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The analysis of regulation on private highway investment under a build-operate-transfer scheme

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Abstract. The build-operate-transfer (BOT) approach has become an attractive instrument for public facility provision, especially for a project that faces difficulty with public finance. This study analyzes the regulation alternatives on private highway investment under a BOT scheme and their impacts on traffic flows, travel costs, toll, capacity, and social welfare (total user-benefit in the traffic system including congestion). For comparison, five cases are analyzed: (1) No BOT with maximizing welfare, (2) No BOT with breaking even on finance, (3) BOT without regulation, (4) BOT with a minimum flow constraint (the total users will not be less than those in Case 1), and (5) BOT with a maximum travel cost constraint (the travel cost for users on a non-tolled road will not exceed the maximum tolerance). After each case is modeled and simulated on some functional forms, we find that the case of BOT with regulations performs between the cases of maximizing welfare and that of maximizing profit. From the perspective of the government, regulation has less power in a project with low elastic demand. Furthermore, even when the regulation is strict, a high cost-efficient firm with BOT could result in a higher level of social welfare than that without a BOT scheme.

1. Introduction

Road congestion is a major urban problem resulting from the externality that consumers (road users) do not take into account when determining their travel decisions. There are many ways to improve problems of congestion. From the demand perspective, a government could levy a Pigouian tax on road users to reflect their costs in order to lessen travel demand, based on Pigou (1920) and Knight (1924).[1] From the supply perspective, increasing road capacity by constructing a new road or expanding the current ones is a direct way to accommodate demand.[2]

Due to financial pressure incurred by the supply approach, optimal capacity with road pricing is also considered.[3] This kind of construction of a new toll road can be regarded as a mix of supply and demand elements. Recently, this tendency toward the introduction of privately-provided public roads has proved to be an efficient way to solve the problem. Through a build-operate-transfer (BOT) approach, the private sector is allowed to build and operate a road at its own expense, and in return it receives the revenue from road toll charges for some set amount of years, and then the road is transferred back to the government.

Build-operate-transfer has turned into an attractive instrument for a public facility provision especially when difficulty occurs with public financing. There are many BOT projects in the world, such as the SR-91 highway in California and the Dulles Toll Road in Virginia in the United States (Viton 1995), the Anglo-French "Chunnel," the Sydney Harbor Tunnel in Australia, some expressways in Malaysia and in Singapore (Tiong 1990), and the High Speed Railroad in Taiwan. Generally, a major reason for the use of BOT is a belief that the private sector is more efficient than the public sector, and therefore it can build and operate facilities at less cost than the public sector (see Gomez-Ibanez et al. 1991, for instance). When a new road is constructed by the BOT approach to an existing network, it might also reduce congestion and benefit the road users on the old road (Mills 1995).

Much research has been done on issues related to BOT. From the viewpoint of a private investor, profitability is the main concern, while from the government side, social welfare should be the main goal. Mills (1995) argues that a profitable project is not guaranteed to be one that adds to welfare.[4] On the other hand, a project with a positive welfare increment is not necessarily profitable either.

Different types of facilities might cause different degrees of private participation. Gomez-Ibanez et al. (1991) investigate toll roads and solid waste disposal facilities in the United States and discover different levels of private sector involvement. From the BOT approach, a long payback period after the initial investment makes it more difficult to predict future revenue and thus makes uncertainty and risk more important than alternative investment options (Nijkamp & Rienstra 1995). Yang and Meng (2000) emphasize the network effect on a BOT project, employing a general road network to examine road capacity and tolls under various market conditions. Viton (1995) investigates the feasibility of private toll-supported roads, in direct competition with a free public expressway. The results show that private sector involvement can succeed under some widely-varying circumstances. Particularly notable is the finding of central-city viability, even under reasonable regulation. Conversely, long-distance projects are more likely to fail.

From the literature on BOT, the consumer's perspective (road users) seems to be neglected. For a toll project, consumers might not want to use that mode of transportation if the charge fee is too high due to the absence of regulation directly or indirectly on the fee, yet the private firm might be under financial pressure if the regulation is too strict and thus the BOT might fail. Therefore, it is important to set up a proper regulation to protect both the consumers and the firms. Recognizing this viewpoint, this paper focuses on the regulation alternatives on private highway investment under a build-operate-transfer scheme.[5] The impact of various circumstances on traffic flow (number of road users), as well as users' cost, profit of the private firm, and the welfare level, are all explored.[6] For comparison, five cases are analyzed:[7] (1) No BOT while maximizing social welfare, (2) No BOT while breaking even on finance, (3) BOT without regulation, (4) BOT with a minimum flow constraint (ensuring the numbers of users), and (5) BOT with a maximum travel time constraint (ensuring the upper bound of user costs). Each case is modeled and simulated on some functional forms.

In this paper welfare is defined as the net benefit of the system. It denotes the users' benefits minus the users' costs (including congestion cost) and producer's construction and maintenance costs.[8] In case 1, the government maximizes social welfare given the road capacity of route 1 with a toll on route 2. In case 2, the government maximizes social welfare given the road capacity of route 1 with a toll on route 2 and looks to break even on the public finance. That is, the revenue collected from the toll on route 2 equals its construction and maintenance costs. In case 3, the firm maximizes its profit by charging a user fee on road 2 without regulations. In case 4, the firm maximizes its profit with a toll on road 2 under the constraint that the total traffic volume of both roads may not be to a certain degree inferior to the total traffic volume of case 1. In case 5, the firm maximizes its profit with a toll on road 2 under the constraint that the user's cost on road 1 will not be higher to some extent than that in case 1.[9]

2. The model

The model is set up with a road providing the transportation supply between two cities. The inverse demand function for the transportation is denoted by $P(N)$,[10] where N is the traffic volume between the two cities. The government plans to invite a private firm to build a new road parallel to the current one to lessen the heavy traffic through a build-operate-transfer approach. That is, the private sector is allowed to build and operate the road at its own expense, and in return it receives the revenue from road toll charges within a set amount of years, and then the road is transferred to the government.

The current road is denoted by 1 and the new road is labeled 2. Users' travel cost is expressed by $C_i(N_i, K_i)$, for $i = 1, 2$, where N_i, K_i denote traffic volume and road capacity on road i, respectively. The road construction and maintenance cost is denoted by $C_{cmi}(K_i)$[11] for $i = 1, 2$. The firm's objective is to maximize its profit under various regulation constraints.

