NPV-AT-RISK METHOD IN INFRASTRUCTURE PROJECT INVESTMENT EVALUATION

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ABSTRACT: Strategic capital investment decisions are crucial to a business firm. The decision to invest in privately financed infrastructure projects requires careful consideration, because they are exposed to high levels of financial, political, and market risks. The project appraisal methods should incorporate analysis of these risks. A number of capital-investment decision methods can take risks into account, but each of them focuses on different factors and has its limitations. Thus, a more vigorous method is needed. A systematic classification of existing evaluation methods shows that it is possible to develop a new method—the net-present-value-at-risk (NPV-at-risk) method—by combining the weighted average cost of capital and dual risk-return methods. The evaluation of two hypothetical power projects shows that the NPV-at-risk method can provide a better decision for risk evaluation of, and investment in, privately financed infrastructure projects.

INTRODUCTION

Strategic capital investment decisions are crucial to a business firm. The decision to become involved in privately financed infrastructure projects such as a build-operate-transfer (BOT) power plant or a toll road requires careful consideration and thorough analysis. Traditionally, investment decisions on infrastructure projects are made by the investing government based on the benefit-cost analysis and economic viability of the projects. But the BOT procurement strategy separates financial benefits from economic benefits, because the primary objective of the private sector is to maximize profits, and their decisions are mainly based on the financial viability of projects.

The most common methods for the assessment of financial viability are the payback period, average accounting rate of return, the net present value (NPV), and the internal rate of return (IRR) methods. Decisions derived from these methods are based on the forecasts of base-case cash flows. However, BOT infrastructure projects are characterized by high capital outlays, long lead times, and long operating periods. These characteristics make the forecasts of cash flows more difficult and expose the private sector to high levels of financial, political, and market risks. This requires the decision to incorporate risk analysis into project appraisal methods.

The common methods of incorporating risk into capital-investment decisions are the dual risk-return and the risk-adjusted discount rate methods. They focus either on measuring risk or determining discount rates. The dual risk-return method is limited by difficulty in determining the size of acceptable deviation, while the risk-adjusted discount rate method failed to provide the confidence level of its results. Furthermore, since financing is a key element of BOT projects, investment decisions also should take financing methods into account. Therefore, a more vigorous investment decision method that incorporates both risk and financing methods is needed.

The need for a more vigorous method was highlighted by the recent Southeast Asia financial crisis. A number of BOT projects suffered disastrous consequences. Some BOT projects under construction had been postponed or abandoned by the promoters, and others had to be bailed out by the host governments. Projects in operation also suffered huge losses resulting from such factors as the depreciation of local currencies or reduction in tariffs by utilities. The experience increased the need to seek more powerful methods of addressing risk in investment decision making. This paper develops a new project evaluation method called the NPV-at-risk and attempts to show that this method can potentially overcome these problems in investment decision making.

METHODS OF CAPITAL INVESTMENT DECISION MAKING

The project evaluation methods may systematically be classified into three categories: methods based on return, methods based on risk, and methods based both on return and risk (Fig. 1). The methods based on return include the payback period, the average accounting rate of return (also called the return on capital employed), NPV, and IRR. The payback period and the average accounting rate of return methods ignore the time value of money, whereas NPV and IRR methods incorporate the time value of money into decision making using discounted cash flow techniques. But all of them are based on the assumption that the cash flows of the project are certain, whereas the project's actual cash flows could differ substantially from the forecast cash flows.

The uncertainties bring risk into capital investment evaluation decisions. This directs attention to the development of risk-rating systems. In risk-rating systems, an investment is evaluated and assigned a grade of risk ratings. The ratings are divided into investment and speculative grades. The decision rule is that the project is eligible for investment if it is rated one of the investment grades, for example, BBB or above in Standard & Poor's ratings or Baa or above in Moody's ratings. The ratings are intended to measure credit risk, not other forms of investment risk (Stimpson 1991). They pertain to investment quality, not investment attractiveness (Hennessy 1986). Investors must determine their own required returns.

The inadequacies of a decision criterion based solely on return or risk show that methods incorporating risk into the measurement of return should be developed. The most common methods are the risk-adjusted discount rate methods such as Capital Asset Pricing Model (CAPM), arbitrage pricing theory (APT), and the weighted average cost of capital (WACC). They focus on the determination of discount rates under uncertainties. The philosophy of these methods is that the risk-adjusted discount rate should consist of risk-free rate and risk premium. A major problem with the methods is that there is no indication of confidence level on the determined discount rate.

