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Determining a reasonable concession period for private sector provision of public works and services

Xueqing Zhang and Simaan M. AbouRizk

Abstract: The concession period is one of the most important issues to be addressed in private sector provision of public works and services through concession arrangements as it, to some extent, demarcates the rights and responsibilities between public and private sectors in a project's life cycle, and it is also critical to the project's sustainable development. This paper proposes a methodology for the determination of an appropriate length of the concession based on a win–win principle for parties involved and exercises simulation techniques in measuring and evaluating construction and economic uncertainties and risks. A case study of a hypothetical infrastructure project is provided to demonstrate the application of the proposed methodology, mathematical model, and simulation techniques.

Key words: build–own–transfer, concession, critical path method, financial management, infrastructure, Monte Carlo simulation, partnerships, procurement, risk analysis.

Résumé : Lorsque des contrats de concession sont utilisés, la période de concession est l'un des enjeux les plus importants à être abordés lors de la fourniture de travaux publics et de services par le secteur privé puisque cette période délimite les droits et les responsabilités entre les secteurs privé et public durant le cycle de vie d'un projet; cela est également important pour le développement durable du projet. Le présent article propose une méthode pour déterminer une durée de concession appropriée en se basant sur le principe gagnant–gagnant pour les parties impliquées et présente des techniques de simulation pour mesurer et raisonner les incertitudes et les risques économiques et de construction. Une étude de cas pour un projet d'infrastructure hypothétique est fournie afin de démontrer l'application de la méthode proposée, du modèle mathématique et des techniques de simulation.

Mots clés : construction–exploitation–transfert, concession, méthode du chemin critique, gestion financière, infrastructure, méthode de Monte Carlo, partenariats, approvisionnement, analyse de risque.

[Traduit par la Rédaction]

Introduction

Governments around the world have, in general, shown interest in private sector finance and provision of public works and services. Since 1985, more than 1370 infrastructure projects with estimated capital costs of over US$575 billion have been developed or are proposed to be developed with private financing in more than 100 countries (Reinhardt Communications Corporation 2000; Ye and Tiong 2003). Build–Operate–Transfer (BOT)-type contractual models have been popular in both developed (National Council for Public-Private Partnerships 2002) and developing countries (International Finance Corporation 1999).

Under a BOT scheme, a project is developed through a concession agreement between a public authority and a private company (the concessionaire), in which the public authority grants the concessionaire the rights to build and operate the project for a certain period (the concession period). The concessionaire pays back the loan (principal and interest), recovers its investment with an expected level of profit through revenues from the project within the concession period, and at the end of the concession agreement transfers the project, which should be in operational condition, to the public authority, usually at no cost (Merna and Smith 1996). The concession agreement also generally specifies the payment structure, covenants restricting the conditions under which the public authority or the concessionaire may terminate the concession, and any compensation to be paid by one party to the other in the event of unilateral termination of the concession. BOT schemes are discussed in detail in Delmon (2000).

In practice, a long-term fixed concession period is the most common approach, although there may be a mechanism for extending it for a limited additional period to compensate the concessionaire for risks it is not prepared to bear, such as force majeure and market demand that is far

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Written discussion of this article is welcomed and will be received by the Editor until 30 September 2006.

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below the expected level. Some countries include the construction phase as part of the concession period, others do not. In the former, the concession period starts when construction begins. For example, the first eight design–build–finance–operate roads in the United Kingdom have a fixed concession period of 30 years (Highways Agency 1997). In the latter, the concession period begins at the completion of the construction. For example, the Shajiao B power project in China has a predetermined construction period of 33 months and operation period of 10 years. The concessionaire can still operate the project for 10 years even if the project is completed behind schedule (Ye and Tiong 2000). There are also a few examples of concessions whose terms are variable depending on the date when the lenders recover their principal and interest and equity holders earn a certain level of return. In addition, some countries have legislative provisions limiting the duration of infrastructure concessions to a maximum number of years and (or) requiring that the concession expire once the debts of the concessionaire have been fully repaid and a certain level of revenue and (or) production and (or) usage has been achieved even if this maximum number of years has not been reached. For example, the Dartford bridge project has a maximum concession period of 20 years, within which facilities of the project are required to be handed back to the government once debt charges and other costs have been recovered (Walker and Smith 1995). A variable concession period is more likely to be used where (1) the scope of the project has not been clearly defined, (2) the project company is financially high-leveraged, (3) construction activities of the project are very complex with substantial risks (e.g., cost and duration overruns), and (4) the cash flows in future operation are very difficult to predict.

Different projects will incur different cash flow profiles during their life cycles. BOT-type projects usually require a great amount of up-front investment in the construction of infrastructure facilities, the recovery of which is through revenues from the project over the concession period. One important issue for the government considering using a BOT scheme to develop a particular infrastructure project is the determination of the appropriate length of the concession period. This length depends on a number of factors, such as the type of the project, the size and complexity of construction activities, operational life of the project facility, the capital structure of the concessionaire company, and the market situation and revenue stream in the future operation. There are many uncertainties and risks in construction and future operation, which have significant impacts on the length of the concession period.

This paper proposes a methodology for the determination of the length of the concession period based on a public–private win–win principle. That is, the concession period should be long enough to enable the concessionaire to achieve a reasonable return on its investment, but not so long that the concessionaire's return is excessive and the interests of the public sector are impaired. A mathematical model has been developed to reflect this reasonable but not excessive concept. A Monte Carlo simulation technique is used to model the impact of risks and uncertainties on the length of the concession period. A case study based on hypothetical data is provided to illustrate the application of the proposed methodology, mathematical model, and simulation techniques.

