	Superstructure	—	—	—	—	—	—	0.61	0.39	—	—	—	—	—	—	—	—	—	—	0.69	0.31	0.72	0.28	—	—
	Wet and dry trades	0.63	0.37	—	—	—	—	0.62	0.38	—	—	—	—	—	—	—	—	—	—	0.70	0.30	—	—	—	—
	Services	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.73	0.27	—	—
	External work	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0.72	0.28	—	—

Note: P=profit; R=risk; and —=same as above.

specific outcome such as the opinion of respondents on the significance of criteria in the construction industry. This type of uncertainty in information can be minimized effectively using the fuzzy method in combination with the entropy method presented herein. This showed that the proposed rating and weighting method could be a better measure for quantification of both possibilities and probabilities by the weighting system as shown earlier.

2. The rating and weighting obtained using conventional entropy can only deal with competitive criteria and their applications have been limited to specifying rates and weights to single-objective problems. Hwang and Yoon (1981) noted the deficiency of the entropy method in a multiobjective environment; the method needs a decision matrix as part of its input, which is an impractical requirement in real life. The inclusion of the factor in the newly defined equation enables the handle of noncompetitive besides competitive criteria. This minimized bias in place of using a decision matrix, enabling the use of the entropy method in a multiobjective environment. Maximum entropy can always be achieved if all the respondents have a similar bias, either toward least or most, on the significance of the criteria. The inclusion of the expected value eliminated the usual deficiency of conventional approach.
3. The major drawback of the conventional entropy is that it depends on the relative values of u_i instead of the absolute value. Thus the various membership functions in a set will have the same entropy that equal to 1. In the newly defined equation, the drawback has been overcome by using the absolute value as the domain and assigning the relative value for the fuzzy method, with the inclusion of the weightings derived from the rating and weighting process, represents a flexible, easily understood method. Its acceptance of qualitative input makes it very suitable to study the linguistic variable content.

For the negative sides

1. The results are weighting depending and the weights used in this study are limited because they are derived from a small sample. Although a large sample of respondents is not a must for using the proposed method. A adequate amount of different types of respondents will be the main subject domain of using this method.
2. The decision matrix may not be functioning effectively for the competitive criteria if the encountered criteria are numerous.