An alternative approach is probabilistic and statistical analyses. This leads to development of decision trees, mean-vari-

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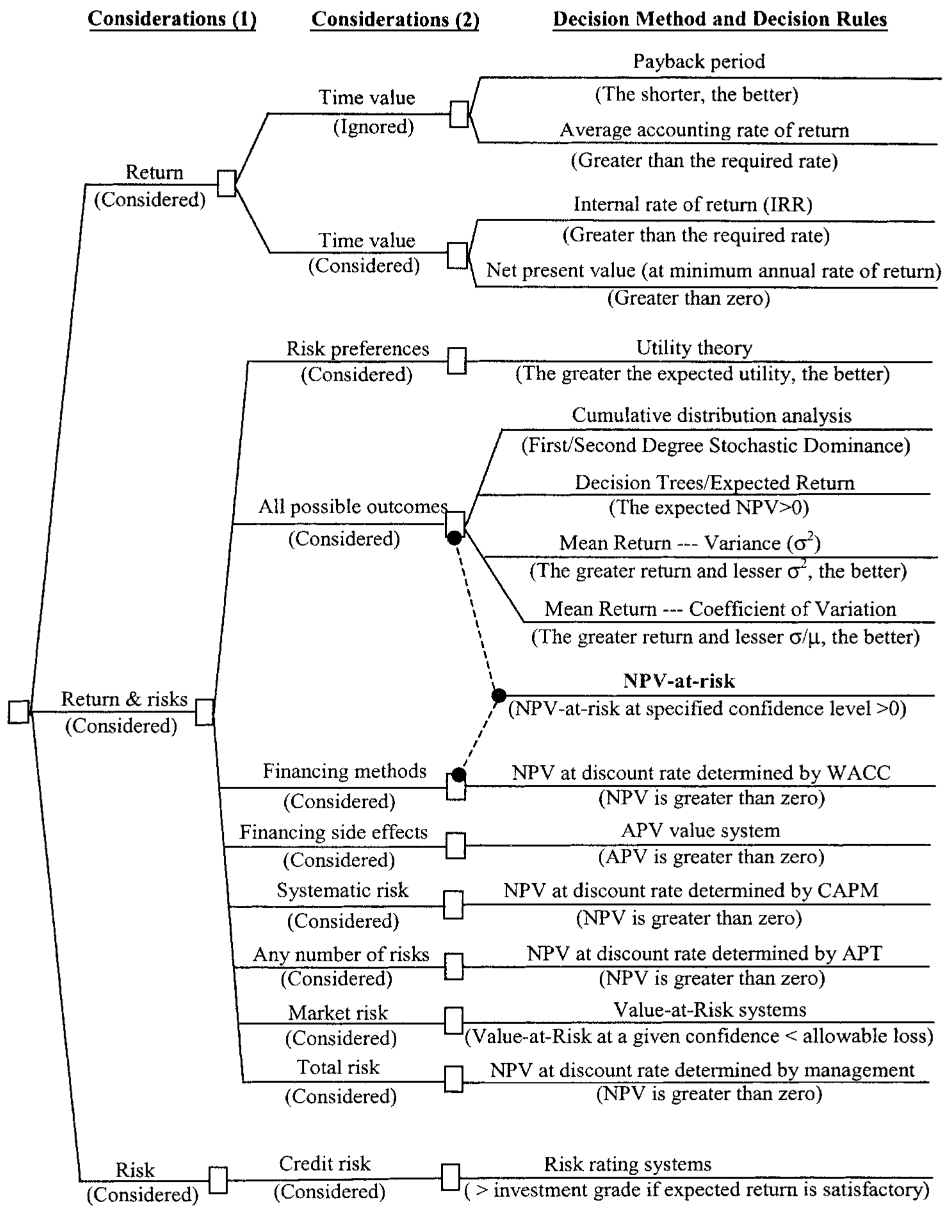

FIG. 1. Methods and Decision Rules for Capital Investment Decisions

ance and expected return-coefficient of variation methods, and cumulative distribution analysis. Decision-tree analysis produces the weighted average of the possible returns, weighted by the probability of the return occurrence. This method ignores the dispersion of returns. The mean-variance and the expected return-coefficient of variation methods measure return and risk separately. Their method is to compute the expected return as the measurement of return and the size of deviation from the expected return as the measurement of risk. They have difficulty determining the size of the allowable deviation. The cumulative distribution analysis is used for analyzing mutually exclusive investments by comparing their entire cumulative distributions of possible returns.

Another method taking return and risk into account simultaneously is the utility theory, including the expected utility model and the generalized expected utility model. The expected utility is the weighted average of the utilities of the possible outcome where the weights are the objective probabilities of each outcome. The decision criterion is that the greater the expected utility, the better the project. This method involves subjectivity in constructing appropriate utility functions, which are based on individual risk preferences. As a result, the operational usefulness of the expected utility model is severely limited (Lumby 1984).