Concession period

Design of the concession period

Ye and Tiong (2003) identified three major elements in the design of the concession period: (1) structure, (2) length, and (3) incentive scheme. There are two period structures. One is the single-period concession that combines the construction period and operation period, and the other is the two-period concession that separates the operation period from the construction period. The single-period concession fixes the length of the concession and thus transfers the construction-time-overrun risk to the concessionaire. This means that the operation period is shorter if the construction period is longer, and vice versa. The concessionaire benefits from revenues generated from earlier operation if the project is completed ahead of schedule or otherwise bears the loss of revenues resulting from delayed and reduced operation time. In the two-period concession, the concessionaire has a fixed operation period regardless of actual completion time of construction. Possible incentive schemes include an early completion bonus (the concessionaire shares a percentage of the revenues generated in the period ahead of the scheduled completion time) or late completion penalty (the concessionaire bears a percentage of the losses resulting from delay of completion).

Essence of the concession period

As mentioned in the previous section, BOT-type projects usually require high project development costs, which are intended to be recovered through revenues in the future operation period. In general, a longer concession period will allow the concessionaire to collect more revenues with reduced interests to the public sector, and vice versa. Therefore, the concession period divides the revenues in the project life cycle between the public and private sectors.

The length of the concession period is determined by two time variables: construction period and operation period. Construction schedules are always estimates because a great number of factors affect construction activities. The operation period is the time needed for the concessionaire to pay back loans (principle and interest) and recover its investment with a certain level of return based on projected revenues, which are subject to market risks. A shorter concession period may mean higher initial tariff or toll levels and (or) future increases of tariff or tolls in the operation period. High tariff or toll levels and their increases often encounter strong public opposition. Therefore, the essence of an appropriate length of the concession period lies in (1) an informed estimation of the project completion time within which an experienced contractor can complete the project on schedule and (2) a sound prediction of the operation period that allows the concessionaire to obtain a reasonable but not excessive level of return.

For the private concessionaire, the length of the concession period should be long enough to allow the concessionaire to recoup its investment costs and obtain a reasonable return within that period, when the scope and severity of risks involved in the particular project and the opportunity costs in the current and future markets are taken into consideration. For the public client, the concessionaire's return should not be excessive compared to its commitments and efforts and benchmarked with information on costs and rates of return that are available in the current and future markets.

In addition, a BOT scheme should achieve a better result than a traditional public procurement approach. This is often examined by introducing a public sector comparator (PSC). The United Kingdom Treasury Taskforce (1999) defines the PSC as a hypothetical, risk-adjusted costing by the public sector as a supplier, to an output specification produced as part of a procurement exercise. The PSC is expressed in net present value terms based on the required output specifications and taking into full account the risks that would be encountered by that style of procurement. The PSC is used (1) to determine if the project is affordable to government by ensuring full life-cycle costing at an early stage; (2) as a means to test whether a public-privately partnered (PPP) project is viable and demonstrates value for money; (3) as a management tool to communicate with partners on such key aspects as output specifications and risk allocation and (4) as a means to encourage broader competition by creating greater confidence in the bidding process (Industry Canada 2003).

Mathematical definition of the concession period

According to the reasonable but not excessive principle, the concession period T is defined as

$$[1] \qquad T = T_c + T_o$$

where T_c is the project completion time, T_o is the operation period, and T_c and T_o satisfy eqs. [2]–[4]

$$[2] \qquad T_c \leq T_{cmax}$$

$$[3] \qquad T_o \leq T_{oc}$$

$$[4] \qquad \mathrm{NPV}_1(1 + R_{min}) \leq \mathrm{NPV}|_{T_o=t} \leq \mathrm{NPV}_1(1 + R_{max})$$

where T_{cmax} is the maximum allowable project completion time, T_{oc} is the designed economic operation life of the project, NPV_1 is the net present value of the total project development cost, R_{min} is the minimum rate of return required by the private sector in the development of a certain type of projects, R_{max} is the maximum rate of return to the total project development cost that is acceptable to the public sector, and $\mathrm{NPV}|_{T_o=t}$ is the net present value of net revenues generated from an operation period $T_o = t$.

All T in which T_c and T_o satisfy eqs. [2]–[4] constitutes the concession interval. Any point within this interval is considered to be an appropriate length of the concession period.

Simulation-based approach

Risks affecting the concession period

From eqs. [1]–[4], it is obvious that the determination of an appropriate concession period T requires a good estimation of the construction period T_c and the operation period T_o. T_c is dependent on the duration of various construction activities, their relationships, planning, and scheduling. Various construction risks may occur in the project site, relationships of contractual parties, contractual arrangements, technical specifications, and other areas. These include archaeological discoveries; delays in resolving site construction problems; adverse environmental conditions such as hazardous wastes; permits and licenses; varying subsurface conditions (e.g., difficult soils, rock, groundwater, and underground utilities); design changes; extreme weather or natural disasters; insufficiency of plans and specifications; construction cost escalation; inadequacy of resources (e.g., labor force, material, funding); changes in legal requirements; delays in delivery of critical equipment and supplies; labor strife and (or) jurisdictional disputes; political involvement and interference; subcontractor capability; protracted disputes; and third-party litigation (American Consulting Engineers Council and Associated General Contractors of America 1998). These risks have significant impact on the project completion time.

