CAPM-Based Valuation of Financial Government Supports to Infeasible and Risky Private Infrastructure Projects

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Abstract: Driven by acute fiscal problems and disenchantment with the performance of publicly delivered quality services, many governments in both industrial and developing countries are now relying on the private sector to deliver infrastructure services. In the context of private infrastructure financing, the provision of governmental support is not uncommon. The present paper discusses the theoretical framework of the valuation of the financial impact of support on the cost of debt, cost of equity, the expected return on equity (ROE), and the project's net present value (NPV), based on the capital asset pricing model (CAPM) within a single-period context. Special focus is placed on a minimum revenue guarantee, a direct cash subsidy, and a subsidized subordinated debt. This paper shows that a guarantee reduces the cost of debt and can increase or decrease the cost of equity depending on the leverage, while a subsidy increases only the cost of equity. A subsidized subordinated debt increases both the cost of senior debt and the cost of equity. For all cases, all the supports improve the NPV because the expected increase of ROE can more than sufficiently offset any change in the cost of equity. This paper is of interest to academics because it provides the theoretical analysis of how a support can affect the rates of return expected by debt and private equity investors from risky and, probably, infeasible privately financed infrastructure projects. Practitioners, particularly those from the public sector, can also benefit from research findings that may inform decision makers about what support to provide.

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Introduction

There has been a global trend toward privatizing infrastructure activities in recent times. The idea itself is not new but rather an old concept that has regained popularity among governments following, among other things, acute fiscal problems and disenchantment with the performance of publicly delivered quality services. Historically, infrastructure networks in water, power, gas, and rail were often developed by private firms that incidentally bore substantial market risk with limited protection from competition in the past two centuries (Kerf et al. 1998). But with time, more and more infrastructure companies were regulated or nationalized, although the pattern varied substantially among countries and within countries and sectors, while wars and economic depression gave another boost to nationalization and stronger regulation, which increased in the 1940s and 1950s (Klein and Roger 1994).

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Pioneered initially by a few countries such as Chile and the United Kingdom in the 1980s, many industrial and developing countries are now relying on the private sector to deliver infrastructure services. The World Bank reported that in 1990–2001 developing countries transferred to the private sector the operating risk for almost 2,500 infrastructure projects, attracting investment commitments of more than $750 billion (World Bank 2003).

In the context of private infrastructure financing, the provision of governmental support is not uncommon. This may take various forms, such as a comfort letter, a grant or a direct subsidy, debt and/or equity capital contribution, tax and customs benefits, exclusivity, and a sovereign guarantee. The examples of support provisions in real-life projects among countries and sectors are given in, for instance, Tiong (1995); Dailami and Klein (1997); Dailami and Leipziger (1997); Lewis and Mody (1998); World Bank (1999); and Abdul-Azis (2001).

The main objective of this paper is to present the theoretical framework of the valuation of the financial impact of some governmental supports on costs of capital and the project's market value based on the capital asset pricing model (CAPM). This paper is primarily motivated by the fact that the issue has not been well researched. It is believed that a guarantee, for instance, can benefit the respective project sponsor, but how this works has not been explained. There is a list of questions that need to be addressed; for example: Is a guarantee cheaper than another form of support? or What factors play an important role in determining the impact of guarantee?

The CAPM is built on the assumptions that all securities in the market are fairly priced and that the market is in equilibrium. In a CAPM world, only market or systematic risks that are nondiversifiable are priced, while project-specific or nonsystematic risks are not considered because they can be eliminated by diversification. Interested readers can consult, for instance, Pollio (1999) and Brealey and Myers (2000) for detailed discussion of critics that challenge the validity of the CAPM. The CAPM is not the only technique that can be used to predict the expected return on a risky asset. Alternatively, one can employ the arbitrage pricing theory (APT) that also considers the macroeconomic factors for determining the expected return. But, unlike the CAPM that captures all systematic risks in a well-defined single factor, the APT does not tell what the underlying factors are (Brealey and Myers 2000).

The discussion is exclusively carried out within a single-period context but the research findings should also be applicable for multiperiod projects. Although special focus is placed only on three types of support—a minimum revenue guarantee, a subsidized subordinated debt, and a direct cash subsidy—the framework is also workable for examining the impact on other support types. This paper is organized as follows. First, it discusses the relations between risk and return based on the CAPM. Second, it presents the framework for determining the cost of debt and the cost of equity and provides the basic definition of a risky and infeasible project. Third, it discusses changes in the cost of debt, the cost of equity, the expected return on equity (ROE), and the net present value (NPV) when the project is supported with a minimum revenue guarantee, a subsidized subordinated debt, and a direct cash subsidy. Finally, it discusses the support effectiveness from the perspectives of both the government and the project sponsor. A hypothetical project is used to illustrate the implementation of the framework.

Academics can benefit from this study as it provides a theoretical analysis of how government support can change the rates of return expected by debt and private equity investors when venturing to support risky and infeasible privately financed infrastructure projects. To some extent, the study findings are of interest to practitioners, in particular to those in the public sector because this paper also provides general information on how the market value of a project can be improved through support provisions so that they can better make a decision on what support to provide.

Literature Review

The available studies about the financial effect of government supports, in particular in the form of guarantees on costs of capital and the project's value, are very rare. To the best knowledge of the writer, only the work of Dias and Ioannou (1995) explicitly deals with the effect of guarantees. The work discusses how the debt capacity of a project changes if the project is equipped with a minimum revenue guarantee and a minimum production guarantee. It examines also how a guarantee affects the optimal capital structure. In general, a guarantee can increase the expected ROE and, thereby, the project's NPV. The only problem with using their model is that there are discontinuities in the curves of the cost of debt, the cost of equity, and the expected ROE related to guarantees. The original writers argue that these discontinuities exist because the boundary level at which the nature of the debt payment changes from nonrisky to risky. On the contrary, the present writer believes that any discontinuity should not occur for the curves of costs of capital and the expected ROE. The reason will be discussed in detail below.