The consumers (road users) can choose one of the two roads for their trips. From Wardrop's first principle (Wardrop 1952), total travel costs on the two roads (including the toll on road 2) are the same in equilibrium; otherwise, road users would shift from one road to the other. That is, the average social cost on road 1 is equal to the average social cost on road 2 plus the fee. Furthermore, both should be equal to the marginal benefit. The toll on road 2 is thus equal to the difference between travel costs on road 1 and road 2. That is, $\tau = C_1(N_1, K_1) - C_2(N_2, K_2)$. Thus, the marginal road user's benefit equals his travel cost in equilibrium. That is, the marginal benefit, $P(N)$, equals the average social cost, $C_1(N_1, K_1)$.

Five cases with various situations are analyzed for comparison. In case 1, the government maximizes the social welfare given the road capacity of road 1 with a toll on road 2. In case 2, the government's objective is the same as in case 1, but with a constraint that the revenue collected from the toll on road 2 is equal to its construction and operating cost. That is, a break-even target[12] on public finance is pursued on road 2. In case 3, the firm then maximizes its profit with a toll on road 2 while road 1 is still owned by the government and there is no toll charge for using road 1. In case 4, the firm maximizes its profit with a toll on road 2 under the constraint that the total traffic volume of both roads may not be to a certain degree inferior to the total traffic volume of case 1. In case 5, the firm maximizes its profit with a toll on road 2 under the constraint that the user cost on road 1 will not be higher to some extent than that in case 1.

The five cases are modeled as follows:

2.1. *Case 1: No BOT (government maximizes social welfare)*

In this case the government maximizes social welfare given the road capacity of route 1 with a toll on route 2.

$$
\begin{aligned}
\max_{N_1, N_2, K_2} \quad & W = \int_0^N P(V)dV - N_1 C_1(N_1, K_1) - \\
& \qquad N_2 C_2(N_2, K_2) - C_{cm1}(K_1) - C_{cm2}(K_2) \\
s.t. \quad & P(N) = C_1(N_1, K_1) \\
& N_1 + N_2 = N \\
& N_1, N_2, K_2 \geq 0.
\end{aligned} \tag{1}
$$

In the formulation, $\int_0^N P(V)\mathrm{d}V$ equals the areas under the demand curve and denotes the total benefit of the road system. The terms $N_1C_1(N_1, K_1)$ and

$N_2C_2(N_2, K_2)$ are total road users' costs on road 1 and on road 2, respectively, while $C_{cm1}(K_1)$ and $C_{cm2}(K_2)$ are construction and operating costs for road 1 and for road 2, respectively.

The necessary first-order conditions for the problem can be expressed as follows:

$$L = W + \lambda[C_1(N_1, K_1) - P(N)] \tag{2}$$

$$\frac{\partial L}{\partial N_1} = P(N) - C_1(N_1, K_1) - N_1 \frac{\partial C_1(N_1, K_1)}{\partial N_1} + \lambda \left[\frac{\partial C_1(N_1, K_1)}{\partial N_1} - P'(N) \right] = 0 \tag{3}$$

$$\frac{\partial L}{\partial N_2} = P(N) - C_2(N_2, K_2) - N_2 \frac{\partial C_2(N_2, K_2)}{\partial N_2} - \lambda P'(N) = 0 \tag{4}$$

$$\frac{\partial L}{\partial K_2} = -N_2 \left[\frac{\partial C_2(N_2, K_2)}{\partial K_2} \right] - \frac{\partial C_{cm2}(K_2)}{\partial K_2} = 0 \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = C_1(N_1, K_1) - P(N) = 0. \tag{6}$$

Equation (2) is the Lagrange function for the problem, while equations (3) and (4) are the derivative of the Lagrange function with respect to N_1 and N_2 being nil, respectively. Equation (5) means that total road user's marginal cost with respect to capacity is equal to the amount of marginal production cost (construction cost and maintenance cost) on road 2. Equation (6) is equivalent to the user's behavior constraint. Let the solution of the problem[13] be N_1^*, N_2^*, K_2^*. The congestion toll on road 2 is thus equal to $P(N^*) - C_2(N_2^*, K_2^*)$, which is the benefit of the marginal road user minus his cost.

2.2. *Case 2: No BOT (government maximizes social welfare with a break-even on the finance of road 2)*

In this case the government maximizes social welfare given the road capacity of route 1 with a toll on route 2 and looks to break even on the public finance. That is, the revenue collected from the toll on route 2, τN_2, equals its construction and maintenance costs, $C_{cm2}(K_2)$.

$$\max_{N_1, N_2, K_2} W = \int_0^N P(V)dV - N_1 C_1(N_1, K_1) - N_2 C_2(N_2, K_2) - C_{cm1}(K_1) - C_{cm2}(K_2)$$

$$\begin{aligned} s.t.\quad & P(N) = C_1(N_1, K_1) \\ & \tau N_2 = C_{cm2}(K_2) \\ & N_1 + N_2 = N \\ & N_1, N_2, K_2 \geq 0. \end{aligned} \tag{7}$$

The toll, τ, could be substituted by the difference between the travel cost on road 1 and that on road 2, $C_1(N_1, K_1) - C_2(N_2, K_2)$. Thus, equation (7) can be reformulated as follows:

$$\max_{N_1, N_2, K_2} W = \int_0^N P(V)dV - N_1 C_1(N_1, K_1) - N_2 C_2(N_2, K_2) - C_{cm1}(K_1) - C_{cm2}(K_2)$$

$$\begin{aligned} s.t.\quad & P(N) = C_1(N_1, K_1) \\ & N_2(C_1(N_1, K_1) - C_2(N_2, K_2)) = C_{cm2}(K_2) \\ & N_1 + N_2 = N \\ & N_1, N_2, K_2 \geq 0. \end{aligned} \tag{8}$$

The necessary first-order conditions of the problem are:

$$L = W + \lambda[C_1(N_1, K_1) - P(N)] + \mu[C_{cm2}(K_2) - N_2(C_1(N_1, K_1) - C_2(N_2, K_2))] \tag{9}$$

$$\frac{\partial L}{\partial N_1} = P(N) - C_1(N_1, K_1) - N_1 \frac{\partial C_1(N_1, K_1)}{\partial N_1} + \lambda\left[\frac{\partial C_1(N_1, K_1)}{\partial N_1} - P'(N)\right] + \mu\left[-N_2 \frac{\partial C_1(N_1, K_1)}{\partial N_1}\right] = 0 \tag{10}$$

$$\frac{\partial L}{\partial N_2} = P(N) - C_2(N_2, K_2) - \lambda P'(N) - N_2 \frac{\partial C_2(N_2, K_2)}{\partial N_2} + \mu\left[C_2(N_2, K_2) + N_2 \frac{\partial C_2(N_2, K_2)}{\partial N_2}\right] = 0 \tag{11}$$

$$\frac{\partial L}{\partial K_2} = -N_2\left[\frac{\partial C_2(N_2, K_2)}{\partial K_2}\right] - \frac{dC_{cm2}(K_2)}{dK_2} + \mu\left[\frac{dC_{cm2}(K_2)}{dK_2} + N_2\frac{\partial C_2(N_2, K_2)}{\partial K_2}\right] = 0 \tag{12}$$

$$\frac{\partial L}{\partial \lambda} = C_1(N_1, K_1) - P(N) = 0 \tag{13}$$

$$\frac{\partial L}{\partial \mu} = C_{cm2}(K_2) - N_2(C_1(N_1, K_1) - C_2(N_2, K_2)) = 0 \tag{14}$$