T_o depends on the project development cost (NPV_1) and the net present value of the net revenues in the operation period ($\mathrm{NPV}|_{T_o=t}$). NPV_1 depends on the costs of various construction activities. The various construction risks mentioned in the above may also greatly increase the project development cost. For example, because of construction risks the construction cost of the Channel Tunnel project doubled although initially it was presumed to be less risky because of its technical simplicity (Finnerty 1996). $\mathrm{NPV}|_{T_o=t}$ depends on the construction period T_c and many risks that may be encountered in future operation of the project, particularly, economic risks such as service and (or) product demand (quantity and price), project operation and maintenance costs, exchange rate (if foreign currency is involved), interest rate, and inflation rate.

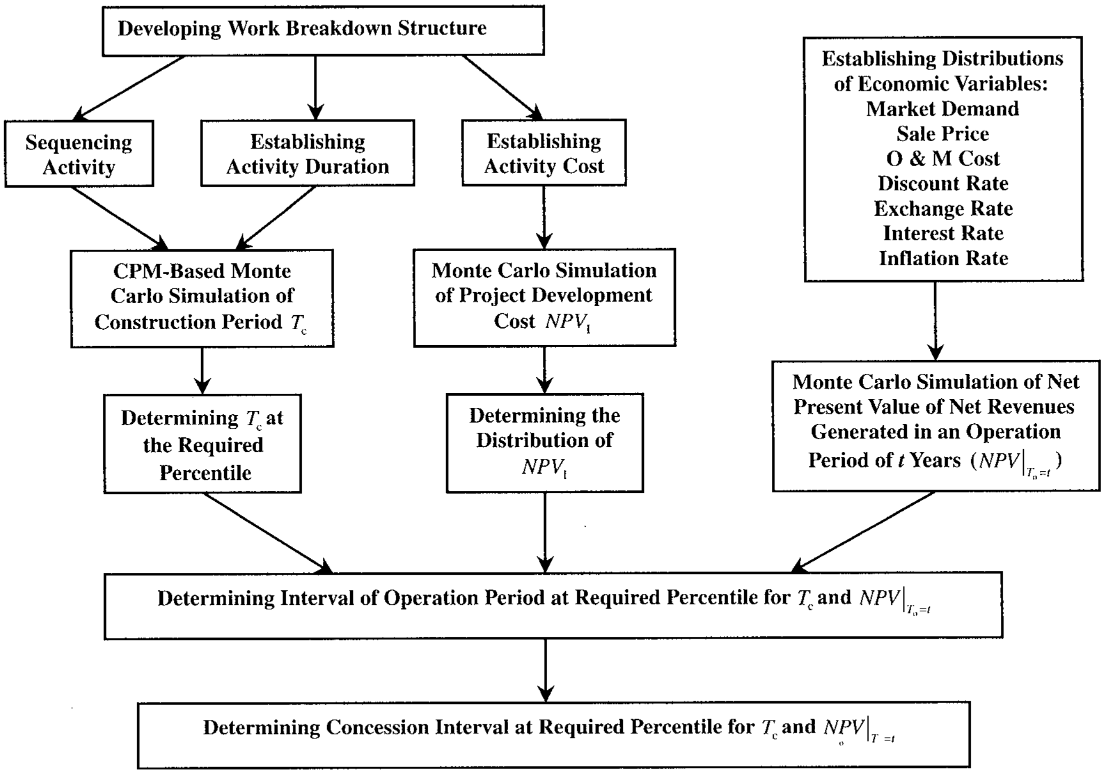
Framework of the simulation-based approach

As mentioned in the above, a PPP infrastructure project is subject to a variety of risks and uncertainties. Therefore, to facilitate decision-making, it is necessary to quantify these risks and model the project development as a stochastic process that behaves according to certain laws of probability. Risk analysis and modeling and consequent re-engineering of the project development process can lead to informed decisions in the procurement of pubic works and services. Computer simulation is a useful tool for decision-making under uncertainties and risks. The advances in simulation methodologies, development of special-purpose simulation languages, and massive computing capabilities of modern computers have made computer simulation one of the most widely used tools in operations research and systems analysis over the last two decades (Banks et al. 2001). For example, computer simulation has been used in many areas of the construction industry including process modeling and simulation, claims analysis and dispute resolution, and project planning, scheduling, estimating, and control.

One useful and often used simulation tool is Monte Carlo simulation (Binder and Heermann 2002), which models a stochastic process with random input data that follows certain statistical distributions. In such a simulation, the computer generates large sets of outputs after running a large number of iterations with random inputs. These outputs are then statistically analyzed to measure their uncertainties and risks. For example, Monte Carlo simulation has been used in the risk analysis of new business ventures (Wright 2002) and in life-cycle costing analysis with uncertainties (Emblemsvag 2003).

In this paper, Monte Carlo simulation is used to quantify and reason with the risks affecting the length of the concession period of a BOT-type project. Project development parameters

Fig. 1. Procedures of the proposed simulation approach.



are assumed to be random variables following certain statistical distributions instead of being deterministic values. Major risk variables considered here are construction period T_c, project development cost NPV_1, market demand, sale price, project operation and maintenance costs, and discount rate (combing interest rate and inflation rate). The procedures of the simulation-based approach are shown in Fig. 1. Details of each step are discussed in the following sections.