Cost of Capital

The cost of capital of a project reflects the hurdle rate that an investor can earn elsewhere on projects with similar risk. In the traditional present worth analysis this cost of capital will then serve as the risk-adjusted discount rate for future risky cash flows. The use of the present worth analysis alone does not suffice for valuing the financial impact of a guarantee because it requires information on the cost of capital that must always be adjusted to reflect changes in risk. This is the underlying reason why the present study employs the CAPM. The CAPM defines the expected return on asset as the sum of the risk-free rate and the product of beta and the expected market risk premium, which is the difference between the expected market return and the risk-free rate. Mathematically

$$E(\tilde{r}) = r_f + \beta[E(\tilde{r}_m) - r_f] \tag{1}$$

where $E(\tilde{r})$=expected return on asset; r_f=risk-free rate; and $E(\tilde{r}_m)$=expected market return; with

$$\beta = \frac{\mathrm{cov}(\tilde{r};\tilde{r}_m)}{\sigma_m^2} \tag{2}$$

For a single-period project, the expected return on asset is the end-of-period asset value divided by the present market value of asset, minus one, or

$$E\left(\frac{\tilde{X}_1}{X} - 1\right) = r_f + \frac{\mathrm{cov}(\tilde{X}_1/X - 1;\tilde{r}_m)}{\sigma_m^2}[E(\tilde{r}_m) - r_f] \tag{3}$$

where $\tilde{X}_1$=end-of-period asset value; and X=asset present value. Because X is not unknown and X does not covary with $\tilde{r}_m$, Eq. (3) can be rearranged as

$$\frac{E(\tilde{X}_1)}{X} - 1 = r_f + \frac{\mathrm{cov}(\tilde{X}_1;\tilde{r}_m)}{\sigma_m^2 X}[E(\tilde{r}_m) - r_f] \tag{4}$$

Solving for X gives

$$X = [E(\tilde{X}_1) - \lambda\,\mathrm{cov}(\tilde{X}_1;\tilde{r}_m)]/(1 + r_f) \tag{5}$$

Lambda (λ) in Eq. (5) is often termed the risk market price, which equals

$$\lambda = [E(\tilde{r}_m) - r_f]/\sigma_m^2 \tag{6}$$

The numerator of Eq. (5) is the certainty equivalence of uncertain cash flows. Next, Eq. (3) can be rewritten as

$$E(\tilde{r}) = r_f + \lambda\,\mathrm{cov}(\tilde{X}_1,\tilde{r}_m)/X \tag{7}$$

Substituting Eq. (5) into Eq. (7) and rearranging the terms gives

$$E(\tilde{r}) = E(\tilde{X}_1)/X - 1 \tag{8}$$

Eq. (8) provides an alternative way for determining the expected return on asset when the expected future asset value and the present asset value are known.

Cost of Debt

Debt is a risky asset. This is particularly true if the project financing is structured on a nonrecourse basis where the debt investor relies solely on the project cash flows and its assets for debt repayment. This approach does not allow the debt investor to seek recourse to the parent firm of the borrowing project sponsor. All else being equal, when a project is more leveraged, the probability of the project being in financial distress that results when it is unable to meet the full debt service will be higher. The worst scenario to result from financial distress is when the project goes bankrupt. In such an event the private equity investor exercises their right to default and project ownership is transferred from the equity investor to the debt investor. There will be costs associated with bankruptcy that the debt investor must pay before the investor can take ownership. The costs may include legal and administrative costs, sales of fixed assets and inventories, etc. In this paper, a bankruptcy is assumed to occur when the net operating revenue is below the promised debt service.

Let $\tilde{R}$ be the operating revenue that obeys a normal distribution with $E(\tilde{R})$ and σ_R as mean and standard deviation, respectively. Dias and Ioannou (1995) assume that the operating cost is proportional to the operating revenue

$$\tilde{C} = c_v \tilde{R} + c_f \tag{9}$$

where $\tilde{C}$=operating cost; $\tilde{R}$=gross operating revenue; c_v=variable cost component; and c_f=fixed cost component. The net operating revenue thus equals

$$\tilde{V} = (1 - c_v)\tilde{R} - c_f \tag{10}$$

where $\tilde{V}$=net operating revenue. Let d_1 be the promised debt service. The revenue level at which the debt investor can receive the full promised debt service can be obtained by setting $\tilde{V}=d_1$, which gives

$$a = (d_1 + c_f)/(1 - c_v) \tag{11}$$

where a=revenue level that secures the full debt service. By definition, the project goes bankrupt when $\tilde{V}<d_1$ and there will be costs associated with it. Kim (1978) models the bankruptcy cost as a linear function of the net operating revenue.

$$\tilde{B} = b_v \tilde{V} + b_f \tag{12}$$

where $\tilde{B}$=bankruptcy cost; b_v=variable bankruptcy cost; and b_f=fixed bankruptcy cost. In the event of bankruptcy, the payment to the debt investor equals the net operating revenue less the bankruptcy cost

$$\tilde{D}_1 = \tilde{V} - \tilde{B} \tag{13}$$

where $\tilde{D}_1$=payment to debt investor. The revenue level that makes the debt investor receive no payment can be determined by setting $\tilde{V}=\tilde{B}$ that yields

$$b = \lfloor b_f + c_f(1 - b_v)\rfloor/[(1 - c_v)(1 - b_v)] \tag{14}$$

where b=revenue level at which the debt investor receives no payment. The payment to the debt investor as a function of $\tilde{R}$ can be formulated as

$$\tilde{D}_1 = d_1 \quad \text{if } \tilde{R} \geqslant a \tag{15a}$$

$$\tilde{D}_1 = (1 - b_v)[(1 - c_v)\tilde{R} - c_f] - b_f \quad \text{if } b \leqslant \tilde{R} < a \tag{15b}$$

$$\tilde{D}_1 = 0 \quad \text{if } \tilde{R} < b \tag{15c}$$

To determine the present value of a risky debt, both the expected debt service and the covariance between the debt service and the market return are required. The expected debt service can be written as the sum of different functions of $\tilde{R}$

$$E(\tilde{D}_1) = d_1 \int_a^\infty f_R(\tilde{R})d\tilde{R} + (1 - b_v)(1 - c_v) \times \int_b^a \tilde{R} f_R(\tilde{R})d\tilde{R} - [(1 - b_v)c_f + b_f]\int_b^a f_R(\tilde{R})d\tilde{R} \tag{16}$$

Eq. (16) deals with the so-called partial moment problem that can be solved using the recursive formula given by Winkler et al. (1972)

$$E_{-\infty}^{\delta}(\tilde{R}^n) = -\sigma_R^2 \delta^{n-1} f_R(\delta) - (n-1)\sigma_R^2 E_{-\infty}^{\delta}(\tilde{R}^{n-2}) + E(\tilde{R})E_{-\infty}^{\delta}(\tilde{R}^{n-1}) \tag{17}$$