Equations (10)–(13) are similar to those in case 1. Equation (14) is set up to ensure a break-even on the finance of road 2. The solutions satisfying the above equations are the result of maximizing welfare with a break-even on the finance of road 2.

2.3. *Case 3: BOT without regulation*

In this case the firm maximizes its profit by charging a user fee τ on road 2 without regulations. This case is a hypothesized one for comparison with other cases.

$$\begin{aligned} \max_{\tau, N_2, K_2} \pi &= \tau N_2 - C_{cm2}(K_2) \\ s.t. \quad & P(N) = C_1(N_1, K_1) \\ & (C_1(N_1, K_1) = C_2(N_2, K_2) + \tau \\ & N_1 + N_2 = N \\ & N_1, N_2, K_2 \geq 0. \end{aligned} \tag{15}$$

It can be reformulated as follows:

$$\begin{aligned} \max_{N_1, N_2, K_2} \pi &= [C_1(N_1, K_1) - C_2(N_2, K_2)]N_2 - C_{cm2}(K_2) \\ s.t. \quad & P(N) = C_1(N_1, K_1) \\ & N_1 + N_2 = N \\ & N_1, N_2, K_2 \geq 0. \end{aligned} \tag{16}$$

The necessary first-order condition can be expressed as follows:

$$\begin{aligned} L \;=\; & [C_1(N_1, K_1) - C_2(N_2, K_2)]N_2 - \\ & C_{cm2}(K_2) + \lambda[C_1(N_1, K_1) - P(N)] \end{aligned} \tag{17}$$

$$\frac{\partial L}{\partial N_1} = \frac{\partial C_1(N_1, K_1)}{\partial N_1}(N_2 + \lambda) - \lambda P'(N) = 0 \tag{18}$$

$$\frac{\partial L}{\partial N_2} = -C_2(N_2, K_2) - N_2\frac{\partial C_2(N_2, K_2)}{\partial N_2} - \lambda P'(N) = 0 \tag{19}$$

$$\frac{\partial L}{\partial K_2} = -N_2\frac{\partial C_2(N_2, K_2)}{\partial K_2} - \frac{dC_{cm2}(K_2)}{dK_2} = 0 \tag{20}$$

$$\frac{\partial L}{\partial \lambda} = C_1(N_1, K_1) - P(N) = 0 \tag{21}$$

$$\lambda, N_1, N_2, K_2 \geq 0.$$

Since the firm's goal is to maximize its profit, equations (18)–(20) describe the behavior of the firm and equation (21) is the behavior of road users.

2.4. *Case 4: BOT with a minimum flow constraint*

In this case the firm maximizes its profit by charging a user fee τ on road 2 under the constraint that the total users on the two roads will not be less than those in the case of maximizing social welfare (case 1) to some extent. This is done so as to secure the welfare of the consumers by keeping the users' number from decreasing too much. A tolerance ratio, ε, is employed to the model to reflect the context that the number of users might decrease. A higher value of the tolerance ration means a lower number of total road users allowed.

$$\begin{aligned} \max_{\tau, N_2, K_2} \pi \;=\; & \tau N_2 - C_{cm2}(K_2) \\ s.t. \quad & P(N) = C_1(N_1, K_1) \\ & C_1(N_1, K_1) = C_2(N_2, K_2) + \tau \\ & N_1 + N_2 \geq (N_1^* + N_2^*)(1 - \varepsilon) \\ & N_1 + N_2 = N \\ & N_1, N_2, K_2 \geq 0. \end{aligned} \tag{22}$$

This can be reformulated as follows:

$$\begin{aligned}
\max_{N_1,N_2,K_2} \pi \ &= \ [C_1(N_1, K_1) - C_2(N_2, K_2)]N_2 - C_{cm2}(K_2) \\
s.t. \quad & P(N) = C_1(N_1, K_1) \\
& N_1 + N_2 \geq (N_1^* + N_2^*)(1 - \varepsilon) \\
& N_1 + N_2 = N \\
& N_1, N_2, K_2 \geq 0.
\end{aligned} \tag{23}$$

The first-order condition (Kuhn-Tucker condition) can be expressed as follows (assume that the solutions for N_1, N_2, and K_2 are not zero):

$$\begin{aligned}
L \ = \ & [C_1(N_1, K_1) - C_2(N_2, K_2)]N_2 - C_{cm2}(K_2) + \\
& \lambda[C_1(N_1, K_1) - P(N)] + \mu[N_1 + N_2 - (N_1^* + N_2^*)(1 - \varepsilon)]
\end{aligned} \tag{24}$$

$$\frac{\partial L}{\partial N_1} = \frac{\partial C_1(N_1, K_1)}{\partial N_1}(N_2 + \lambda) + \mu - \lambda P'(N) = 0 \tag{25}$$

$$\frac{\partial L}{\partial N_2} = -C_2(N_2, K_2) - N_2\frac{\partial C_2(N_2, K_2)}{\partial N_2} - \lambda P'(N) + \mu = 0 \tag{26}$$

$$\frac{\partial L}{\partial K_2} = -N_2\frac{\partial C_2(N_2, K_2)}{\partial K_2} - \frac{dC_{cm2}(K_2)}{dK_2} = 0 \tag{27}$$

$$\frac{\partial L}{\partial \lambda} = C_1(N_1, K_1) - P(N) = 0 \tag{28}$$

$$\frac{\partial L}{\partial \mu} = N_1 + N_2 - (N_1^* + N_2^*)(1 - \varepsilon) \geq 0 \tag{29}$$

$$\mu\frac{\partial L}{\partial \mu} = \mu[N_1 + N_2 - (N_1^* + N_2^*)(1 - \varepsilon)] = 0 \tag{30}$$

λ, μ, N_1, N_2, $K_2 \geq 0$

Equations (29) and (30)[14] describe the tolerance ration on flow in order to ensure that the number of road users does not decrease too much due to the provision of the private sector. The other equations are similar to those in case 3.

