ELSEVIER

Electrical Power and Energy Systems 23 (2001) 213–218

ELECTRICAL POWER & ENERGY SYSTEMS

www.elsevier.com/locate/ijepes

A game-theoretical model of private power production

W. Xing*, F.F. Wu

Center for Electrical Energy Systems, Department of Electrical & Electronic Engineering, Pokfulam Road, The University of Hong Kong, Hong Kong SAR, People's Republic of China

Received 22 March 2000; revised 3 July 2000; accepted 24 July 2000

Abstract

Private power production has sprung up all over the world. The build–operate–transfer (BOT) arrangement has emerged as one of the most important options for private power production, especially in developing countries with rapidly growing demand and financial shortages. Based on oligopoly theory, the paper proposes a Stackelberg game model between a BOT investor and an electric utility whereby they can negotiate a long-term energy contract. Asymmetric pricing schemes are taken into account such that a host utility purchases electricity from a BOT company at its "avoided cost", and sells its electricity to end users at its "average cost". Our Stackelberg game model is transferred into a two-level optimization problem, and then solved by an iterative algorithm. The game model is demonstrated by an illustrative example. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Private power production; Build–operate–transfer; Game theory; Two-level optimization

1. Introduction

The large amount of private power production has emerged as worldwide electrical power industry entering into a new era — deregulation. As a result of rapidly growing demands and a shortage of financial investment by governments, private power production is encouraged in the traditionally monopolistic power industry in developing countries, among which the build–operate–transfer (BOT) arrangement is becoming the most popular option, such as the 700 MW Shajiao-B power stations in China, the 1200 MW Hab River project in Pakistan, the 300 MW coal-fired projects in Philippines and the 1000 MW Aliaga project in Turkey [1–3].

A BOT arrangement is one where a consortium of private companies finance, build and operate an infrastructure project for a relative long specific period, and at the end of this concessionary period, when all the estimated investment costs have been recouped and a profit returned, the project title is transferred from the private consortium to the host. The long-term nature of the BOT arrangement gives rise to a number of concerns regarding future uncertainties and risks facing the BOT investors and the host [4,5]. To mitigate against future risks, BOT investors usually ask for an important contract, an "energy contract", which stipulates how much energy from the BOT power plant is to be delivered, at what price and during what time periods. In the BOT energy contract, a minimal annual energy from the BOT power plant must be guaranteed for delivery, by which the fixed investment cost and operating cost of a BOT power plant will be paid back.

In this paper the amount of annual energy and the energy price for a BOT power plant are assumed to be fixed, and from the point of view of oligopoly theory, a Stackelberg game model is proposed to describe the interaction between a BOT investor and a host utility in the BOT energy contract negotiation. In addition, asymmetric pricing schemes are taken into account, whereby the host utility purchases electricity from the BOT power plant at a rate of "avoided cost" but itself sells its electricity to the end users at the average cost. In fact, if both the BOT power plant and the host utility sell their electricity at their respective marginal costs, the social welfare optimal will be reached at the same time. Different pricing schemes make the bargaining more complicated.

Game theory is a discipline that is used to analyze problems of conflict among interacting decision makers. Game theory has already been used for the analysis of electricity pricing and bargaining [6–11] in recent years. For example, Ruusunen et al. [6] applied cooperative game theory to analyze electricity exchange in a power pool, and a two-level hierarchical algorithm was proposed to solve the problem. Haurie et al. [7] modeled the interaction

---

* Corresponding author. Tel.: +852-2857-8422; fax: +852-2548-8532.
E-mail address: wgxing@eee.hku.hk (W. Xing).

PII: S0142-0615(00)00059-4

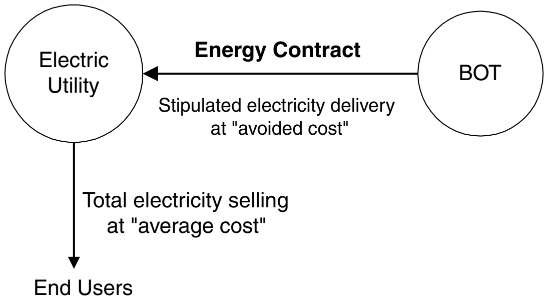

Fig. 1. Energy contract between a utility and a BOT power plant.

between a utility company and electricity cogenerators via a game-theoretic, systems analysis approach, and a bilevel optimization technique was developed to compute the equilibrium.