Hence the expected debt service can be written as

$$E(\tilde{D}_1) = d_1[1 - F_R(a)] + (1 - b_v)(1 - c_v)\{E(\tilde{R})[F_R(a) - F_R(b)] + \sigma_R^2[f_R(b) - f_R(a)]\} - [(1 - b_v)c_f + b_f][F_R(a) - F_R(b)] \tag{18}$$

where $F_R(\cdot)$=operator of cumulative normal distribution function and $f_R(\cdot)$=operator of probability normal distribution function. The next task is to calculate the covariance between the debt service and the market return. By definition

$$\mathrm{cov}(\tilde{D}_1, \tilde{r}_m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{D}_1(\tilde{R}) \cdot \tilde{r}_m \cdot f_{R,r_m}(\tilde{R},\tilde{r}_m)d\tilde{r}_m d\tilde{R} - \int_{-\infty}^{\infty} \tilde{D}_1(\tilde{R}) f_R(\tilde{R})d\tilde{R} \int_{-\infty}^{\infty} \tilde{r}_m f_{rm}(\tilde{r}_m)d\tilde{r}_m \tag{19}$$

where $f_{R,r_m}(\tilde{R},\tilde{r}_m)$=joint distribution function of $\tilde{R}$ and $\tilde{r}_m$. Eq. (19) can be rewritten as

$$\mathrm{cov}(\tilde{D}_1, \tilde{r}_m) = \int_{-\infty}^{\infty} \tilde{D}_1(\tilde{R}) f_R(\tilde{R}) \left[\int_{-\infty}^{\infty} \tilde{r}_m f_{rm|R}(\tilde{r}_m|\tilde{R} = R)d\tilde{r}_m - E(\tilde{r}_m)\right] d\tilde{R} \tag{20a}$$

or

$$\mathrm{cov}(\tilde{D}_1, \tilde{r}_m) = \int_{-\infty}^{\infty} \tilde{D}_1(\tilde{R}) f_R(\tilde{R})[E(\tilde{r}_m|\tilde{R} = R) - E(\tilde{r}_m)]d\tilde{R} \tag{20b}$$

If the market return is assumed to be normally distributed and if $\tilde{R}$ and $\tilde{r}_m$ are jointly normally distributed (Benjamin and Cornell 1970)

$$E(\tilde{r}_m|\tilde{R} = R) = E(\tilde{r}_m) + \mathrm{cov}(\tilde{R},\tilde{r}_m)/\sigma_R^2[\tilde{R} - E(\tilde{R})] \tag{21}$$

Substituting Eq. (21) into Eq. (20b) and rearranging the terms gives

$$\text{cov}(\tilde{D}_1,\tilde{r}_m) = \frac{\text{cov}(\tilde{R},\tilde{r}_m)}{\sigma_R^2}\left\{ d_1 \int_a^\infty [\tilde{R} - E(\tilde{R})] f_R(\tilde{R}) d\tilde{R} + (1-b_v)(1-c_v)\int_b^a [\tilde{R}^2 - \tilde{R}E(\tilde{R})] f_R(\tilde{R}) d\tilde{R} - [(1-b_v)c_f + b_f]\int_b^a [\tilde{R} - E(\tilde{R})] f_R(\tilde{R}) d\tilde{R} \right\} \tag{22}$$

The following relationships are obtained from Eq. (17):

$$\int_b^a [\tilde{R}^2 - \tilde{R}E(\tilde{R})] f_R(\tilde{R}) d\tilde{R} = \sigma_R^2 [b f_R(b) - a f_R(a) + F_R(a) - F_R(b)] \tag{23}$$

$$\int_b^a [\tilde{R} - E(\tilde{R})] f_R(\tilde{R}) d\tilde{R} = \sigma_R^2 [f_R(b) - f_R(a)] \tag{24}$$

Substituting Eqs. (23) and (24) into Eq. (22) and rearranging the terms yields

$$\text{cov}(\tilde{D}_1,\tilde{r}_m) = \text{cov}(\tilde{R},\tilde{r}_m)\{(d_1 b_v + b_f) f_R(a) + (1-b_v)(1-c_v) \times [F_R(a) - F_R(b)]\} \tag{25}$$

The debt market present value, D, can thus be computed as

$$D = 1/(1+r_f)[E(\tilde{D}_1) - \lambda \, \text{cov}(\tilde{D}_1,\tilde{r}_m)] \tag{26}$$

The cost of debt can be calculated from Eq. (8) by replacing $E(\tilde{X}_1)$ with $E(\tilde{D}_1)$ and X with D. The interest rate promised to the debt investor is equal to

$$i = d_1/D - 1 \tag{27}$$

If debt is a riskless asset, the expected debt service will equal the promised debt service and the debt market value will equal the expected debt service discounted at the risk-free rate. Therefore, in the case of a risk-free debt, both the interest rate and the cost of debt equal the risk-free rate.

Cost of Equity

The payment the equity investor receives is subject to taxes. Because the project lasts only for one period, making depreciation that reduces taxable income is equal to the total project cost seems to be justifiable. Another cash flow that also provides tax savings or a tax shield is interest payment. Therefore

$$\tilde{T} = t_c\{[(1-c_v)\tilde{R} - c_f] - A - iD\} \tag{28}$$

where $\tilde{T}$=taxes; t_c=tax rate; and A=total project cost. The taxes must be treated as random because they are not certain and depend on whether the equity investor can enjoy payment. Because the private equity investor can receive payment only when the obligation to the debt investor has been met, the after-tax equity payment can be formulated as

$$\tilde{S}_1 = (1-t_c)\{(1-c_v)\tilde{R} - c_f - d_1\} + t_c(A-D) \quad \text{if } \tilde{R} \geq a \tag{29a}$$

$$\tilde{S}_1 = 0 \quad \text{if } \tilde{R} < a \tag{29b}$$

where $\tilde{S}_1$=after-tax equity payment. To determine the equity market value, the information on the expected equity payment and the covariance between the payment and the market return are required. Applying similar procedures to derive the expected debt service gives

$$E(\tilde{S}_1) = (1-t_c)(1-c_v)\{E(\tilde{R})[1-F_R(a)] + \sigma_R^2 f_R(a)\} - [(1-t_c)(c_f+d_1) - t_c(A-D)][1-F_R(a)] \tag{30}$$

Likewise, the covariance between the equity payment and the market return can be obtained from

$$\text{cov}(\tilde{S}_1,\tilde{r}_m) = \text{cov}(\tilde{R},\tilde{r}_m)\{(1-t_c)(1-c_v)[1-F_R(a)] + t_c(A-D) f_R(a)\} \tag{31}$$

Hence the market value of equity equals

$$S = [E(\tilde{S}_1) - \lambda \, \text{cov}(\tilde{S}_1,\tilde{r}_m)]/(1+r_f) \tag{32}$$

The cost of equity can be obtained using Eq. (8) by replacing $E(\tilde{X}_1)$ with $E(\tilde{S}_1)$ and X with S. The expected return on equity is defined here as the ratio of the expected equity payment to the net capital requirement that the equity investor must meet, minus one. Mathematically

$$E(\text{R}\tilde{\text{O}}\text{E}) = E(\tilde{S}_1)/(A-D) - 1 \tag{33}$$

where $E(\text{R}\tilde{\text{O}}\text{E})$=expected return on equity. For all cases, the market debt value should not be allowed to exceed the total cost, $D < A$. For an extreme case where the debt value equals the total cost, the expected return on equity will theoretically be indefinite.