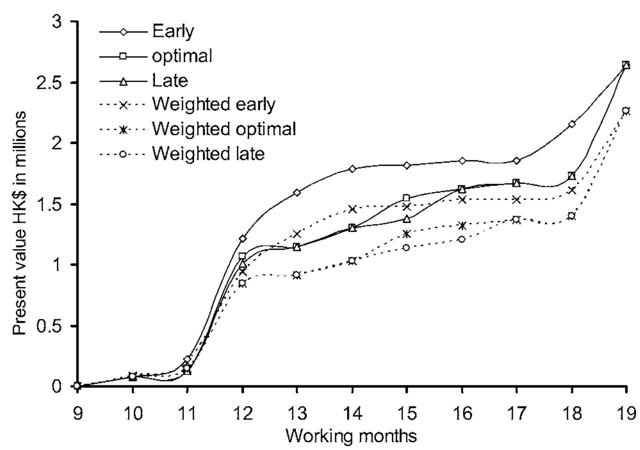

Fig. 11. Cash flow for Project 1

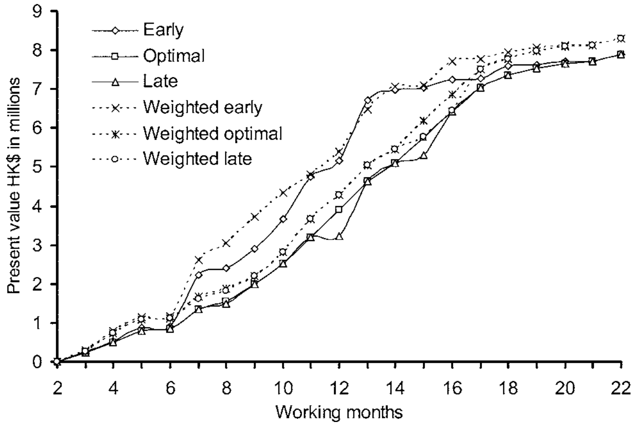

Fig. 12. Cash flow for Project 2

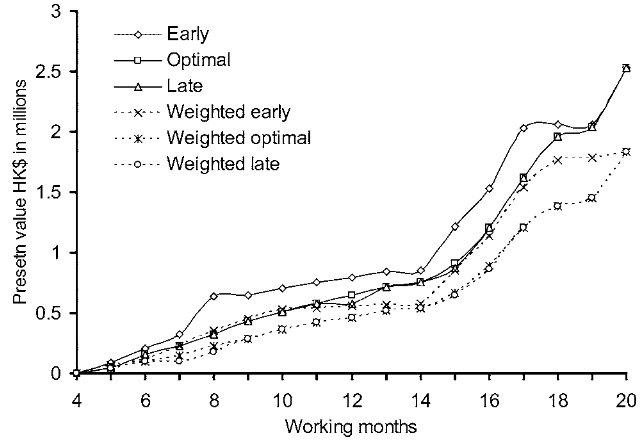

Fig. 13. Cash flow for Project 3

Cash Flow in Projects 1–4

The data describing Projects 1–4 were plotted as S curves for the analysis as shown in Figs. 11–14. For all projects, the completion dates were generally the same regardless of the introduction of the weightings. For Projects 1 and 3, the overall profiles of the weighted and nonweighted curves were almost the same, whereas for Project 4, they were exactly the same. This was due to the fact that Project 4 had the highest profit weight and the lowest risk weight and hence the first priority in terms of resource allocation.

For Project 1, the nonweighted curves were located higher than their corresponding weighted curves after Working Month 12. This also happened for Project 3 after Working Month 6. This was due to the relatively low profit weights for Project 3 throughout the year as shown in Table 2. The relatively low contract sum and profit margin for Projects 1 and 3 further delayed progress by a certain percentage. Thus, the sets of weighted curves for these two projects were shifted down. This showed that there was a lower profit than expected for these two projects.

For Projects 2 and 4, the three weighted curves were generally located at a slightly higher position than that of the nonweighted. This was due to the relatively high profit weights in these projects throughout the year as shown in Table 2. Their profit margins and contract sums were also comparatively higher than the other two projects. At the end, the overall cash flow profiles of the weighted curves were shifted upward in these two projects.

For Working Months 2–4, only Project 2 was running. Therefore, the weighted and nonweighted curves should have been overlapped with each other. However, Fig. 14 showed that the curves were separated. Since the weightings were introduced into the model at the very beginning. The results were totally different. For Working Months 7–8, only Projects 2 and 3 were running. Fig. 15 showed that the nonweighted optimal and late curves overlapped, but that the weighted curves did not. However, the profit weight of Project 3 (0.40) was not high. The weighted optimal curve should go to a late schedule instead of shifting upward. This abnormal phenomenon was due to the high profit margin and contract sum of Project 2. In order to release more resources for Project 2, Project 3 which has a lower profit weight, should have been driven toward an early schedule.

In Working Month 12 of Project 1, a difference appeared after the introduction of weightings. The nonweighted optimal curve was higher than the nonweighted late curve whereas the weighted optimal one overlapped with the nonweighted late one. The drop in the weighted optimal curve was due to the relatively high risk weight (0.63) of Project 1 on December in the cycle as shown in Table 2. In order to avoid such a high risk, the project progress should be driven to a late schedule. This also occurred for Projects 2 and 3 in the same working month, which had a relatively high risk weight (0.56 for Project 2 and 0.69 for Project 3).