In the applications of game theory, oligopoly theory has recently been applied more and more because it has been realized that power markets are no more perfectly competitive than oligopolistic markets. Obviously the game between a BOT investor and a host utility is the very case where the utility owns a leadership and a BOT power plant is a price taker.

In addition, most of the electricity pricing and bargaining models in the literature have mainly focused on real-time operation, and then discussed game players' behaviors. The BOT arrangement is quite different; the model and analysis must involve long-term effects on the operation and planning of a power system. Therefore in our game model long-term generation expansion planning (GEP) is adopted as a suitable tool to evaluate a BOT power project [4].

The remainder of the paper is arranged as follows: in Section 2 the game model is presented. In Section 3 the breakeven cost is explained. In Section 4 the game model is transferred to a two-level optimization problem. In Section 5 an iterative algorithm is developed to solve this optimization problem. In Section 6 the game model is applied to an illustrative example, and some analyses are conducted. Section 7 concludes the paper.

2. Energy contract and game model

Criticisms often arise about BOT projects in developing countries after the projects are in operation; these relate to their high electricity prices and/or annual energy outputs that are agreed upon by the host utilities, and the effect of a BOT energy contract on long-term system planning and scheduling. Therefore, a rational decision-support model that can be used to analyze the efficiency of the contract is needed. In this paper, oligopoly theory is used for the negotiation and bargaining of a BOT energy contract.

We use a simplified model that involves a single BOT power plant and a single utility. An energy contract between them is needed to negotiate. The BOT power plant can only sell its electricity to the host utility company, and the host utility company has an obligation to accept it at a rate of "avoided cost". This kind of avoided cost should include the capacity and energy cost savings of the utility, and thus long-term marginal cost is a closer concept. Here, the breakeven cost proposed in Ref. [4] is adopted as the "avoided cost" because it involves the effect of a BOT power plant on the long-term GEP. It is obvious that the breakeven cost and the annual energy of a BOT power plant are interrelated, and an equilibrium (breakeven cost, annual energy) for a BOT energy contract should be agreed upon.

Actually the host utility company dominates the electricity market where it sells all its electricity to end users at a price of "average cost". The relationship between a utility and a BOT power plant is illustrated in Fig. 1. Here we simplify the scenario by assuming enelastic demands that are free of uncertainties (some of which in fact can be taken into account in long-term GEP, such as demand uncertainty).

This BOT arrangement can be modeled as a Stackelberg game that can be depicted as follows.

Notations:

n	a year in horizon period
N	horizon period of GEP
D^n	demand in year n
L^n	peak load in year n
R^n	system reserve in year n
Q_{BOT}	minimal annual electricity delivery from the BOT power plant stipulated in the energy contract
Q_{U}^n	total electricity supply of the utility in year n
X_{U}^n	accumulative capacity of the utility until year n
X_{BOT}^n	accumulative capacity of the BOT power plant until year n
$\mathrm{FC}_{\mathrm{BOT}}$	fixed investment cost of the BOT power plant
$\mathrm{VC}_{\mathrm{BOT}}$	variable cost of the BOT power plant
$\mathrm{FC}_{\mathrm{U}}^n$	fixed investment cost of the utility in year n
$\mathrm{VC}_{\mathrm{U}}^n$	variable cost of the host utility in year n
P_{B}	breakeven cost for BOT's electricity
P_{A}	average cost of the host utility
T_{BOT}	lifetime of the BOT power plant
C_0	total cost of GEP without the BOT contract
C_{BOT}	total cost of GEP with the BOT contract

Payoff function:

Utility:

$$f_{\mathrm{U}} = \sum_{n=1}^{N} [D^n \times P_{\mathrm{A}} - (\mathrm{FC}_{\mathrm{U}}^n + \mathrm{VC}_{\mathrm{U}}^n \times Q_{\mathrm{U}}^n)] - N \times Q_{\mathrm{BOT}} \times P_{\mathrm{B}} \tag{1}$$

BOT:

$$f_{\mathrm{BOT}} = N \times Q_{\mathrm{BOT}} \times P_{\mathrm{B}} - \sum_{n=1}^{N} (\mathrm{FC}_{\mathrm{BOT}}^n + \mathrm{VC}_{\mathrm{BOT}}^n \times Q_{\mathrm{BOT}}) \tag{2}$$

Based on the asymmetric pricing schemes, each player wants to maximize its payoff. The BOT power plant proposes the amount of annual energy output, then the utility will agree to a price based on the breakeven cost concept. Due to the correlation between the annual energy and price, the BOT will change its annual energy output to look for maximum payoff. Thus, the game between a utility and a BOT power plant is a Stackelberg game. After interaction between the two players in the game, a Nash equilibrium point (P_B,Q_{BOT}) can be solved.