The traditional way of calculating the NPV from the equity investor's perspective dictates that the expected equity payment must be discounted at the risk-adjusted rate, the cost of equity, less the net capital requirement; that is

$$\text{NPV}_e = E(\tilde{S}_1)/[1 + E(\tilde{r}_e)] - A + D \tag{34}$$

where NPV_e=NPV from equity point of view. Because the first term of the right-hand side of Eq. (34) is the market value of equity, the NPV from equity's perspective is identical with the project's NPV. From Eq. (33)

$$A - D = E(\tilde{S}_1)/[1 + E(\text{R}\tilde{\text{O}}\text{E})] \tag{35}$$

Substituting Eq. (34) into Eq. (35) gives

$$\text{NPV} = E(\tilde{S}_1)/[1 + E(\tilde{r}_e)] - E(\tilde{S}_1)/[1 + E(\text{R}\tilde{\text{O}}\text{E})] \tag{36}$$

The project must be rejected if the project's NPV is negative, which occurs if the expected return on equity is lower than the cost of equity.

Cost of Subordinated Debt

Subordinated debt is debt that ranks lower than the senior debt but higher than equity in the payment claim. Let d_1^{sub} and d_1^{sen} the promised payoffs of subordinated debt and senior debt, respectively. The project is assumed to go bankrupt when the subordinated debt service cannot fully be made. The revenue level that can protect the project from bankruptcy must equal

$$\alpha = (d_1^{\text{sen}} + d_1^{\text{sub}} + c_f)/(1 - c_v) \tag{37}$$

In case the net operating revenue is less than this level, the subordinated debt investor is allowed to receive payment only if any obligation to senior debt has been met. The revenue level at which the subordinated debt investor receives no payment is

$$\gamma = [d_1 + b_f + c_f(1 - b_v)]/[(1 - b_v)(1 - c_v)] \tag{38}$$

Furthermore, the subordinated debt service can mathematically be written as

$$\tilde{D}_1^{\text{sub}} = d_1^{\text{sub}} \quad \text{if } \tilde{R} \geqslant \alpha \tag{39a}$$

$$\tilde{D}_1^{\text{sub}} = (1 - b_v)[(1 - c_v)\tilde{R} - c_f] - b_f - d_1 \quad \text{if } \gamma \leqslant \tilde{R} < \alpha \tag{39b}$$

$$\tilde{D}_1^{\text{sub}} = 0 \quad \text{if } \tilde{R} < \gamma \tag{39c}$$

and the senior debt repayment can be written as

$$\tilde{D}_1^{\text{sen}} = d_1^{\text{sen}} \quad \text{if } \tilde{R} \geqslant \gamma \tag{40a}$$

$$\tilde{D}_1^{\text{sen}} = (1 - b_v)[(1 - c_v)\tilde{R} - c_f] - b_f \quad \text{if } b \leqslant \tilde{R} < \gamma \tag{40b}$$

$$\tilde{D}_1^{\text{sen}} = 0 \quad \text{if } b > \tilde{R} \tag{40c}$$

Following the same procedures to determine the expected debt service and the covariance between the debt service and the market return gives

$$E(\tilde{D}_1^{\text{sub}}) = d_1^{\text{sub}}[1 - F_R(\alpha)] + [F_R(\alpha) - F_R(\gamma)] \times \{(1 - b_v)[(1 - c_v)\tilde{R} - c_f] - b_f - d_1\} + (1 - b_v)(1 - c_v)\sigma_R^2[f_R(\gamma) - f_R(\alpha)] \tag{41}$$

$$\text{cov}(\tilde{D}_1^{\text{sub}}, \tilde{r}_m) = \text{cov}(\tilde{R}, \tilde{r}_m)\{d_1^{\text{sub}} f_R(\alpha) + (1 - b_v)(1 - c_v) \times [(\gamma - \alpha) f_R(\alpha) + F_R(\alpha) - F_R(\gamma)]\} \tag{42}$$

Given the expected subordinated debt service and the covariance between the debt service and the market return, the subordinated debt market value and the cost of subordinated debt can be computed. The expected senior debt service can be computed from Eq. (18) by replacing d_1 with d_1^{sen} and a with γ while the covariance can be computed by applying the following formula:

$$\text{cov}(\tilde{D}_1^{\text{sen}}, \tilde{r}_m) = \text{cov}(\tilde{R}, \tilde{r}_m)\{d_1^{\text{sen}} f_R(\gamma) + (1 - b_v)(1 - c_v) \times [(b - \gamma) f_R(\gamma) + F_R(\gamma) - F_R(b)]\} \tag{43}$$

Minimum Revenue Guarantee

Let g be the minimum guaranteed level. From the previous study by Dias and Ioannou (1995), it has been noted that if g is less than b, the guarantee has no impact on both debt and equity market values, so the discussion is limited only for cases where g is equal to or greater than b.