In the Working Month 16 of Projects 1 and 2, an opposite phenomenon occurred. The nonweighted optimal curve overlapped the late one. However, the weighted optimal curves were higher than their corresponding late curves for these two projects. This was caused by two factors. First, the net late progress was higher than the net early progress. For example, the net late progress percentage for the superstructure (32.20) was greater than that of net early progress (14.94) in Project 1; the net late progress percentage of wet and dry trades was 20, but 18.99 for net early progress percentage in Project 2. Second, the risk weights for all activities in these projects were still high. After introducing the weightings, the progress might have been expected to shift to an early schedule to reduce the work done.

In the Working Month 18 of Project 1, the distance between the early and the late schedules in the nonweighted case was greater than that of the weighted one. The shifting down of the weighted early curve narrowed the distance. This distance actually reflects the amount of float in the project schedule. It implied that Project 1 should have been finished early so as to release resources to the other projects, which had relatively high profit margins and contract sums. For the case in Project 3, the distance increased between the weighted early curve and the weighted late one. More float days should have been provided, as the risk weight was the highest (0.75) for Project 3 in this month. This would have shifted up the weighted early curve instead. In the Working Month 19 of Project 3, the case was the same with a less vigorous up-shifting of the weighted early curve.

With the introduction of the weighting system, the total profit was totally different along with the approach to the actual value (HK$ 0.38 million less for Project 1; HK$ 0.41 million more for Project 2; HK$ 0.70 million less for Project 3, and HK$ 0.55 million more for project 4). This actually provides a more realistic picture for the cash-flow forecasting and planning of contractors.

Overall Cash Flow

The data from all projects were combined to plot an S curve as shown in Fig. 15. Basically, a compromise solution of the construction schedule in each project was achieved. Fig. 15 showed that the weighted or nonweighted optimal progress curves approached the late schedules. This meant that the contractor had

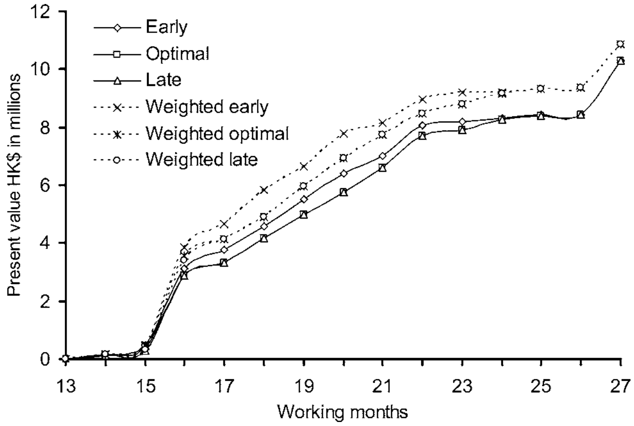

Fig. 14. Cash flow for Project 4

inadequate resources to take on a new project during the construction period. The introduction of weightings on profit and risk in the overall cash-flow model had no effect on the completion month of the multiple projects. However, the overall cash flow was totally different with weightings introduced when compared to the situation with no weighting as shown in Fig. 15. The weighted curves were generally higher than the nonweighted curves, except in Working Months 11–16. In the early stages, only Projects 2 and 3 were operating. As Project 2 had a high profit weight, it therefore became dominant in the overall cash flow. Thus, the weighted curves were pushed upward at that time. From Working Months 11–16, the weighted curves shifted down a little. This was because two relatively low-profit-weighted projects, Projects 1 and 3, were operating. After Working Month 16, the introduction of Project 4 became dominant. As Project 4 had the highest profit margin and profit weight, it caused the whole weighted profile to move upward. The weighted overall profit at the end was HK$ 0.11 million less than that of the nonweighted profit that was found to be approach to the actual value of final profit. A realistic profile allows contractors to decide the best time to invest in new projects and to optimize their resource allocation. Thus the corporate cash flow is improved, reducing the chance of insolvency or bankruptcy.