CPM-based Monte Carlo simulation of project completion time, T_c

The critical path method (CPM) is the most commonly used technique in the determination of the minimum possible duration of a construction project. The CPM breaks down a construction project into distinct work activities, arranges them into a logical sequence, estimates the duration of each activity, and displays the work plan using either a precedence diagram, arrow diagram, or conditional diagram. It then determines the minimum possible construction duration using forward pass and backward pass calculations based on the logic and criticality of the activities (Project Management Institute 2000). The CPM is a deterministic tool in that it assumes only one value for the duration of each activity and thus it does not provide a measure of uncertainty associated with the estimate of a particular milestone or of the overall project completion time. Monte Carlo simulation can eliminate the limitations of the CPM in addressing risks and uncertainties (Ahuja et al. 1994). Instead of determining the path criticality of a construction project as in the CPM, Monte Carlo simulation examines activity criticality based on the statistical distribution of the duration of each activity. The criticality of an activity is measured by the ratio of number of runs in which this activity is critical to the number of total simulation runs. The higher the ratio is, the more critical the activity. Therefore, once the project schedule network is finalized and the time distribution of each activity in the network is established based on the historical data and (or) expert knowledge, Monte Carlo simulation can be used to establish the statistical distribution of the project completion time using the CPM based on a randomly generated set of durations of all work activities. Then, the project completion time at a particular percentile can be calculated using this established distribution. Furthermore, the distribution of the project completion time also provides a basis on which the maximum allowable project completion time ($T_{c\max}$) is determined.

Monte Carlo simulation of operation period, T_o

T_o must satisfy the condition

$$\mathrm{NPV}_1(1+R_{\min}) \le \mathrm{NPV}|_{T_o=t} \le \mathrm{NPV}_1(1+R_{\max})$$

To determine an appropriate operation period T_o requires good estimation of the total project development cost NPV_1 and efficient prediction of $\mathrm{NPV}(T_o)$. The procedures for simulating NPV_1 and $\mathrm{NPV}(T_o)$, and for determining T_o are discussed in the following sections.

Simulation of NPV_1

A Monte Carlo simulation technique can be employed to determine the probability of achieving an estimate of the total project development cost that is within a certain range based on the statistical cost distributions of major project development activities. The following steps are taken in this simulation analysis: (1) define the project scope and establish its work

Table 1. Construction cost (US$) and duration distributions of different activities.

Activity	Cost distribution	Duration distribution
1	Normal distribution, with mean $150 000 000 and standard deviation $15 000 000	Triangular distribution, with most likely duration of 1.5 years, minimum duration of 1 year, and maximum duration of 2 years
2	Normal distribution, with mean $200 000 000, and standard deviation $30 000 000	Uniform distribution, with minimum duration of 1 year, and maximum duration of 2 years
3	Triangular distribution, with most likely value of $200 000 000, minimum value of $100 000 000, and maximum value of $300 000 000	Normal distribution, with mean of 1.5 years and standard deviation of 0.2 years
4	Uniform distribution, with minimum value of $100 000 000 and maximum value of $300 000 000	Triangular distribution, with most likely duration of 1 year, minimum duration of 0.5 years, and maximum duration of 1.5 years

breakdown structure; (2) classify the work items of each work package into two groups: group one – work items with a high degree of cost certainty and group two – work items with uncertain costs; (3) establish or assume the statistical cost distributions of uncertain work items; (4) establish the statistical cost distribution of each work package; (5) establish the statistical distribution of the total construction cost of the project; and (6) calculate the total project development cost at a required percentile (Zhang 2005).

Simulation of NPV$|_{T_o=t}$

NPV$|_{T_o=t}$, the net present value of the net revenues generated in a specific operation period $T_o = t$, is calculated using the following formula:

$$[5]\qquad \mathrm{NPV}|_{T_o=t} = \frac{1}{(1+r)^{T_c}} \sum_{i=1}^{t} \frac{\mathrm{NCF}_i^o}{(1+r)^i} = \frac{1}{(1+r)^{T_c}} \sum_{i=1}^{t} \frac{(I_i^o - C_i^o)}{(1+r)^i}$$

$$[6]\qquad I_i^o = Q_i^o \times P_i^o$$

where NCF$_i$ is net cash flow, I_i^o is income, C_i^o is operation and maintenance cost, Q_i^o is quantity of demand, P_i^o is sale/service price in the ith year of operation, and r = annual discount rate.

NPV$|_{T_o=t}$ is dependent on T_c, I_t^o, C_t^o and r. As discussed in the above, the distribution of T_c is estimated using CPM-based Monte Carlo simulation. T_c corresponding to a specific percentile a_c can be calculated based on this established distribution. If the statistical distributions of I_t^o, C_t^o and r can be established based on historical data, or reasonably assumed based on expert knowledge, then the statistical distributions of NPV$|_{T_o=t}$ can be established using Monte Carlo simulation. NPV$|_{T_o=t}$ can be reasonably assumed as a normal distribution with mean μ_o and standard deviation σ_o. μ_o and σ_o can be determined by a large number of simulation runs. NPV$|_{T_o=t}$ corresponding to a specific percentile a_o can be calculated based on this established normal distribution.

Interval of operation period

NPV$|_{T_o=t}$ corresponding to different percentiles can be calculated based on the established distributions of NPV$|_{T_o=t}$. Let $(T_o^L\ T_o^U)|_{\alpha_o}^{\alpha_I}$ denote the interval of the operation period at α_I percentile of NPV$_I$ and α_o percentile of NPV$|_{T_o=t}$. Then, T_o^L is the minimum t that satisfies $\mathrm{NPV}_I^{\alpha_I}(1+R_{\min}) \le \mathrm{NPV}^{\alpha_o}|_{T_o=t}$ and T_o^U is the maximum t that satisfies $\mathrm{NPV}^{a_o}|_{T_o=t} \le \mathrm{NPV}_I^{\alpha_I}(1+R_{\max})$, where $\mathrm{NPV}_I^{\alpha_I}$ is the net present value of the total project development cost at α_I percentile and $\mathrm{NPV}^{\alpha_o}|_{T_o=t}$ is the net present value of the total annual net cash flows from operation year 1 to t at α_o percentile.