Cost of Debt

Two cases are discussed: $g \geqslant a$ and $b \leqslant g < a$. The full debt service can always be made in the first case because the guaranteed revenue level is higher than the revenue level that suffices to make a full debt service. The appropriate discount rate for debt equals therefore the risk-free rate. For the second case, a guaranteed project can default on servicing debt but the risk is lower than it should be if the project is not guaranteed. Fig. 1 demonstrates how a minimum revenue guarantee reduces the debt risk. If $\tilde{R}$ is equal to or greater than a, debt is riskless. As $\tilde{R}$ moves a little further to the left, the project defaults because the net operating revenue has been below the promised debt service. The net revenue continues to decline until some point where $\tilde{R}$ equals g. As $\tilde{R}$ is lower than g, the net revenue increases again because the project sponsor receives a constant level of revenue, g, while the operating cost declines. At some point where $\tilde{R} = b' = (g - c_f - d_1)/c_v$, the net revenue equals the promised debt service. As $\tilde{R}$ becomes less than b', the debt is riskless and the equity investor can receive payment again. Mathematically, the uncertainty of debt service for $b \leqslant g < a$ can be written as

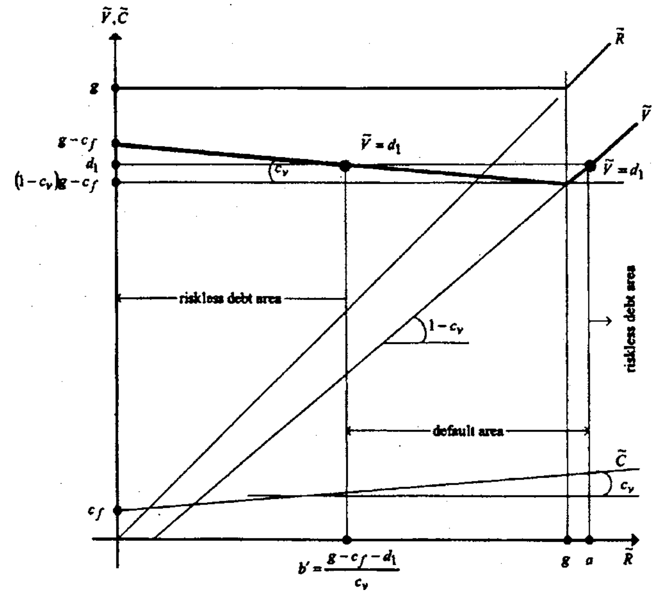

Fig. 1. Default risk in the presence of a guarantee

$$\tilde{D}_1^g = d_1 \quad \text{if } \tilde{R} \geqslant a \tag{44a}$$

$$\tilde{D}_1^g = (1 - b_v)[(1 - c_v)\tilde{R} - c_f] - b_f \quad \text{if } g \leqslant \tilde{R} < a \tag{44b}$$

$$\tilde{D}_1^g = (1 - b_v)(g - c_v\tilde{R} - c_f) - b_f \quad \text{if } b' \leqslant \tilde{R} < g \tag{44c}$$

$$\tilde{D}_1^g = d_1 \quad \text{if } \tilde{R} < b' \tag{44d}$$

where $\tilde{D}_1^g$ = debt service of a guaranteed project. The expected debt service and the covariance between the debt service and the market return of a guaranteed project can be determined from the following formulas:

$$E(\tilde{D}_1^g) = d_1[1 - F_R(a) + F_R(b')] + (1 - b_v)(1 - c_v) \times [E(\tilde{R}) - \sigma_R^2 f_R(a)] + (1 - b_v)\{F_R(g)[g - E(\tilde{R})] + F_R(b')[c_v E(\tilde{R}) - g] + \sigma_R^2[f_R(g) - c_v f_R(b')]\} - [(1 - b_v)c_f + b_f][F_R(a) - F_R(b')] \tag{45}$$

$$\mathrm{cov}(\widetilde{D}_1^g,\widetilde{r}_m)=\mathrm{cov}(\widetilde{R},\widetilde{r}_m)\{d_1[f_R(a)-f_R(b')]+(1-b_v)(1-c_v)\times\{F_R(a)-a[f_R(a)-f_R(b')]\}+(1-b_v)[c_vF_R(b')-F_R(g)]-[(1-b_v)c_f+b_f][f_R(b')-f_R(a)]\}\quad(46)$$

The debt market value of a guaranteed project can be written as follows

$$D_g=[E(\widetilde{D}_1^g)-\lambda\,\mathrm{cov}(\widetilde{D}_1^g,\widetilde{r}_m)]/(1+r_f)\quad(47)$$

where D_g=debt market value of a guaranteed project. Given the debt market value and the expected debt service, the cost of debt can be computed. As already illustrated in Fig. 1, the presence of a guarantee reduces debt risk. The direct implication of the risk reduction must be a reduction in the cost of debt because the debt service becomes less correlated with the market volatilities while the expected debt service increases.

Cost of Equity

For the case $b\leqslant g<a$, the after-tax equity payment is

$$\widetilde{S}_1^g=(1-t_c)[(1-c_v)\widetilde{R}-c_f-d_1]+t_c(A-D_g)\quad\text{if }\widetilde{R}\geqslant a\quad(48a)$$

$$\widetilde{S}_1^g=(1-t_c)(g-c_v\widetilde{R}-c_f-d_1)+t_c(A-D_g)\quad\text{if }\widetilde{R}\leqslant b'\quad(48b)$$

where $\widetilde{S}_1^g$=equity payment of a guaranteed project. The expected after-tax equity payment and the covariance between the equity payment and the market return are given in Eqs. (49) and (50), respectively.

$$E(\widetilde{S}_1^g)=(1-t_c)(1-c_v)\{[1-F_R(a)][E(\widetilde{R})-a]+\sigma_R^2f_R(a)\}+t_c(A-D_g)[1-F_R(a)+F_R(b')]-c_v(1-t_c)\{F_R(b')[E(\widetilde{R})-b']-\sigma_R^2f_R(b')\}\quad(49)$$

$$\mathrm{cov}(\widetilde{S}_1^g,\widetilde{r}_m)=\mathrm{cov}(\widetilde{R},\widetilde{r}_m)\{(1-t_c)(1-c_v)[1-F_R(a)]-c_v(1-t_c)F_R(b')+t_c(A-D_g)[f_R(a)-f_R(b')]\}\quad(50)$$

For the case $g\geqslant a$, the uncertainty of the after-tax equity payment can be formulated as follows:

$$\widetilde{S}_1^g=(1-t_c)[(1-c_v)\widetilde{R}-c_f-d_1]+t_c(A-D_g)\quad\text{if }\widetilde{R}\geqslant g\quad(51a)$$

$$\widetilde{S}_1^g=(1-t_c)(g-c_v\widetilde{R}-c_f-d_1)+t_c(A-D_g)\quad\text{if }\widetilde{R}<g\quad(51b)$$

The expected after-tax equity payment and the covariance between the equity payment and the market return are given in Eqs. (52) and (53), respectively.

$$E(\widetilde{S}_1^g)=(1-t_c)\{(1-c_v)[E(\widetilde{R})-a]+F_R(g)[g-E(\widetilde{R})]+\sigma_R^2f_R(g)\}+t_c(A-D_g)\quad(52)$$

$$\mathrm{cov}(\widetilde{S}_1^g,\widetilde{r}_m)=\mathrm{cov}(\widetilde{R},\widetilde{r}_m)\{(1-t_c)[1-F_R(g)-c_v]\}\quad(53)$$