The data were developed further into a cash-flow diagram (saw-tooth diagram) as shown in Figs. 16 and 17 by consideration of the difference between the cost (value curves in Fig. 15 with the deduction in profit) and income (released retention from clients) at the start and end of each working month. The optimal, cumulative saw-tooth curves were within the region of early and late progress. Additional resources are required when the projects are driven to a higher rate of progress. This reflected the fact that no additional resources were required. The adopted project method was more suitable for high-profit-margin and high-contract-sum projects. This was because the relative cost of the adopted project could also be increased. The break-even point that showed the minimum or maximum cash requirement and the occurrence of final profit was different when weights were introduced. From the overall saw-tooth diagram in Fig. 16 (without weights) and in Fig. 17 (with weights), the break-even point occurred in Working Months 21 and 19, respectively. This meant that a positive cash flow actually occurred two working months earlier. It was also the most suitable time for starting new projects. However, the appropriate number of new projects depends on the contractor's financial position, the resources available, and the business environment, etc.

The results are weighting dependent and the weights used in this study are limited because they are derived from a small sample. But it was well enough to exercise the power of the proposed model by taking a sample of 26 different respondents for the four projects in the case study. Besides, it is also noted that the weightings obtained here are restricted to the multiple projects of a medium-sized construction company in Hong Kong. For international construction companies or other special projects, different weightings will result whether to use and more or less input risk variables and risk factor elements should be considered. The proposed system can deal with unlimited projects and variables. Moreover, modeling for future events makes the validation of the model difficult as the real world changes with time. Up-to-date information should be provided to the decision makers. Nevertheless, the merits of the rating and weighting process actually provide a method for determining the weights of multiobjective functions such that the quantitative problem in complex decision-making problems can be solved.