Case study

A hypothetical BOT infrastructure project is used to demonstrate the application of the proposed methodology, mathematical model, and simulation-based approach discussed in the above. Please note that this project is intentionally simplified for the purpose of demonstration. In this case study, the package *CRYSTAL BALL* (Moore and Weatherford 2001) was used for conducting Monte Carlo simulations. A total of 20 000 simulation analyses were conducted in each required simulation variable, such as construction time, project development cost, and the accumulative net present value of the net revenues up to a particular operation year in the designed economic operation life of the project.

Statistical distributions of key project variables

The estimates on key project variables are given probability distributions. These variables are project development cost, activity duration, market demand, sale price, operation and maintenance (O&M) cost, and discount rate.

Activity costs and durations

The project is divided into four major work activities (1, 2, 3, and 4). It is assumed that the distributions of the costs (in million dollars at the beginning of the first year of construction) and durations of the four activities are already established based on historical data, using the methods mentioned in the sections Simulation of NPV$_I$ and CPM-based Monte Carlo simulation of project completion time T_c, respectively. These distributions are shown in Table 1.

Market demand and price

The designed annual production capacity of the project is 1.0 × 10^9 units. In the operation period, the annual market demand of the product follows a normal distribution, with mean value of 8 × 10^8 units and standard deviation of 2 × 10^8 units. The sale price of the product follows a normal distribution with a mean of US$0.4/unit and a standard deviation of US$0.04/unit.

Fig. 2. Frequency chart of total construction time.

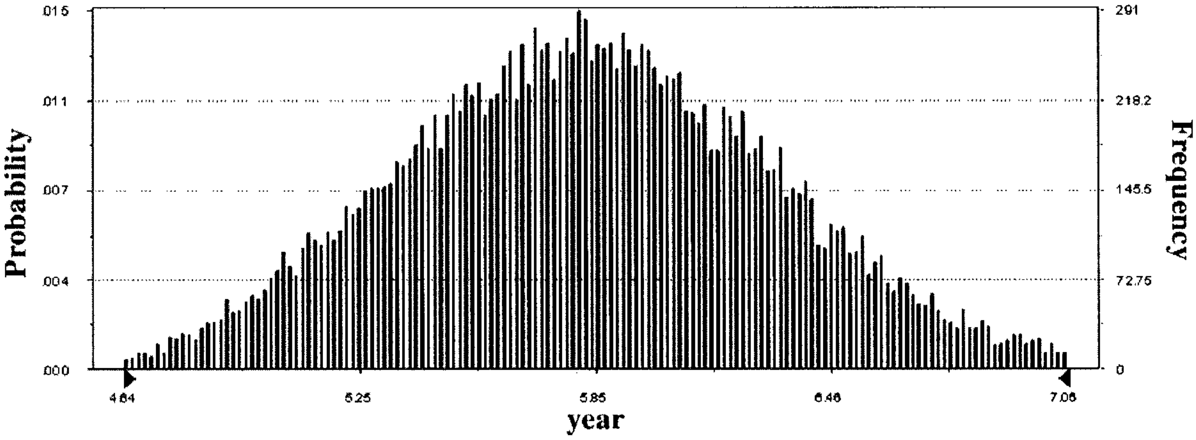
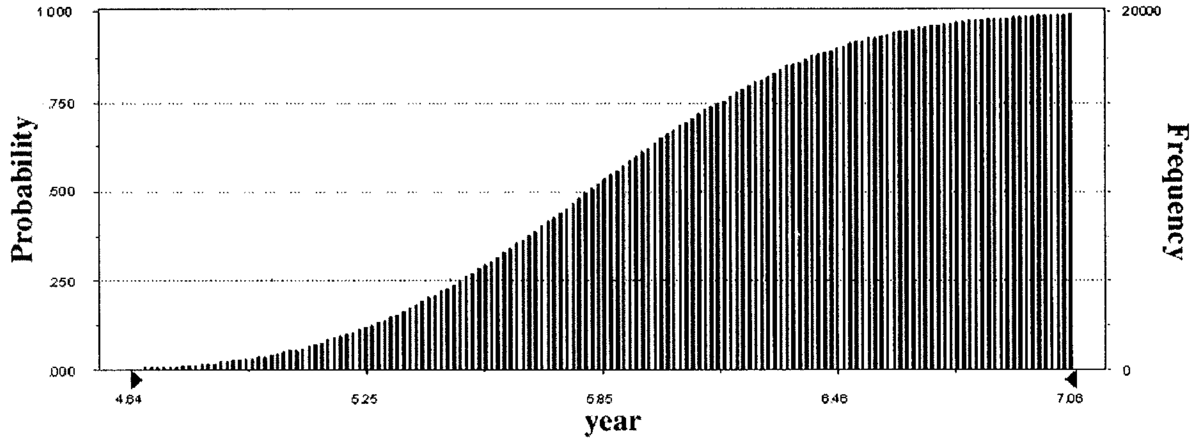


Fig. 3. Cumulative chart of total construction time.



Operation and maintenance cost

The designed economic operation life of the project is 30 years. It is assumed that the operation and maintenance (O&M) cost is increasing over this operation life. For simplicity, it is assumed that the annual O&M cost is 20% of the total annual sales revenue in the first 10 years of operation, 30% in the second 10 years, and 40% in the third 10 years. As the annual quantity of demand and sale price are random variables, the annual O&M cost is also random.