2.5. *Case 5: BOT with a maximum travel time constraint*

In this case the firm maximizes its profit by charging a user fee τ on road 2 under the constraint that the user cost on road 1 will not exceed that in case 1 to some extent. This is done so as to secure the welfare of the consumers by keeping their travel costs from increasing too much. A tolerance ration, r, is employed to reflect the context that the user cost on road 1 might increase. A higher value of the tolerance ration means a lower service level on road 1 is allowed.

$$\begin{aligned}\max_{\tau, N_2, K_2} \pi \ &= \ \tau N_2 - C_{cm2}(K_2) \\ s.t. \quad & P(N) = C_1(N_1, K_1) \\ & C_1(N_1, K_1) = C_2(N_2, K_2) + \tau \\ & C_1(N_1 + K_2) \leq C_1(N_1^* + K_1^*)(1 + r) \\ & N_1 + N_2 = N \\ & N_1, N_2, K_2 \geq 0.\end{aligned} \tag{31}$$

This can be reformulated as follows:

$$\begin{aligned}\max_{N_1, N_2, K_2} \pi \ &= \ [C_1(N_1, K_1) - C_2(N_2, K_2)]N_2 - C_{cm2}(K_2) \\ s.t. \quad & P(N) = C_1(N_1, K_1) \\ & C_1(N_1, K_1) \leq C_1(N_1^*, K_1) + (1 + r) \\ & N_1 + N_2 = N \\ & N_1, N_2, K_2 \geq 0.\end{aligned} \tag{32}$$

The necessary first-order conditions can be expressed as follows (assume that the solutions for N_1, N_2 and K_2 are not zero):

$$\begin{aligned}L \ = \ & [C_1(N_1, K_1) - C_2(N_2, K_2)]N_2 - C_{cm2}(K_2) + \\ & \lambda[C_1(N_1, K_1) - P(N)] + \mu[C_1(N_1^*, K_1)(1 + r) - C_1(N_1, K_1)]\end{aligned} \tag{33}$$

$$\frac{\partial L}{\partial N_1} = \frac{\partial C_1(N_1, K_1)}{\partial N_1}(N_2 + \lambda - \mu(1 + r)) - \lambda P'(N) = 0 \tag{34}$$

$$\frac{\partial L}{\partial N_2} = -C_2(N_2, K_2) - N_2 \frac{\partial C_2(N_2, K_2)}{\partial N_2} - \lambda P'(N) = 0 \tag{35}$$

$$\frac{\partial L}{\partial K_2} = -N_2 \frac{\partial C_2(N_2, K_2)}{\partial K_2} - \frac{dC_{cm2}(K_2)}{dK_2} = 0 \tag{36}$$

$$\frac{\partial L}{\partial \lambda} = C_1(N_1, K_1) - P(N) = 0 \tag{37}$$

$$\frac{\partial L}{\partial \mu} = C_1(N_1^*, K_1)(1 + r) - C_1(N_1, K_1) \geq 0 \tag{38}$$

$$\mu \frac{\partial L}{\partial \mu} = \mu[C_1(N_1^*, K_1)(1 + r) - C_(N_1, K_1)] = 0 \tag{39}$$

$\lambda, \mu, N_1, N_2, K_2 \geq 0$

Equations (38) and (39)[15] describe the tolerance ration on the cost function to ensure that the user cost on road 1 does not increase too much due to the provision of the private sector. The other equations are similar to those in case 3.

3. Simulations and analyses

Due to the complexity of the analytical solutions, we apply simulation skills to these cases for analysis. The functional forms for the model are selected as follows:

Demand function: $N = Q_0 - bP$.

Travel cost: $C_i(N_i, K_i) = \bar{t}_i \left(1 + \alpha \left(\frac{N_i}{K_i}\right)^{\beta}\right)$ for$i = 1, 2$.

Road construction and maintenance cost: $C_{cm1}(K_1) = k_1 K_1$, $C_{cm2}(K_2) = k_2 K_2$.

For simplicity and without loss of generality, a linear form for the demand function and for the road construction and maintenance cost is employed in the simulation analysis. That is, the slope of the demand function is constant. This means the maximum traffic volume is Q_0 when the road users face a zero price and this traffic volume decreases b units when the price P (including the travel cost and toll) increases one unit. In addition, the marginal cost of road construction and maintenance in each route is constant.

The term K_1 denotes the road capacity of the existing road and K_2 denotes the road capacity of the new road (by BOT). The terms k_1 and k_2 denote the unit construction and maintenance cost for road 1 and road 2, respectively. The travel cost function used in the Urban Transportation Planning Package (UTPP) is employed[16] where C_i represents the travel cost for each route and is a function of the travel flows; $\overline{t_i}$ is the cost of free flow time for this trip (if there is no congestion on the road); α and β are the default setting parameters for the travel time cost functions. Therefore, the congestion effect is reflected in this function form.

3.1. *Basic analysis*

The basic values for the parameters are as follows:

$Q_0 = 7000$ and $b = 200$ denote the demand of transportation;
$\alpha = 0.15$ and $\beta = 4$ denote the typical user cost function;[17]
$\overline{t_1} = \overline{t_2} = 0.9652$ denote the travel cost without congestion;[18]

$k_1 = 1.824$,[19] and $K_1 = 2000$[20] denote the unit construction and maintenance cost and the road capacity for one line in each direction.

In addition, $k_2 = 0.9\ k_1$ for case 1 and case 2, $k_2 = 0.5\ k_1$ for case 3 to case 5, $\varepsilon = 0.005$ for case 4,[21] and $r = 1.0$ for case 5. The smaller value of unit construction and operating cost (k_2) for case 3 to case 5 reflects the cost efficiency of the private sector compared to the public sector. The formulation for case 1 could be expressed as follows:

$$\begin{aligned}\max_{N_1,N_2,K_2} W \ = \ & \int_0^N \left(\frac{Q_0}{b} - \frac{V}{b}\right) dV - N_1\bar{t}_1 \left(1 + \alpha\left(\frac{N_1}{K_1}\right)^{\beta}\right) - \\ & N_2\bar{t}_2 \left(1 + \alpha\left(\frac{N_2}{K_2}\right)^{\beta}\right) - k_1K_1 - k_2K_2 \qquad (40)\\ s.t. \quad & \frac{Q_0}{b} - \frac{N}{b} = \bar{t}_1 \left(1 + \alpha\left(\frac{N_1}{K_1}\right)^{\beta}\right)\\ & N_1 + N_2 = N\\ & N_1, N_2, K_2 \geq 0.\end{aligned}$$