For both cases, the equity market value can be computed from

$$S_g=[E(\widetilde{S}_1^g)-\lambda\,\mathrm{cov}(\widetilde{S}_1^g,\widetilde{r}_m)]/(1+r_f)\quad(54)$$

where S_g=equity market value of a guaranteed project. Whether the cost of equity of a guaranteed project decreases or increases when compared to that of an unguaranteed project depends on leverage. At low levels of leverage, i.e., when g is greater than a, the risk to the equity investor must be reduced because the equity payment can always be made. Even if g is exactly equal to a, the probability that the equity investor will receive payment must be greater with guarantee than without guarantee. This makes the expected equity payment increase and the covariance between the payment and the market return decreases so that in total the cost of equity must be lower with guarantee than without guarantee. At moderate levels of leverage, whether the cost of equity is lower with guarantee than without guarantee depends on whether the additional possibility of receiving payment at low operating revenue can outweigh the decreasing interest tax saving. Because of an increase in the debt market value when a project is guaranteed, the tax saving from interest payments must be decreased. This is the "negative" impact of a guarantee on equity. At the same time, however, the presence of a guarantee allows the equity investor to receive payment although the operating revenue is very low, which never happens if the project is not guaranteed. This should decrease the covariance between the equity payment and the market return. If the "negative" impact of a guarantee on equity that reduces the interest tax savings can be well-compensated for by the positive impact of a guarantee that reduces the equity payment uncertainty, the cost of equity will be lower with guarantee than without a guarantee for a given value of d_1; otherwise, the cost of equity will be higher. At high levels of leverage, the probability that the equity investor can receive payment at low operating revenue becomes less significant. In this case the cost of equity will be higher with guarantee than without guarantee. A higher cost of equity should not automatically lead to a decrease in the NPV. Because the guarantee increases the debt market value, the net equity capital requirement will decrease, thereby increasing the expected ROE; and this increase can more than sufficiently offset any increase in cost of equity so that the project's NPV improves.

Cost of Guarantee

A guarantee is a risky cash flow from the government's perspective. The guarantee payment can be modeled as follows:

$$\widetilde{G}_1=0\quad\text{if}\quad\widetilde{R}\geqslant g\quad(55a)$$

$$\widetilde{G}_1=g-\widetilde{R}\quad\text{if}\quad\widetilde{R}<g\quad(55b)$$

where $\widetilde{G}_1$=guarantee payment. The government cost can be obtained using the following formula:

$$G=1/(1+r_f)\{[g-E(\widetilde{R})]F_R(g)+\sigma_R^2f_R(g)+\mathrm{cov}(\widetilde{R},\widetilde{r}_m)F_R(g)\}\quad(56)$$

where G=government cost when providing a guarantee.

Table 1. Summary of Financial Impacts of Different Supports

Type of support	Cost of debt	Cost of equity	Expected return on equity	Net present value
Guarantee	(−)	(+)/(−)	(+)	(+)
Subsidy	(=)	(+)	(+)	(+)
Subsidized subordinated debt	(+)	(+)	(+)	(+)

Note: (+): increase; (−): decrease; and (=): unchanged.

Direct Cash Subsidy

The presence of a cash subsidy does not alter risk to the debt investor but changes risk to the equity investor. On the one hand, a decrease in the total net cost reduces the depreciation tax savings. This makes the equity market value decrease, and the cost of equity becomes higher with subsidy than without subsidy. On the other hand, if a project is subsidized, the equity capital requirement will be reduced. The consequence is that the expected ROE must increase significantly and any increase in the cost of equity must be well-compensated for by an increase in the expected ROE.

Subsidized Subordinated Debt

Suppose that the government provides a loan of $Z million. The government allows the loan repayment plus interest to be made if any obligation to the senior debt investor has been met. Let i_s be the interest rate charged by the government. The promised subordinated debt payoff is therefore equal to $$Z(1+i)$ million. A subordinated debt by the government is said to contain a subsidy when the subordinated debt market value is lower than $Z million. It must be borne in mind that to make the developed formulations mathematically applicable, α must be equal to or greater than γ. In other words, there will be the maximum limit of d_1^{sen}. This is justifiable because the government can require the project sponsor to limit the maximum level of debt service because a higher promised debt payoff will put the government at a higher risk. Under the assumptions that the project goes bankrupt when the project fails to meet any financial obligations, the presence of a subordinated debt should increase the cost of senior debt because the probability of the project going bankrupt increases. As with a subsidy, the presence of a subsidized subordinated debt reduces the net total cost so that the depreciation tax savings declines. This makes the equity market value decrease, and the cost of equity must increase. At the same time, the expected ROE must increase significantly and this increase can more than sufficiently offset any increase in the cost of equity. Table 1 summarizes the financial impact of different types of support on the costs of capital, the expected ROE, and the project's NPV.

Numerical Example

A hypothetical project with key information given in Table 2 is presented. Fig. 2 shows the cost of debt, the cost of equity, the expected ROE, and the NPV for different values of d_1. As shown, the cost of debt and the cost of equity increase with the increased promised debt service, but the latter increases more rapidly than the former. It makes sense when the equity investor requires the highest return because, in the payment claim, the equity payment ranks the lowest; that is, the equity investor can receive payment only if all financial obligations to the debt investor have been met. In this case, structuring the capital cannot help make the project financially feasible because the expected ROE is lower than the cost of equity for all possible values of d_1. If the project sponsor's objective is to maximize the expected ROE, the optimal capital structure occurs at $d_1 = \$50$ million, which generates the ROE of 12.5%, the cost of equity of 15.3%, and the project's NPV of $-\$1.3$ million. But if the ultimate objective is to maximize the equity investor's wealth, the optimal capital structure occurs at $d_1 = \$25.0$ million that provides the equity investor with the project's NPV of $-\$0.8$ million. At this capital structure, the cost of debt, the cost of equity, and the expected ROE are, respectively, 6.3, 12.4, and 11.3%.

Table 2. Key Project Information

Project	Market
$E(\tilde{R}) = \$175.0$	$E(\tilde{r}_m) = 12\%$
$\sigma_R = \$43.8$	$\sigma_m = 20\%$
$A = \$100.0$	$r_f = 6\%$
$b_v = 0.3$; $b_f = \$5.0$	$\rho_{R,rm} = 0.7$
$e_v = 0.2$; $e_f = \$25.0$	
$T = 0.35$	

Note: Dollar values in millions.