When a utility determines the price for a BOT's electricity, some constraints must be considered. The utility has the obligation to meet future demand and system security.

Constraints:

1. Energy balance

$$D^n = Q_{BOT} + Q_U^n \qquad n = 1, 2, ..., N \tag{3}$$

For each year, the total amount of electricity generated by the BOT power plant and the host utility should be exactly equal to the total demand. The detailed operational scheduling is not taken into account in our model even though it is a tough problem.

2. Capacity requirement

$$X_U^n + X_{BOT}^n \geq L^n + R^n \qquad n = 1, 2, ..., N \tag{4}$$

In order to meet the demand each year, adequate system capacity is required. In addition, to maintain a certain degree of system security, a system reserve margin of capacity must be kept. The utility can make full use of the BOT capacity.

3. Other constraints

In many cases, there are limits on investments for both the BOT investor and the host utility, and sometimes these constraints will have a big effect on the decision-making progress of each player. Nevertheless, for simplicity, these factors are not considered in this paper.

3. Definition of breakeven cost

Usually in developing countries electric utilities price their electricity based on their average costs, but in order to encourage private investment, an "avoided cost" pricing mechanism is adopted. This kind of asymmetric pricing phenomenon makes the problem complicated.

The breakeven cost for a BOT power plant is defined as follows:

$$P_B = \text{Breakeven cost} = \frac{C_0 - C_{BOT}}{T_{BOT} Q_{BOT}} \tag{5}$$

The breakeven cost implies that the host utility purchases the BOT's electricity in such a way that the utility's total generating cost in the horizon period of GEP should not change before and after the entry of the BOT power plant. Of course, the breakeven cost is the basic cost for the BOT power plant without a returned profit. Moreover, the utility usually makes some changes to the breakeven cost according to corresponding policies in order to attract private investors.

To calculate the breakeven cost for the BOT, GEP must be performed, and the original GEP problem will be complicated by the BOT's constraints. To facilitate the inclusion of constraints introduced by BOT plants in GEP, a genetic algorithm (GA) approach is utilized in GEP. The implementation of a long-term GEP with BOT electricity and the calculation of breakeven cost have been developed and discussed in our previous paper [4].

4. A two-level optimization formulation

With the game model in hand, our interest is in the equilibrium (Q_{BOT}^*,P_B^*) for negotiating the energy contract. In this game, each player is concerned with the maximization of its benefit, i.e. each one maximizes its payoff function under the constraints. From the first order of necessary conditions, each palyer should sell its electricity at its respective marginal cost. Moreover, the two objective functions can be combined into a global payoff function, and an optimal social welfare will be obtained. In fact, the electricity transaction between the BOT power plant and the utility will be cancelled out in the global payoff function. Here, we transfer our game model to a two-level optimization problem for computing the equilibrium.

A Stackelberg game is a dynamic game model in which a dominant (or leader) firm moves first and a subordinate (or follower) firm moves second. It is straightforward to extend what follows to allow for more than one 'follower' firm. Usually static Stackelberg problems can be treated as a class of multi-level optimization problems.

However, the price coupling mechanism is endogenous and requires the evaluation of both the average and the breakeven cost of the utility, so that common bilevel optimization techniques are difficult to apply directly to the present problem. Therefore we restrict the equilibrium analysis of our model under the constraints of two existing pricing schemes, and formulate our game model as a two-level optimization problem as follows.

Objective: GEP with buying BOT electricity at breakeven cost

s.t.

$$\max f_{BOT}$$

$$D^n = Q_{BOT} + Q_U^n \qquad n = 1, 2, ..., N$$

$$X_U^n + X_{BOT}^n \geq L^n + R^n \quad n = 1, 2, ..., N$$

In our game model, the utility wants to maximize its net income by selling its electricity at average cost, thus it should behave as if it only wants to minimize the total cost to satisfy the demand without any consideration of

Table 1
Existing plant (15 units)

Plant type	Energy cost ($/MWh)	Max. capacity (MW * No.)
Oil #1 (heavy oil)	24	200 * 1
Oil #2 (heavy oil)	27	200 * 1
Oil #3 (heavy oil)	30	150 * 1
LNG G/T #1	43	50 * 3
LNG C/C #1	38	400 * 1
LNG C/C #2	40	400 * 1
LNG C/C #3	35	450 * 1
Coal #1 (anthracite)	23	250 * 2
Coal #2 (bituminous)	19	500 * 1
Coal #3 (bituminous)	15	500 * 1
Nuclear #1	5	1000 * 1
Nuclear #2	5	1000 * 1