The necessary first-order condition for the problem is expressed as follows:

$$\frac{Q_0}{b} - \frac{N}{b} - \bar{t}_1\left(1 + \alpha\left(\frac{N_1}{K_1}\right)^{\beta}\right) - (N_1 - \lambda)\bar{t}_1\alpha\beta N_1^{\beta-1}K_1^{-\beta} + \frac{\lambda}{b} = 0 \quad (41)$$

$$\frac{Q_0}{b} - \frac{N}{b} - \bar{t}_2\left(1 + \alpha\left(\frac{N_2}{K_2}\right)^{\beta}\right) - N_2\bar{t}_2\alpha\beta N_2^{\beta-1}K_2^{-\beta} + \frac{\lambda}{b} = 0 \quad (42)$$

$$N_2\bar{t}_2\alpha\beta N_2^{\beta}K_2^{-\beta-1} - k_2 = 0 \quad (43)$$

$$\frac{Q_0}{b} - \frac{N}{b} - \bar{t}_1\left(1 + \alpha\left(\frac{N_1}{K_1}\right)^{\beta}\right) = 0 \quad (44)$$

The simulation result of case 1 is: traffic volumes on road 1 and on road 2 are 2,500 vehicles per hour and 4,301 vehicles per hour, respectively, and the total traffic volume is 6,801 vehicles per hour; the road capacity of the new road (road 2) is 3,496 vehicles per hour; user costs on road 1 and road 2 are \$1.3286 and \$1.3061, respectively; the toll level on road 2 is thus \$0.0225 (the difference between the user costs on road 1 and on road 2); revenue minus cost on road 2 is –\$5,642 (i.e. the government has to subsidize \$5,642 on road 2); the welfare level is \$144,876.

The formulations of the other cases could be expressed similarly to case 1.[22] The simulation results of the five cases are shown in Table 1.

Table 1. Simulation results of basic analysis.

Case	Case 1	Case 2	Case 3	Case 4	Case 5
b	200	200	200	200	200
Effc	0.9	0.9	0.5	0.5	0.5
LoSc				0.05	1
N_1	2,514	3,678	4,574	3,886	3,732
N_2	4,219	2,794	1,434	2,503	2,720
$N_1 + N_2$	6,733	6,472	6,008	6,390	6,452
K_2	3,433	2,271	1,310	2,288	2,486
C_1	1.3364	2.6403	4.9617	3.0520	2.7409
C_2	1.3061	1.3061	1.1809	1.1809	1.1809
τ	0.0303	1.3342	3.7808	1.8711	1.5600
π_2	–5,502	0	4,226	2,597	1,975
W	104,174	101,067	90,808	101,016	102,392

Effc = k_2/k_1 = 0.9 for case 1 and case 2; Effc = k_2/k_1 = 0.5 for case 3 to case 5, LoSc = ε = 0.05 for case 4, LoSc = r = 1.0 for case 5.

The table lists the traffic volumes (road users) on road 1 and road 2 (N_1, N_2), total traffic volume ($N_1 + N_2$), road capacity of road 2 (K_2), user cost on road 1 and road 2 (C_1, C_2), toll on road 2 (τ), the profit from constructing and operating road 2 (π_2), and the welfare (W).

From these results, some findings are demonstrated as follows:

1. Under the goal of maximizing welfare (case 1), the toll on road 2 reflects the real cost on users. The toll collected from road users on road 2 is less than the construction and operating costs. That is, the government has to subsidize road 2 if it wishes to pursue welfare maximizing under this set of parameters.
2. To obtain a break-even on finance for road 2 and to maximize welfare (case 2), it is necessary to have a low unit cost of constructing and operating road 2. That is, a high unit cost of construction and operation will yield no feasible solutions.
3. When pursuing maximum profits without any regulation (case 3), the private firm will provide a small road capacity for road 2. This pushes the road users to road 1 and results in a high toll, a high user cost, and a low welfare level.
4. BOT with regulation (case 4 and case 5) obtains the results between case 1 (maximizing welfare) and case 3 (maximizing profit). That is, it provides a larger capacity of road 2 (thus a larger number of road users), but a smaller user cost (and thus smaller tolling) with a smaller positive profit compared to case 3. The tolerance ration in these two cases plays an important role (see the analysis in 3.2 section).
5. Comparing the five cases, the result of case 1 yields the largest traffic volume (road users) on road 2, the largest total traffic volume, the largest capacity of road 2, the lowest tolling on road 2, the lowest profit level (on road 2), and the highest welfare. On the contrary, case 3 yields the smallest traffic volume (road users) on road 2, the smallest total traffic volume, the smallest capacity of road 2, the highest toll on road 2, the highest profit level (on road 2), and the lowest welfare.

3.2. *Analysis of various costs and tolerance rations*

In order to compare the results of various economic environments and various regulations, the changes in (1) construction and operating costs and (2) tolerance ration on the traffic flow constraint in case 4 and that on user cost on road 1 in case 5 are analyzed. For the changes in construction and operating cost, k_2 equals 0.8 k_1 for case 1 and case 2 and k_2 equals 0.2 k_1 for case 3 to case 5. This reflects the cost reduction on constructing and operating road 2 and a higher efficiency from the private sector. For the changes in the tolerance ration, ε changes from 0.005 to 0.001 for case 4 to reflect a stricter regulation on total traffic flow requirement, and r changes from 1.0 to 0.5 for case 5 to reflect a better level of service regulation from a user's cost perspective. The results are shown in Table 2.

From these results, some findings are demonstrated as follows:

1. When construction and operating costs decrease, total traffic volume (road users) increases, capacity of road 2 increases, user cost decreases, tolling on road 2 decreases, profits (subsidies) on road 2 increase (decrease), and welfare increases in every case.
2. When the government sets a strict regulation (a smaller tolerance ration on total flow in case 4 or user cost of road 1 in case 5), the private firm might incur a negative profit and cause the BOT project to fail. However, a loose regulation (a larger tolerance ration) will degenerate the results of case 4 and case 5 to case 3 (BOT without regulation). That is, a high tolerance ration will have no confining power to protect the consumers.
3. For case 4, only a small range for ε will obtain a feasible solution that is different from the result of case 3. However, the range of tolerance ration (r) in case 5 is larger than that in case 4. It seems that the regulation on user cost has a larger range to set the tolerance ration than that on total traffic flow.
4. The BOT project might succeed if the private firm has high efficiency (low building and operating costs, k_2) due to cost reduction and a positive profit even if the regulation is strict. From another perspective, the government could set up a stricter regulation to protect road users and increase the welfare when the private firm has higher efficiency.