Minimum Revenue Guarantee

It is assumed that the host government will provide a minimum guarantee of 70% of the expected revenue, or $g = \$122.5$ million. If the gross operating revenue falls short of the guaranteed level, the government pays the project sponsor the amount of the difference between actual and guaranteed levels; otherwise, the government pays nothing. Fig. 3 shows the computational results for different values of d_1. Under this guarantee scheme, debt is risk-free when the promised debt service is equal to or less than $73.0 million. This is d_1 that sets a equal to g. In this case, the cost of debt equals the risk-free rate. If the promised debt service is higher than this level, the cost of debt increases with d_1. For instance, at $d_1 = \$100.0$ million, the cost of debt is 10.8%, but this

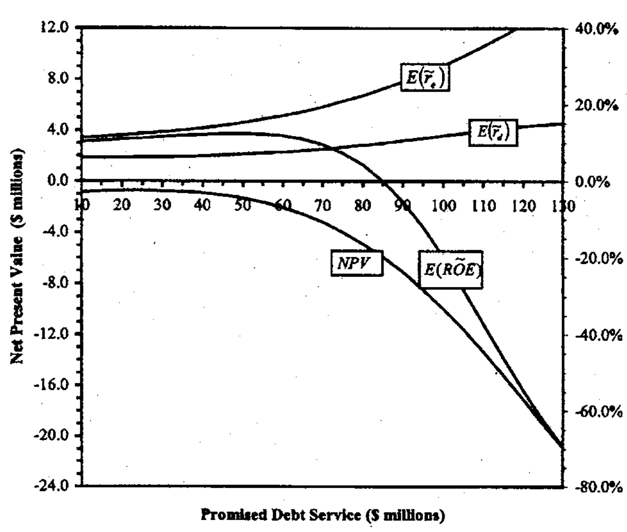

Fig. 2. Financial parameters of a project without guarantee

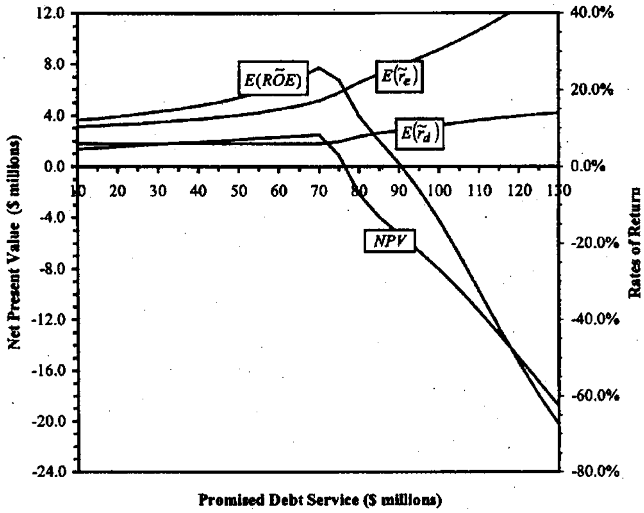

Fig. 3. Financial parameters of a guaranteed project

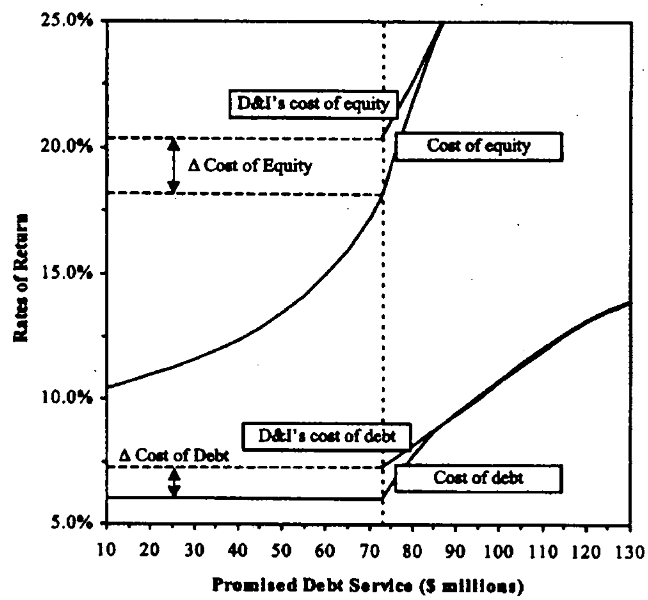

Fig. 4. Present model versus (Dias and Ioannou's model 1995)

figure is lower than the cost of debt of 11.9% when the project is not guaranteed. At low and moderate levels of leverage, the cost of equity is lower with guarantee than without guarantee. For instance, at $d_1 = \$75.0$ million, the cost of equity with and without guarantee are, respectively, 19.3 and 20.9%; but at $d_1 = \$90$ million, for instance, the cost of equity with and without guarantee are 26.2 and 26.0%, respectively. At this borrowing level, however, the expected ROE increases from −5.4 to 0.7% while the project's NPV increases from −$7.1 million to −$5.3 million. The optimal capital structure occurs at $d_1 = \$73.0$ million, which gives the maximum NPV of $3.9 million. At this capital structure, the cost of debt, the cost of equity, and the expected ROE are 6.0, 17.2, and 25.8%, respectively.

Present Model versus Dias and Ioannou's Model

In some cases, the present model gives different results from those of Dias and Ioannou's (1995) model. Fig. 4 shows the difference. Dias and Ioannou's model suggests two different values when g is close to a (or d_1 close to $73.0 million). From the right-hand side, $g \to a^+$, for instance, at $d_1 = \$72.99$ million the cost of debt and the cost of equity are 6.0 and 18.1%, respectively. From the left-hand side, $g \to a^-$, for instance, at $d_1 = \$73.01$ million the cost of debt and the cost of equity are 7.4 and 20.5%, respectively. These are remarkable differences. A further investigation reveals, however, that any discontinuity may occur because the possibility of debt being risk-free for cases of low operating revenue is ignored in the original model.

Direct Cash Subsidy

It is assumed that the cash injected by the government for the project implementation is $10.0 million. The net capital requirement is therefore equal to $90.0 million. Fig. 5 shows the computational results for different values of d_1. Because the presence of a direct subsidy does not alter risk to the debt investor, the costs of debt with and without subsidy for a given value of d_1 are equal. The subsidy increases the cost of equity, however. For instance, at $d_1 = \$80.0$ million, the cost of equity of a project increases from 22.4 to 23.4% but, at the same time, the expected ROE increases from 4.3 to 36.9%, improving the project's NPV from −$4.9 million to $2.5 million. The optimal capital structure occurs at $d_1 = \$25.0$ million, which provides the project sponsor with the maximum project's NPV of $6.0 million.