Annual discount rate

Discount rate can be seen as the interest rate charged by financial institutions for the use of their money. It is used to discount cash flows to reflect risks and the time value of money. The discount rate r can be calculated in the following formula (Brealey et al. 2003):

$$[7] \qquad r = (1 + r_r)(1 + r_I) - 1$$

$$[8] \qquad r \approx r_r + r_I$$

where r_r is real interest rate; and r_I is inflation rate.

Here it is assumed that the annual discount rate r follows a normal distribution with mean of 10% and standard deviation of 1%.

Table 2. Statistics of total construction time (year).

Statistics	Time (year)
Mean	5.83
Median	5.83
Standard deviation	0.48
Variance	0.23
Skewness	0.08
Kurtosis	2.78
Coefficient of variability	0.08
Range minimum	4.25
Range maximum	7.56
Range width	3.31
Mean standard error	0.00

Simulation of project completion time, T_c

Assume that the four activities follow finish–start relationships from activity 1 to activity 4, then, T_c is a stochastic variable whose value is the summation of the randomly generated values of the durations of activities 1 to 4. The statistics of T_c are shown in Table 2. Figures 2 and 3 are the frequency and cumulative charts of T_c. Based on the statistics and shapes of the frequency and cumulative charts, it is reasonable to assume that T_c follows normal distribution, with mean of 5.83 years and standard deviation of 0.48 years.

Fig. 4. Frequency chart of total construction cost.

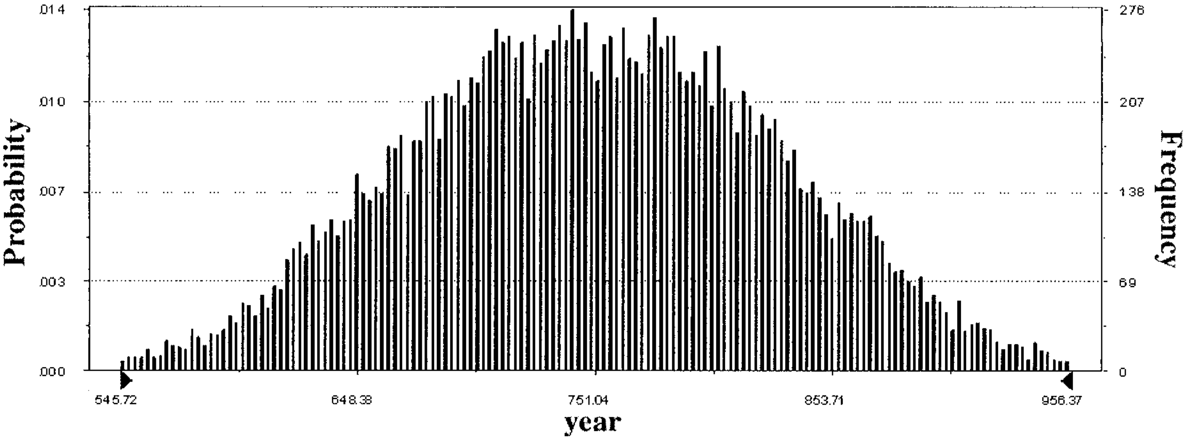
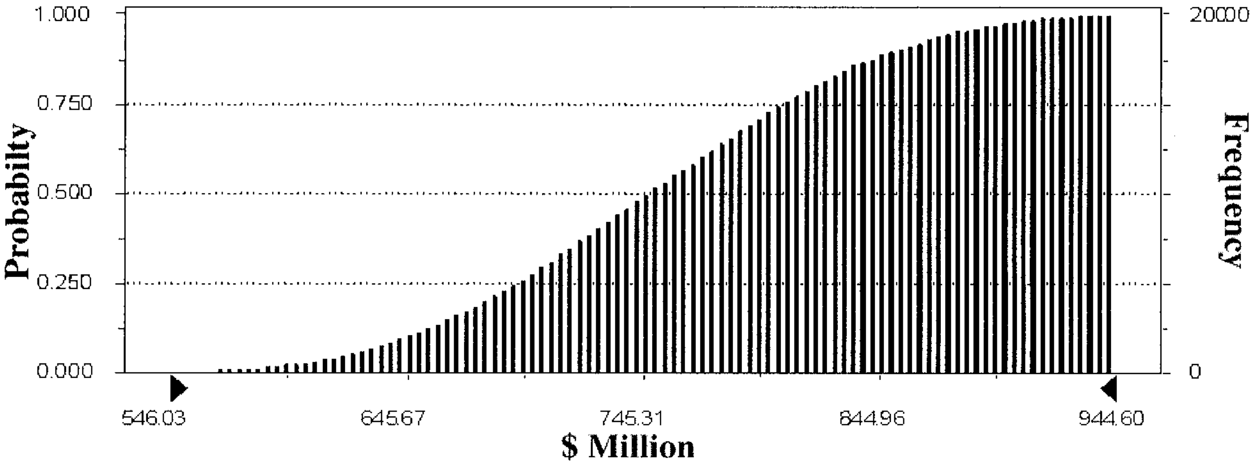


Fig. 5. Cumulative chart of total construction cost.



Let $T_c|_a$ denote the ath percentile of the random variable T_c, then

$$[9] \quad T_c|_a = \overline{T_c} + z_a\sigma$$

where $\overline{T_c}$ is the mean of T_c; z_a is the critical value of standard normal distribution at the specified percentile value a; and σ is standard deviation of T_c.