Table 2. The impact of efficiency and tolerance factor on variables.

Case	Case 1	Case 2	Case 3	Case 4		Case 5	
b	200	200	200	200	200	200	200
Effc	0.8	0.8	0.2	0.2	0.2	0.2	0.2
LoSc				0.01	0.005	1	0.5
N_1	2,451	3,592	4,476	2,996	2,807	3,732	3,302
N_2	4,288	2,910	1,598	3,663	3,885	2,720	3,287
$N_1 + N_2$	6,739	6,502	6,074	6,659	6,692	6,452	6,589
K_2	3,569	2,421	1,754	4,022	4,267	2,987	3,610
C_1	1.3016	2.4904	4.6318	1.7068	1.5386	2.7409	2.0557
C_2	1.2761	1.2761	1.0726	1.0726	1.0726	1.0726	1.0726
τ	0.0285	1.2143	3.5592	0.6342	0.4660	1.6684	0.9831
π_2	–5,098	0	5,046	855	254	3,448	1,915
W	104,811	102,040	93,621	108,051	108,573	103,865	106,800

Effc = k_2/k_1 = 0.8 for case 1 and case 2, Effc = k_2/k_1 = 0.2 for case 3 to case 5, LoSc = for ε case 4, and LoSc = r for case 5.

3.3. *Analysis of various elasticities of demand*

To compare the results for various demand, the parameter of the slope of demand function, b, is changed. Parameter b reflects the elasticity of demand,[23] i.e., the elasticity of demand increases as b increases. We explore the impact of various elasticities of demand on traffic volumes, road capacity, user cost, tolling, and welfare. The simulation results for various values of b are shown in Table 3 (b = 150) and Table 4 (b = 250).

Table 3. Simulation results of low elasticity of demand.

Case	Case 1	Case 2	Case 3	Case 4	Case 5
b	150	150	150	150	150
Effc	0.9	0.9	0.5	0.5	0.5
LoSc				0.05	1
N_1	2,500	3,678	4,730	4,131	3,723
N_2	4,301	2,926	1,440	2,325	2,869
$N_1 + N_2$	6,801	6,604	6,170	6,456	6,592
K_2	3,496	2,378	1,316	2,125	2,623
C_1	1.3286	2.6403	5.5344	3.6268	2.7230
C_2	1.3061	1.3061	1.1809	1.1809	1.1809
τ	0.0225	1.3342	4.3535	2.4458	1.5421
π_2	–5,642	0	5,069	3,749	2,032
W	144,876	141,726	128,311	139,033	143,213

Effc = k_2/k_1 = 0.9 for case 1 and case 2; Effc = k_2/k_1 = 0.5 for case 3 to case 5, LoSc = ε = 0.05 for case 4, LoSc = r = 1.0 for case 5.

Table 4. Simulation results of high elasticity of demand.

Case	Case 1	Case 2	Case 3	Case 4	Case 5
b	250	250	250	250	250
Effc	0.9	0.9	0.5	0.5	0.5
LoSc				0.05	1
N_1	2,527	3,678	4,447	3,716	3,742
N_2	4,137	2,662	1,418	2,606	2,568
$N_1 + N_2$	6,664	6,340	5,865	6,322	6,310
K_2	3,363	2,164	1,296	2,382	2,348
C_1	1.3442	2.6403	4,5390	2.7108	2.7796
C_2	1.3061	1.3061	1.1809	1.1809	1.1809
τ	0.0381	1.3342	3.3581	1.5299	1.5787
π_2	–5,362	0	3,580	1,814	1,913
W	79,806	76,741	68,734	78,109	77,900

Effc = k_2/k_1 = 0.9 for case 1 and case 2; Effc = k_2/k_1 = 0.5 for case 3 to case 5, LoSc = ε = 0.05 for case 4, LoSc = r = 1.0 for case 5.

The impacts on the above variables for various slopes of demand, *b*, for the five cases are shown from Figure 1 to Figure 6.

From these results, some findings are demonstrated as follows:

1. When the elasticity of demand decreases[24] (*b* decreases), traffic volumes (road users) on each road increase and thus the total traffic volume increases, the capacity of road 2 increases, and welfare increases in every case.

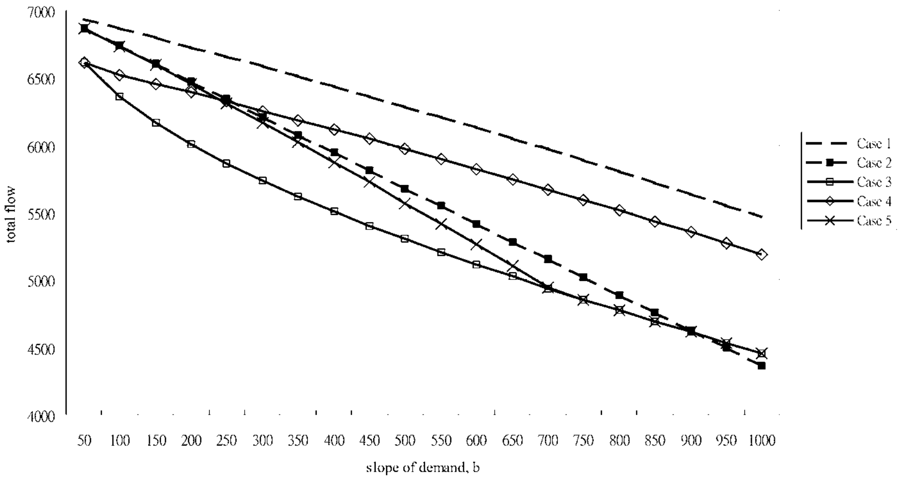

Figure 1. Total flow with various slopes of demand.