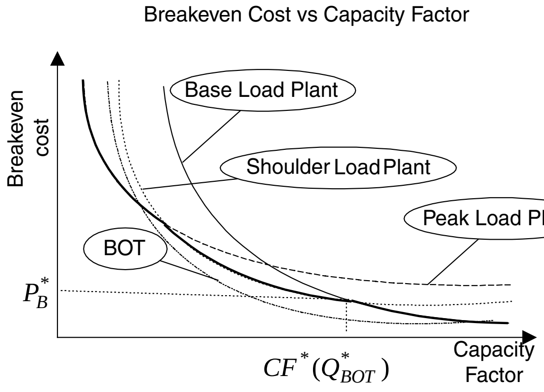

Fig. 2. Breakeven cost as a function of capacity factor.

Table 2
Proposed generation plant (60 units)

Plant type		Energy cost ($/MWh)	Capacity cost ($/kW)	Max. capacity (MW * No.)
1	Nuclear (PHWR)	3	1750.0	700 * 3
2	Nuclear (PWR)	4	1625.0	1000 * 3
3	Coal	14	1062.5	500 * 18
4	Oil	21	812.5	200 * 18
5	LNG C/C	35	500.0	450 * 18

the electricity sale revenues. Therefore long-term GEP is suitable for the utility to evaluate a BOT power plant, and the breakeven cost is calculated to price the electricity of a BOT power plant.

To calculate the equilibrium (Q_{BOT}^*,P_{B}^*), the relationship between Q_{BOT} and P_{B} should be examined. Let us look at a simple case where there are only three types of existing power plants existing and one allowing for future expansion, whose average cost curves are illustrated in Fig. 2.

The average cost of a plant is equal to its fixed (capital) cost per unit of energy production summed with its variable (operating) cost, and may be computed according to the following expression:

$$\text{average cost} = \text{opcost} + \frac{\text{capital cost}}{\text{annual energy}}$$

$$= \text{opcost} + \frac{\text{capital cost}}{\text{capacity factor} \times 8760} \qquad (6)$$

where opcost is the plant's operating cost, and the capacity factor of the BOT power plant stands for the percentage of its energy output with respect to its capacity.

Furthermore, it is assumed that the capacity of each type of power plant can be added continuously in GEP. Then it is easy to get the breakeven cost versus capacity factor curve, i.e. the black curve in Fig. 2. It can be explained that the breakeven cost will be equal to the average cost of peak load plants if the capacity factor of the BOT power plant is smaller, and equal to the average cost of base load plants if it is larger.

We further assume that a BOT power plant is a shoulder load plant with cheaper fixed and variable cost than normal shoulder load plants, and its average cost is shown in Fig. 2. Then the equilibrium point is easy to find for this simple case, and it can be proven that the equilibrium point is unique. On comparison with the slope of the BOT curve, the slope of the breakeven cost curve is smaller at the left of the equilibrium point and larger at the right. This property can be used as a criterion to search for the equilibrium point. Of course, calculated by practical GEP such as in Ref. [4], the breakeven cost versus capacity factor curve will be much more complicated, but the shape of the curve should be similar.

5. Equilibrium computation

Based on the above observation, an iterative algorithm for computing the equilibrium is developed.

For the master optimization problem, a long-term GEP which integrates BOT constraints is used to calculate the

Table 3
Load duration curve

Interval	Peak load (MW)	Base load (MW)	$L^{-1}(x) = (x-d)^2/c$	
			$c \times 10^3$	$d \times 10^3$
Present	5000	2500	0.285	5