The project completion time can be derived according to eq. [9] based on the risk tolerance of the decision maker. For example, if a decision maker of low risk tolerance sets the project completion time at the 95th percentile, denoted by $T_c|_{a=95\%}$, then $T_c|_{a=95\%} = \overline{T_c} + z_a\sigma = 5.83 + 1.645 \times 0.48 = 6.62$ years

Simulation of NPV$_I$

The total project development cost NPV$_I$ is a stochastic variable, whose value is the summation of the randomly generated values of the costs of the four activities. The statistics of NPV$_I$ are shown in Table 3. Figures 4 and 5 are the frequency and cumulative charts of NPV$_I$. Based on the statistics and shapes of the frequency and cumulative charts, it is reasonable to assume that NPV$_I$ follows normal distribution, with mean of US$751.04 million and standard deviation of US$78.97 million.

Table 3. Statistics of total project development cost (US$ million).

Statistics	Cost (US$ × 10^6)
Mean	751.04
Median	750.27
Standard deviation	78.97
Variance	6236.21
Skewness	0.01
Kurtosis	2.59
Coefficient of variability	0.11
Range minimum	494.91
Range maximum	994.96
Range width	500.05
Mean standard error	0.56

If the total project development cost is set at the 95th percentile, denoted by NPV$_I|_{a=95\%}$, then NPV$_I|_{a=95\%}$ = 751.04 + 78.97 × 1.645 = US$880.95 million.

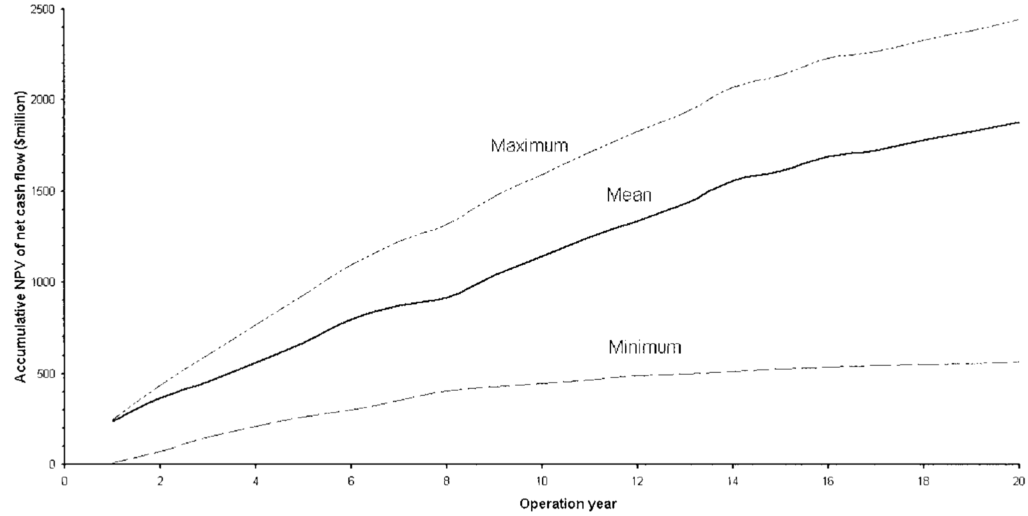
Simulation of NPV$|_{T_o=t}$

As shown in eq. [5], NPV$|_{T_o=t}$ is a stochastic variable that depends on stochastic variables T_c, I_t^o, C_t^o, and r. Here, T_c is

Table 4. Statistics of likely $NPV|_{T_o=t}$ (US$ million).

Year	Mean	Stand. Dev.	Minimum	Maximum	Range width	75th percentile
1	122.02	31.20	11.82	247.79	235.97	143.92
2	232.61	44.92	71.60	437.51	365.90	262.48
3	333.56	56.39	151.30	604.66	453.36	370.56
4	425.35	66.59	209.30	769.43	560.13	469.39
5	508.70	76.35	258.80	928.91	670.11	558.33
6	584.84	85.88	296.79	1096.07	799.28	640.09
7	654.16	95.25	350.30	1224.02	873.72	715.70
8	717.30	104.43	401.43	1318.71	917.28	784.36
9	774.58	113.03	428.91	1468.52	1039.61	846.30
10	826.85	121.50	446.48	1590.76	1144.28	902.97
11	874.64	129.93	464.06	1711.03	1246.96	956.60
12	918.15	138.01	486.70	1825.46	1338.76	1004.59
13	957.73	145.89	499.07	1931.43	1432.37	1048.79
14	993.77	153.37	512.72	2067.69	1554.97	1089.07
15	1026.82	160.60	526.52	2134.81	1608.29	1125.71
16	1056.82	167.52	537.69	2227.87	1690.17	1159.10
17	1084.12	174.20	545.47	2268.51	1723.03	1190.33
18	1109.11	180.59	551.05	2330.29	1779.24	1218.37
19	1131.84	186.70	556.37	2382.94	1826.57	1245.13
20	1152.63	192.55	562.83	2441.72	1878.88	1268.99

Fig. 6. Mean, minimum and maximum of NPV(T_o).



set at the 95th percentile, that is, 6.62 years as calculated in a previous section. According to the assumption made in the section *Operation and maintenance cost*, for year 1 to year 10 of the operation period, $NCF_i^o = I_i^o - C_i^o = I_i^o - 0.2I_i^o = 0.8I_i^o$; for year 11 to year 20 of the operation period, $NCF_i^o = I_i^o - C_i^o = I_i^o - 0.3I_i^o = 0.7I_i^o$; and for year 21 to year 30 of the operation period, $NCF_i^o = I_i^o - C_i^o = I_i^o - 0.4I_i^o = 0.6I_i^o$.