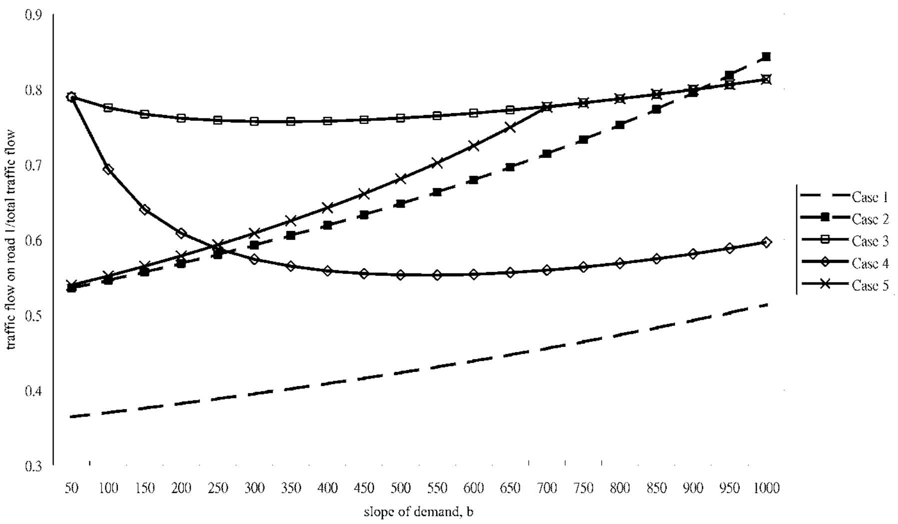

Figure 2. Flow ratio with various slopes of demand.

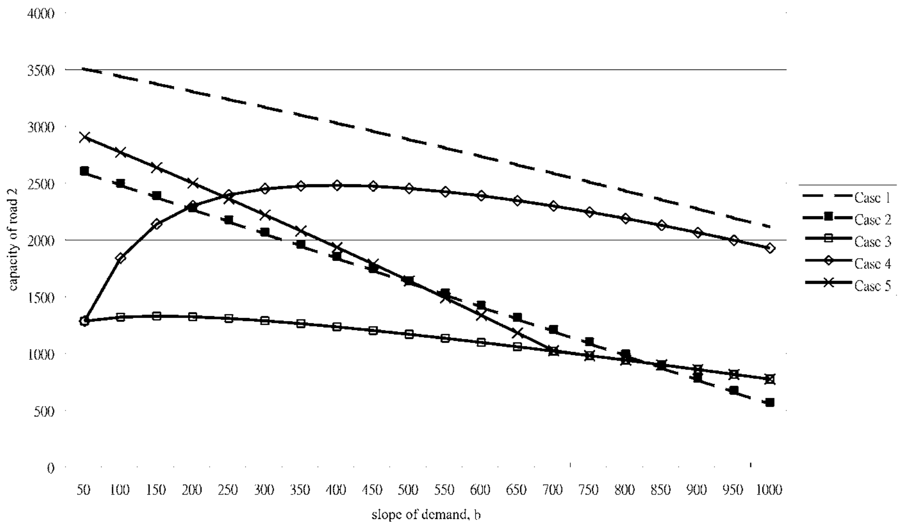

Figure 3. Capacity of road 2 with various slopes of demand.

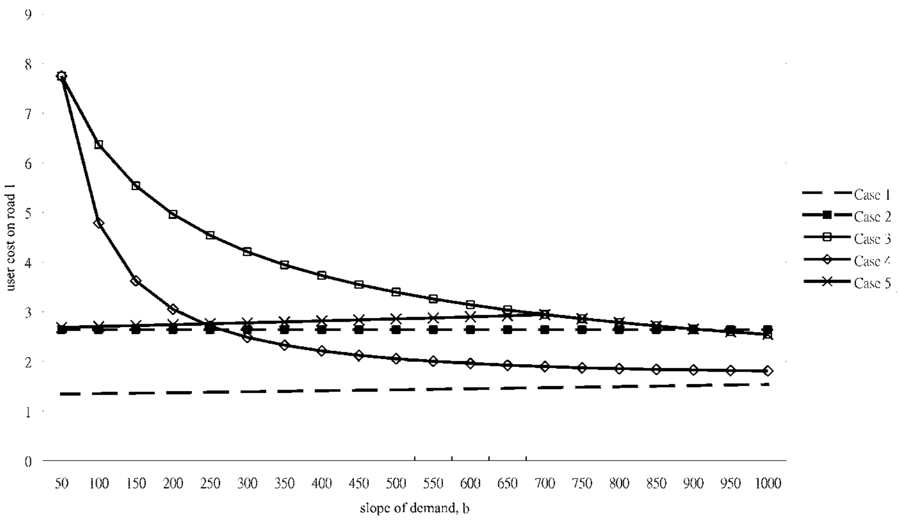

Figure 4. User cost on road 1 with various slopes of demand.

2. When the elasticity of demand decreases (b decreases), the subsidy on road 2 increases in case 1; and the profit level on road 2 increases in case 3 to case 5. At the same time, user travel cost on road 1 increases in case 1 and case 3, remains unchanged in case 2, and is ambiguous in case 4 and case 5 due to the regulation, while user travel cost on road 2 (not including the toll) stays unchanged in every case. The tolling on road 2 thus

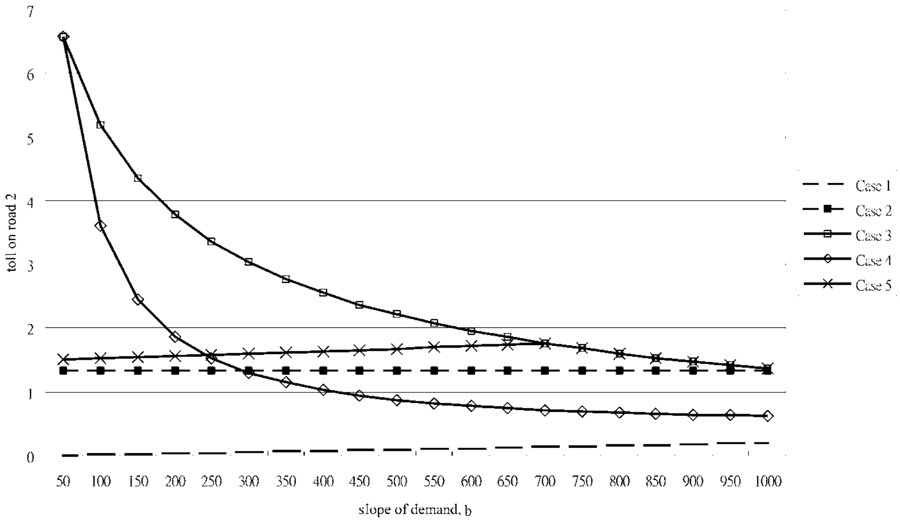

Figure 5. Toll on road 2 with various slopes of demand.