BOT breakeven cost. For the slave optimization problem facing the BOT power plant, a Newton-like solution is used in our algorithm. For convenience and clarity, the annual energy of the BOT power plant is replaced by its capacity factor in the description of our algorithm.

```
Step 1: Set $i = 0$.
    Choose initial $\mathrm{CF}^0_{\mathrm{BOT}}$ to its biggest capacity factor (because a BOT power plant usually wants to generate as much energy as possible).

Step 2: For the given $\mathrm{CF}^i_{\mathrm{BOT}}$, calculate breakeven cost $P^i_{\mathrm{B}}$ and its left-hand and right-hand local derivatives, $S_{\mathrm{B}}^{i^-}$ and $S_{\mathrm{B}}^{i^+}$, with respect to $\mathrm{CF}_{\mathrm{BOT}}$ using the approach in Ref. [4].

Step 3: For the given $\mathrm{CF}^i_{\mathrm{BOT}}$, calculate the average cost $\mathrm{AC}^i_{\mathrm{BOT}}$ and local derivative $S^i_{\mathrm{BOT}}$ with respect to $\mathrm{CF}_{\mathrm{BOT}}$ on the BOT average cost curve (which is first-order smooth).

Step 4: If $S_{\mathrm{B}}^{i^-} \le S^i_{\mathrm{BOT}} \le S_{\mathrm{B}}^{i^+}$, stop.
    Otherwise,
    If $S^i_{\mathrm{BOT}} \le S_{\mathrm{B}}^{i^-}$,
        $\mathrm{CF}^{i+1}_{\mathrm{BOT}} = \mathrm{CF}^i_{\mathrm{BOT}} - (\alpha + |P^i_{\mathrm{B}} - \mathrm{AC}^i_{\mathrm{BOT}}|)^*(S_{\mathrm{B}}^{i^-})^{-1}$
    Else,
        $\mathrm{CF}^{i+1}_{\mathrm{BOT}} = \mathrm{CF}^i_{\mathrm{BOT}} + (\alpha + |P^i_{\mathrm{B}} - AC^i_{\mathrm{BOT}}|)^*(S_{\mathrm{B}}^{i^+})^{-1}$
    $\alpha$ is a small number to prevent a zero value of $|P^i_{\mathrm{B}} - AC^i_{\mathrm{BOT}}|$.

Step 5: Set $i = i + 1$, go to step 2.
```

In the algorithm, the left-hand and right-hand local derivatives are approximately calculated by changing $\mathrm{CF}_{\mathrm{BOT}}$ by a small amount at a time and finding the new costs.

Usually a BOT power plant has lower fixed and variable costs due to its efficient management. So at the equilibrium point, $P^*_{\mathrm{B}} \ge \mathrm{AC}^*_{\mathrm{BOT}}$. Even if $P^*_{\mathrm{B}} \le \mathrm{AC}^*_{\mathrm{BOT}}$, higher return rates or allowances from the host utility based on some policies, will still make a BOT investor benefit from a BOT project. The policy issues are outside the scope of this paper.

6. Numerical examples

The proposed model and approach have been applied to an example system as described in Ref. [4]. The initial system, proposed plants and load data are listed in Tables 1–3, respectively and a 20-year planning period, which is divided into 5 time-stages each of four years duration, is considered.

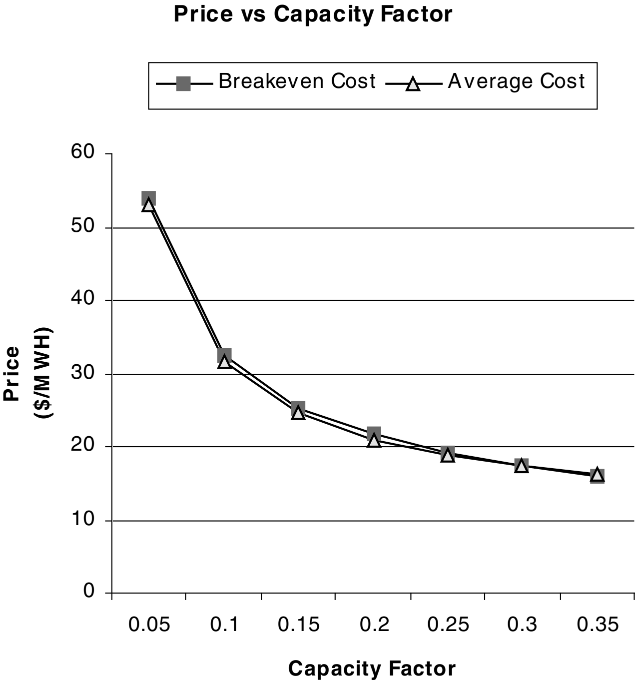

Fig. 3. Breakeven cost and average cost versus capacity factor of the BOT power plant.

In Table 2, plant types 1 and 2 are base plants while 3–5 are shoulder and peak plants. The load duration curves are approximated with a second-order function of loads. Peak loads and base loads are assumed to increase by 10% a year. The reserve is 1%. In addition, an annual discount rate of 10% for both capital and operating expenses is used.