In the simulation process, the following condition is satisfied:

$$Q_i^o = q_i^o \quad \text{if } q_i^o \le 1.0 \times 10^9$$

$$Q_i^o = 1.0 \times 10^9 \quad \text{if } q_i^o > 1.0 \times 10^9$$

where q_i^o is the randomly generated quantity of demand for the *i*th year of operation.

For simplicity, it is assumed that there is no penalty to the concessionaire for not being able to satisfy a total demand that is beyond the designed capacity of the project. The mean, standard deviation, minimum, maximum, range width, and 75th percentile of $NPV|_{T_o=t}$ for $t = 1$ to 20 are shown in Table 4. Figure 6 shows the mean, minimum and maximum of $NPV|_{T_o=t}$.

Determination of concession interval

Assume the government decides to use the 95th percentile value of T_c and NPV_1, and the 75th percentile value of $NPV|_{T_o=t}$. As the project completion time $T_c|_{\alpha=95\%}$ is already derived, the concession interval is known if the lower and upper limits (T_o^L and T_o^U) of the operation period are known.

Lower limit of operation period, T_o^L

Assume R_{min} = 12%, then, the minimum total net revenue required by the concessionaire as discounted at the beginning of the first year of construction is calculated as follows:

$$\begin{aligned}\mathrm{NPV}_1|_{a=95\%}(1+R_{\min}) &= 880.95\times(1+0.12)\\ &= \mathrm{US\$}986.67\times10^6\end{aligned}$$

From Table 4, it is known that $\mathrm{NPV}|_{T_o=11}$ = US\$956.60 million and $\mathrm{NPV}|_{T_o=12}$ = US\$1 004.59 million. Therefore, T_o^L is between 11 and 12 years. Assume there is a linear relationship between $\mathrm{NPV}|_{T_o=t}$ and t in this short duration, then T_o^L is calculated as follows

$$\begin{aligned}\mathrm{NPV}_1|_{a=95\%}(1+R_{\min}) = \mathrm{NPV}|_{T_o=11}\\ + \frac{(T_o^L-11)}{(12-11)}(\mathrm{NPV}|_{T_o=12}-\mathrm{NPV}|_{T_o=11})\end{aligned}$$

$$T_o^L = 11 + \frac{\mathrm{NPV}_1|_{a=95\%}(1+R_{\min})-\mathrm{NPV}|_{T_o=11}}{\mathrm{NPV}|_{T_o=12}-\mathrm{NPV}|_{T_o=11}}$$

$$= 11 + \frac{986.67-956.60}{1\,004.59-956.60} = 11.63$$

Upper limit of operation period, T_o^U

Assume R_{max} = 20%, then, the maximum total net revenue allowed by the government as discounted at the beginning of the first year of construction is calculated as follows:

$$\begin{aligned}\mathrm{NPV}_1|_{a=95\%}(1+R_{\max}) &= 880.95\times(1+0.2)\\ &= \mathrm{US\$}1057.14\times10^6\end{aligned}$$

From Table 4, it is known that $\mathrm{NPV}|_{T_o=13}$ = US\$1048.79 million, and $\mathrm{NPV}|_{T_o=14}$ US\$1089.07 million. Therefore, T_o^U is between 13 and 14 years. Again, assume there is a linear relationship between $\mathrm{NPV}|_{T_o=t}$ and t in this short duration, then T_o^U is calculated as follows

$$\begin{aligned}\mathrm{NPV}_1|_{a=95\%}(1+R_{\max}) = \mathrm{NPV}|_{T_o=13} + \frac{(T_o^U-13)}{(14-13)}\\ \times(\mathrm{NPV}|_{T_o=14}-\mathrm{NPV}|_{T_o=13})\end{aligned}$$

$$T_o^U = 13 + \frac{\mathrm{NPV}_1|_{a=95\%}(1+R_{\max})-\mathrm{NPV}|_{T_o=13}}{\mathrm{NPV}|_{T_o=14}-\mathrm{NPV}|_{T_o=13}}$$

$$= 13 + \frac{1057.14-1048.79}{1089.07-1048.79} = 13.21$$

Therefore, the concession interval is $(T_c + T_o^U, T_c + T_o^L)$ = (6.62 + 11.63, 6.62 + 13.21) = (18.25, 19.83).

Conclusions

The length of the concession is an important issue in infrastructure development through BOT-type arrangements as the concession period divides the rights and responsibilities between the public and private sectors in the life cycle of the project. The essence of the methodology proposed in this paper is that the concession should integrate construction and operation to encourage innovations, efficiency, cost savings, and early project completion. The project completion time should allow a competent contractor to complete the project on schedule and the operation period should be long enough to enable the concessionaire to achieve a reasonable return, but not too long such that the concessionaire's return is excessive and the public sector's interests are sacrificed.

Informed assessments and analysis of risks and uncertainties are a prerequisite to the determination of an appropriate length of concession. Monte Carlo simulation is a useful tool to measure uncertainties and reason with construction and economic risks, including project development cost, project completion time, market demand and price of project services/products, operation and maintenance cost, interest rate, and inflation rate.

The proposed methodology, mathematical model, and simulation-based approach would facilitate the public sector in the determination of a suitable concession period for a particular infrastructure project, and the private sector in determining whether to bid for a concession solicited by a public client. It would also facilitate the private sector to develop unsolicited concession proposals for potential infrastructure projects and the public sector to evaluate such unsolicited proposals.

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