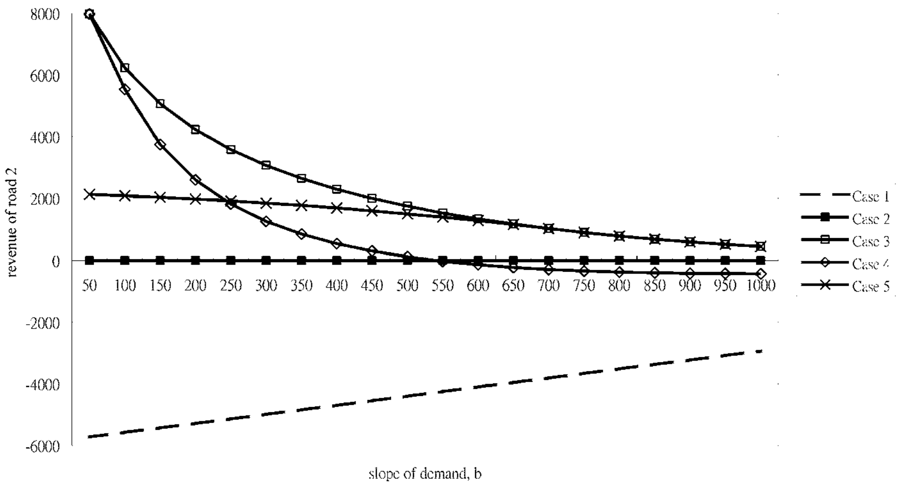

Figure 6. Revenue of road 2 with various slopes of demand.

increases in case 1 and case 3, is unchanged in case 2, and is not certain in case 4 and case 5 due to the tolerance ration that the government regulates.

3. When the elasticity of demand increases (b increases), the results almost show the phenomenon in reverse to those above. However, the feasible solution is more difficult to obtain in case 2 (break-even on finance for road 2).

4. The results of case 4 and case 5 are closer to that of case 3 in Table 3 than in Table 4. That is, government regulation has less power in situations with low elastic demand.

IV. Conclusions

This article explores the regulation alternatives on private highway investment under a build-operate-transfer scheme. The impact of various circumstances on traffic flow (number of road users), users' cost, profit of the private firm, and the welfare level are all explored. For comparison, five cases are analyzed: (1) No BOT while maximizing social welfare, (2) No BOT while breaking even on finance, (3) BOT without regulation, (4) BOT with a minimum flow constraint (ensuring the numbers of users), and (5) BOT with a maximum travel time constraint (ensuring the upper bound of user costs). Each case is modeled and simulated on some functional forms.

Some important finding are summarized as follows:

1. The case of BOT with regulations performs between the cases of maximizing welfare and that of maximizing profit.
2. A smaller tolerance ration might incur a negative profit and cause the BOT project to fail, but a higher tolerance ration might have no confining power to protect the consumers.
3. Government regulation has less power in situations with low elastic demand.
4. A high efficiency level in the private sector could result in a higher level of welfare than that without BOT even when the regulation is strict.

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Notes

1. Walters (1961), Broadman and Lave (1977), Inman (1978), Kraus (1982), Small (1983), and Tsai and Jeng (1999) are some examples employing this approach. See Johansson and Mattsson (1995) for a detailed survey.
2. A combination of a demand approach and a supply approach is commonly employed in some cities to improve the congestion problems in an indirect way such as parking policies, increase in transit supply, and Park&Ride facilities as from a referee's suggestion.
3. Mohring and Harwitz (1962), Vickery (1969), Keeler and Small (1977), Wilson (1983), D'Ouville and McDonald (1990), and Gronau (1994) are some examples along this line.

4. Mills' analysis focuses on the change of annual welfare, which has three components: the annual road cost, each journey's time saving value, and the net value of its generated traffic.
5. One example is the SR 91 Expressway in Orange County, California in the US. This is a privately-built, privately-operated tollway on the median strip of SR 91.
6. We discuss only the peak period in this paper. However, our model could be extended to incorporate the low-flow situation by utilizing the concept of two routes and two time-periods model as in Liu and McDonald (1999). In this stage we like to keep our model this way so as to avoid having the analysis too complicated and also to have more insight into the impacts of BOT regulation.
7. The reason for not considering the option "toll on existing road 1 and no construction of new road 2" is mainly due to political feasibility. When the demand for the transportation is high, then keeping the existing toll on only road 1 will be difficult to be accepted by the public. Thus, constructing a new road with a toll is more politically feasible, because road users on the existing road could still use the road without a toll and users preferring a higher speed could use the new road with a toll.
8. This form of welfare could be seen in Small (1983), Verhoef et al. (1995), McDonald (1995), and Liu and McDonald (1999).
9. The reason for the formulation in case 5 in this way is that it addresses the user costs of road 1 only and not the user costs of road 2 and is for comparison purposes. The user costs of road 2 are dependent on the toll. Thus, the user costs of road 1 are a better candidate for the comparison especially on various simulation results. Note that the total user costs on both roads are equal in equilibrium.
10. Due to this demand function being elastic, the induced traffic resulting from the construction of the new road will be reflected in the demand from the lower user cost.
11. The cost here is obtained by converting the total construction cost into a daily base plus the daily maintenance cost. That is, the time period in the cost function is consistent with that in the demand function.
12. From footnote 11, the impact of the interest rate could be added to adjust the time period in order to obtain the break-even target.
13. The solution should satisfy the first-order conditions. In the simulation analysis, we use mathematical package Matlab © 5.3 for the calculations.
14. Equation (30) is the complementary slackness condition. It means that at least one of the two terms, μ and $\partial L/\partial\mu$, is zero.
15. The role of equation (39) is the same as equation (30).
16. This form is attributed by Kraus et al. (1976) to Solow and Vickery (1971); see Branston (1976), for a detailed account of history and theory.
17. These values for the parameters are used in many research studies, such as Liu and McDoland (1999).
18. Travel cost lies between \$2.25 and \$4.50 per vehicle-hour in 1972 (see Keeler & Small 1977, p. 441). A reasonable conversion to the present obtains the value between \$0.1621 and \$0.3241 per vehicle-minute. Our travel cost parameter takes the value in-between for four minutes.
19. The value for unit construction and maintenance cost is based on Keeler and Small (1977).
20. This is a typical value of road capacity for one lane (see Liu & McDonald 1999, for instance).
21. A higher value for ε will result in no confining power on the firm. That is, the minimum flow constraint will not be binding for a higher value of ε.
22. However, we have omitted these formulations to prevent the paper from being tediously too long.
23. The change on parameter b is not exactly the measure of elasticity of demand, but the direction of change is the same.

24. In economic terms, the elasticity takes the absolute value. Therefore, a high elasticity of demand means the percentage change of demand (decrease) is larger than the percentage change of price (increase).

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