Subsidized Subordinated Debt

The government is assumed to provide a loan in the amount of $40 million. This loan is interest-free and the payment can be made only if the project sponsor has satisfied any financial obligation to the debt investor. In this case, the market value of senior debt is not allowed to exceed the net capital requirement, i.e., $60.0 million. The computational results are presented in Fig. 6. For instance, if the promised senior debt service is $60 million, the cost of equity and the cost of debt are 33.1 and 8.6%, respectively. Without any support, the cost of equity and the cost

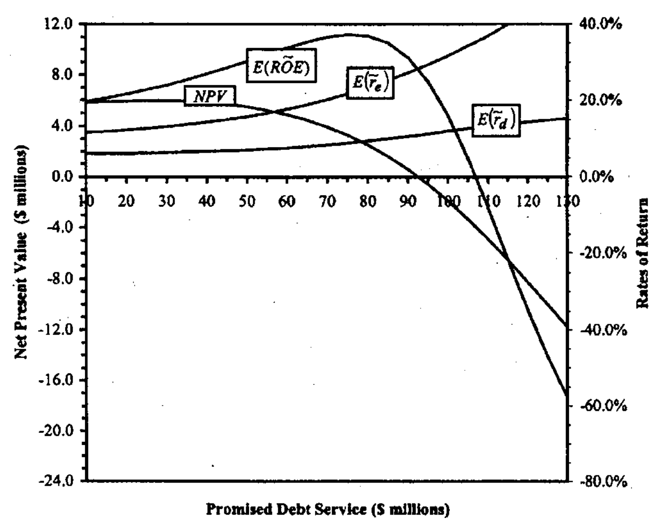

Fig. 5. Financial parameters of a project with cash subsidy

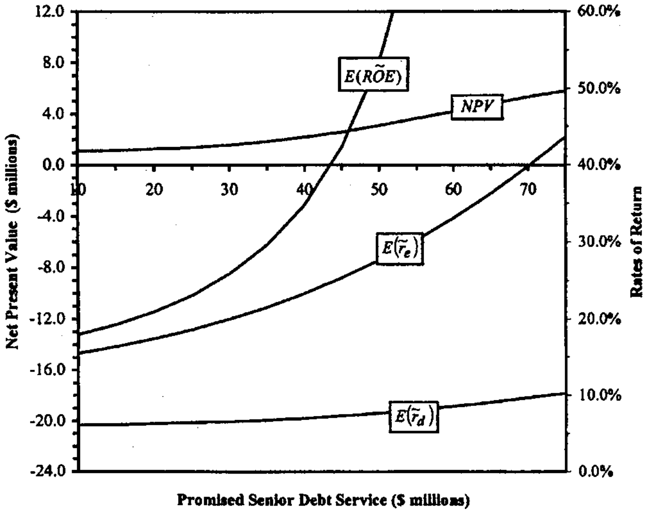

Fig. 6. Financial parameters of a project with subordinated debt

of debt should be 17.1 and 7.6%, respectively. The expected ROE well increases from 11.9 to 100.0% and the project's NPV improves from $-\$2.1$ million to \$4.2 million. At this leverage, the subsidy is equivalent to \$17.6 million. If the government opts to provide a direct cash subsidy in the amount of \$17.6 million, the project's NPV is increased to \$10.2 million. In this case, the subsidized subordinated debt is less effective than the direct cash subsidy because the former generates less NPV for the equity investor than the latter at the same cost to the government.

The comparison can also be made between a minimum revenue guarantee and a direct cash subsidy, as shown in Fig. 7. The curve coded with G90 represents the NPV when the minimum revenue is guaranteed at 90% of the expected value, and S90 is the NPV when the project is supported with a direct cash subsidy equivalent to the government cost when providing a minimum revenue guarantee that covers 90% of the expected revenue. At low and moderate levels of leverage, the project's NPV is higher with guarantee than with subsidy. If the project is highly leveraged, however, the NPV is higher with subsidy than with

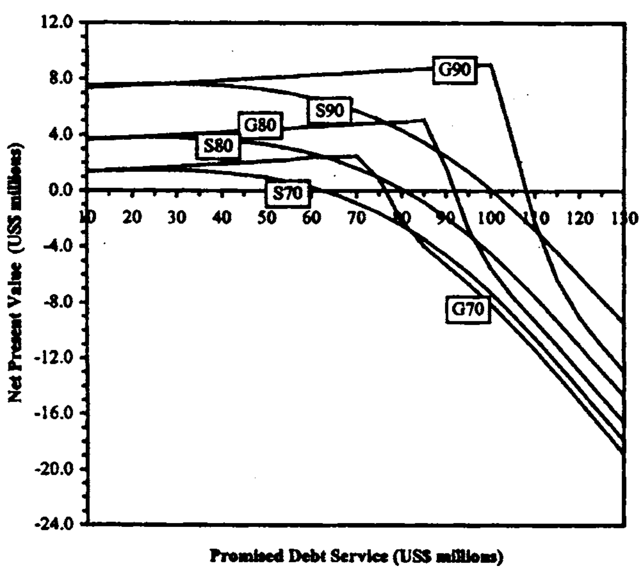

Fig. 7. Financial impact of subsidy and guarantee on NPV

guarantee. Fig. 7 also suggests that a higher guarantee level allows the project sponsor to use more debt financing to maximize the NPV. For instance, if the guaranteed level is 70% of the expected revenue, the optimal capital occurs at $d_1 = \$73.0$ million while if the guarantee level is increased to 90% of the expected level, $d_1 = \$101.0$ million.

Conclusion

This paper presents a theoretical framework of the valuation of the financial impact of a minimum revenue guarantee, a direct cash subsidy, and a subsidized subordinated debt on the costs of capital, the expected ROE, and the NPV based on the CAPM. It has been shown that the presence of a guarantee reduces the cost of debt. It is possible that a guarantee can increase the cost of equity, but any increase has been well-compensated for by an increase in the expected ROE so that the NPV increases. In contrast to a guarantee, a direct cash subsidy does not alter the debt risk. The subsidy increases the cost of equity because the depreciation tax shield declines following the decrease in the net total cost. As with a guarantee, any increase in cost of equity is more than sufficiently offset by an increase in the expected ROE so that the NPV improves. A subsidized subordinated debt increases the senior debt cost because the probability of the project going bankrupt increases. The subsidized subordinated debt also increases the cost of equity but this increase has been compensated for by a higher expected ROE.

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