In this example, a type 5 plant is selected as a BOT power plant (peak load plant), which is introduced in the second stage and whose capacity is 450 MW. Moreover, we assume its capacity cost is by 35% less than that of a power plant of the same type. As breakeven cost only prices the basic cost, the utility is willing to pay back to the BOT power plant; the 35% reduction in the energy cost of the BOT power plant is to make its payoff more positive in order to examine our model and approach.

Using our game model and solution, the equilibrium is reached at $(\mathrm{CF}^*_{\mathrm{BOT}}, P^*_{\mathrm{B}}) = (0.19, 22.26)$. Some of the results in calculating the equilibrium point are shown in Table 4

Table 4
Breakeven cost and average cost versus capacity factor of the BOT power plant

Capacity factor	0.05	0.1	0.15	0.2	0.25	0.3	0.35
P_{B} ($/MWh)	53.91	32.49	25.36	21.64	19.13	17.27	15.9
$\mathrm{AC}_{\mathrm{BOT}}$ ($/MWh)	53.05	31.6	24.5	20.9	18.75	17.32	16.3
BOT payoff ($M)	2.712	5.613	8.199	9.46	5.992	−0.79	−8.8

and Fig. 3. In fact, in the long-term GEP the BOT power plant takes the place of a type 5 plant in the original GEP without the BOT entry. However, that plant has different capacity factors in different years, which range from 0.098 to 0.31, while the annual capacity factors of the BOT power plant are fixed. This kind of non-dispatchability presented by a BOT power plant causes many economic dispatch problems, and in Ref. [12] this issue is discussed and some approaches are proposed to communicate the value of dispatchability for non-utility generation projects.

It has been shown that the electricity price and fixed annual energy of a BOT power plant can be determined simultaneously in our game model, and the annual capacity factor of the BOT power plant is forced to become close to one that of a power plant of the same type in the original GEP.

7. Conclusions

In this paper, we show how game theory can be used to evaluate a popular kind of private power production: the BOT arrangement. Based on oligopoly theory, a Stackelberg game model between a BOT investor and a utility is proposed, which is applied to the negotiation and bargaining of a long-term energy contract. In addition, asymmetric pricing schemes of "avoided cost" and "average cost" are considered. To compute the equilibrium point of this Stackelberg game, it is transferred to a two-level optimization problem, and solved by an iterative algorithm. Finally our model and approach are demonstrated by applying it to a test system.

Acknowledgements

The research is supported by a grant from the Research Grant Council of HKSAR.

References

[1] Walker C, Smith AJ. Privatized infrastructure: the build–operate–transfer approach. Springfield, IL: Thomas Telford, 1995.

[2] Arikan Y. Build–operate–transfer model for new power plants for Turkey. Proceedings of the 7th Mediterranean Electrotechnical Conference, 1994.

[3] Asian Development Bank. Technical Assistance to the People's Republic of China for the BOT CHANGSHA Power Project, December 1996.

[4] Xing W, Wu FF. Cost–benefit analysis of BOT power plants. Proceedings of the IEEE PES Winter Meeting, Singapore, January 23–27, 2000.

[5] David AK. Risk modelling in energy contracts between host utilities and BOT plant investors. IEEE Trans. Energy Conver 1996;11(2): 359–66.

[6] Ruusunen J, Ehtamo H, Hamalainen RP. Dynamic cooperative electricity exchange in a power pool. IEEE Trans Systems, Man Cybernet 1991;21(4):758–66.

[7] Haurie A, Loulou R, Savard G. A two-level game model of power cogeneration in New England. IEEE Trans Automat Control 1992;37(9):1451–6.

[8] Siddiqi SN, Baughman ML. Optimal pricing of non-utility generated electric power. IEEE Trans Power Systems 1994;9(1):397–403.

[9] Kuwahara A, Asano H. Utility–cogenerator game for pricing power sales and wheeling fees. IEEE Trans Power Systems 1994;9(4):1875–9.

[10] Weber JD, Overbye TJ. A two-level optimization problem for analysis of market bidding strategies. Proceedings of IEEE PES Winter Meeting, New York, January 31–February 4, 1999.

[11] Singh H. (Ed.) IEEE tutorial on game theory applications in electric power markets. IEEE PES Winter Meeting, New York, 1999.

[12] Logan DM, Baylor JS, Cotcher D. Communicating the value of dispatchability for non-utility generation projects. IEEE Trans Power Systems 1995;10(3):1048–413.