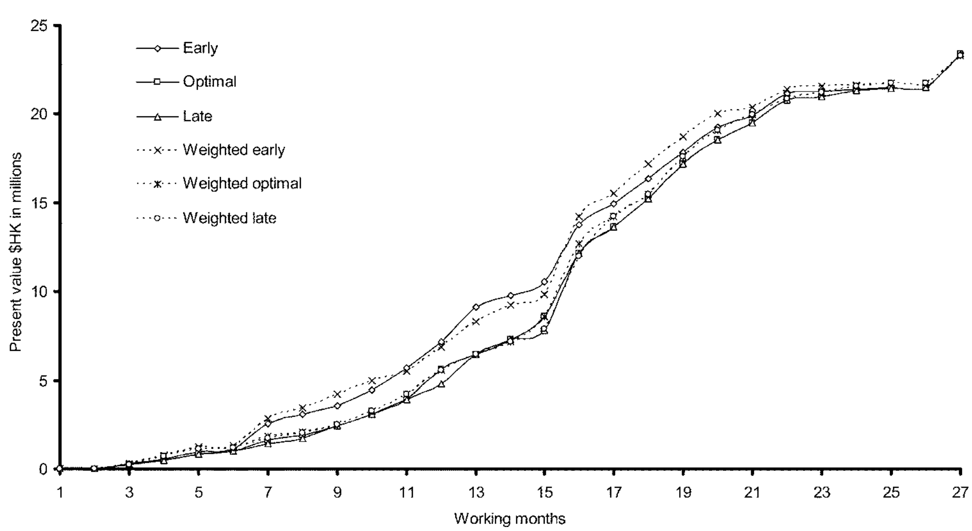

Fig. 15. Overall cash flow

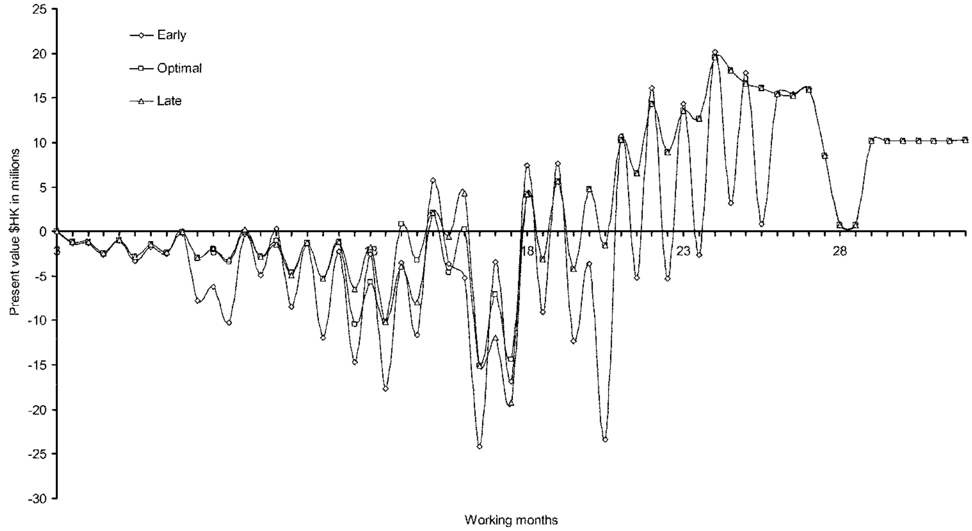

Fig. 16. Overall cash flow—saw-tooth diagram (without weights)

Conclusions

A fuzzy and entropy-based rating and weighting model has been presented with the aim of providing objective and reasonable weightings for the optimization of complex decision-making problems relating to financial management process. The proposed model was divided into four main steps for determining priorities. The use of fuzzy numbers and the entropy method to reflect the vagueness of respondents' biased assessments were sufficient. The weightings in the study were then evaluated using the overall cash-flow model in which the maximization of profit and the minimization of risk cost were the two main conflicting objectives. Those sets of weights showed that the contractor was risk averse for all projects.

Finally, a compromise solution for the construction schedule was achieved in each project. The introduction of weightings on profit and risk in the overall cash-flow model had no effect on the completion month of the multiple projects. However, the overall cash flow was totally different. With the introduction of the weighting system, the weighted overall profit at the end was HK$ 0.11 million less than that of the nonweighted alternative. Through cash-flow forecasting, the decision maker of a contracting firm can identify likely cash flow problems well in advance and may consider not accepting a project even if there is a high profit margin. The situation can be identified from the profit and risk weights cycle derived from the proposed rating and weighting model. In addition, resource allocation can be optimized and the best time to start a new project can be identified in the progress curve. A positive cash flow actually occurred two working months early when weights were introduced and thus the best time to invest in new projects was identified accurately. The application of entropy in multiproject cash-flow situations is viable and a more accurate, objective, reliable, and realistic decision thus can be provided facilitating the cash-flow management decision problem.

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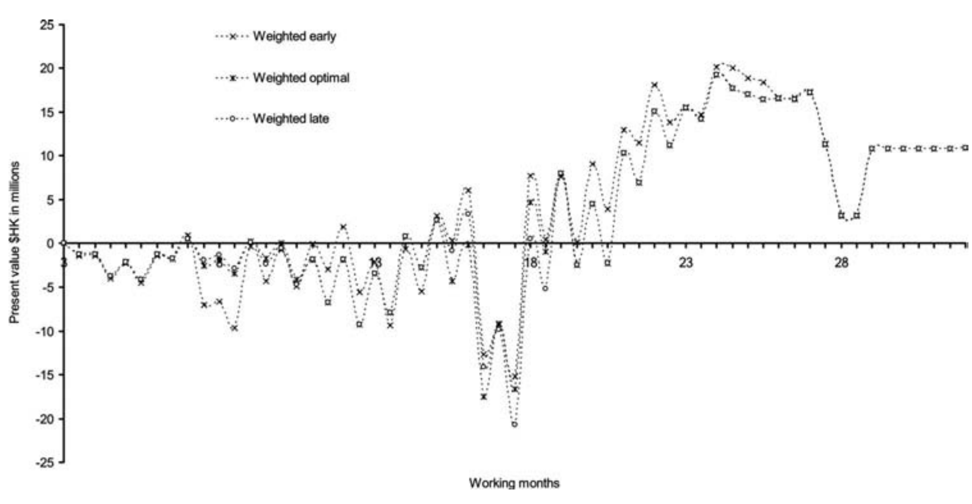

Fig. 17. Overall cash flow—saw-tooth diagram (with weights)

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