The value-at-risk systems provide a decision criterion with a confidence level. However, they were first developed for dealing with market risks and extended to deal with other risks such as credit, liquidity, and cash flow (Dowd 1998). Furthermore, they do not take financing methods and time value into consideration. Adjusted present value (APV) can handle financial side effects. The fundamental idea behind APV is to unbundle components of value, analyze each separately, and then add them back up (Luehrman 1997). It therefore provides more detailed information than WACC, but it also fails to provide a confidence level.

The systematic classification shows that a more rigorous method to consider the time value of all the possible outcomes and financing methods in a decision-making process is both necessary and possible. One of the possible approaches is to synthesize the WACC and the expected NPV method together to form a minimum expected NPV. This leads to the development of the NPV-at-risk method.

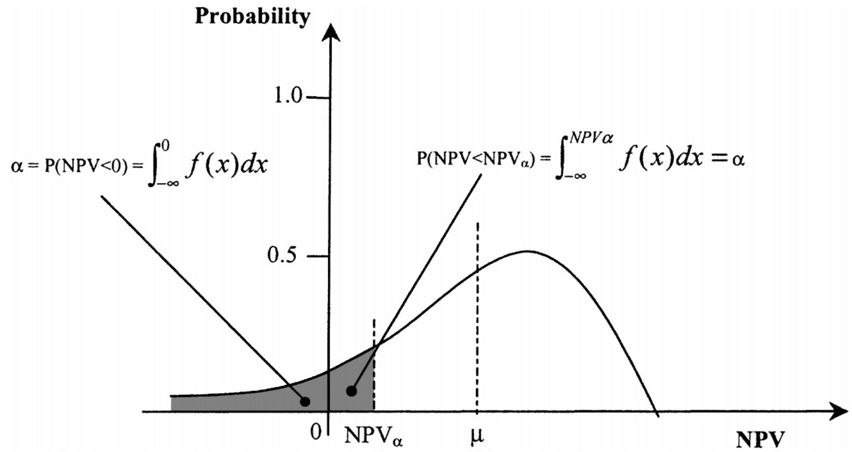

FIG. 2. Calculation of NPV-at-Risk and Confidence Level Based on Probability Distribution Function

DEVELOPMENT OF NPV-AT-RISK METHOD

One of the six definitions of risk listed by Vlek and Stallen (1981) is that risk is the semivariance of the distribution of all consequences, taken over negative consequences only, and with respect to some adopted reference value. The semivariance alone as the measure of risk is not sufficient to make a decision, but it can be combined with the expected NPV to form a new decision rule—a project is acceptable if the mean NPV minus the standard deviation is greater than zero. However, this decision rule fails to provide decision-makers with a confidence level. Instead of calculating the mean NPV and standard deviation, the NPV_α at a given confidence level is computed so that the decision rule is that the project is acceptable with the given confidence level if the NPV_α is greater than zero. Based on this consideration, NPV-at-risk is defined as a particular NPV that is generated from a project at some specific confidence level, that is, the minimum expected NPV with the given confidence level. In other words, NPV-at-risk is the value at which α% of possible NPVs are smaller and $1 - \alpha$% were larger.

According to the definition of NPV-at-risk, the following decision rules can be derived: the project is acceptable with a confidence level of $1 - \alpha$ if NPV-at-risk at the given confidence is greater than zero; otherwise, it is unacceptable. Alternatively, the project is acceptable if the computed confidence level at the point of zero NPV is equal to or greater than the predetermined confidence level; otherwise, it is unacceptable.

The NPV-at-risk method aims to calculate the value that the project's NPVs will be greater than, with the probability corresponding to the given confidence level. It involves the determination of discount rate and the generation of cumulative distribution of possible NPVs. To calculate NPV, the key task is to determine an appropriate discount rate. There are various methods of determining a discount rate, each of which has its own application. Since the portfolio theory and subsequent CAPM model, and APT were developed for financial markets, their application to stand-alone projects suffers from the difficulty in determining appropriate beta. Moreover, the discount rate determined by these methods may overemphasize (double-count) the impact of risk exposure because the NPV-at-risk method will also take the risk into account.

Unlike CAPM and APT, WACC is the cost of various financial sources weighted by their corresponding proportions in the overall pool of financing. The cost of a financial source is the return expected by investors. According to Tiong (1995), the equity of a BOT project is usually about 20–30% of total investment, and the remainder is debt finance. The equity required return is assumed to be the hurdle rate of sponsors, whereas the debt required return is assumed to be the average market interest rate. Determined in this way, WACC does not sufficiently reflect the required risk premium. Moreover, according to Farid et al. (1989), WACC is the only practicable option at the present time. But that does not mean that WACC alone is good at handling risk. Therefore, the use of WACC for determining the discount rate in the NPV-at-risk method is practicable without overestimating risk. In addition, WACC enables the NPV-at-risk method to take financing methods into account.

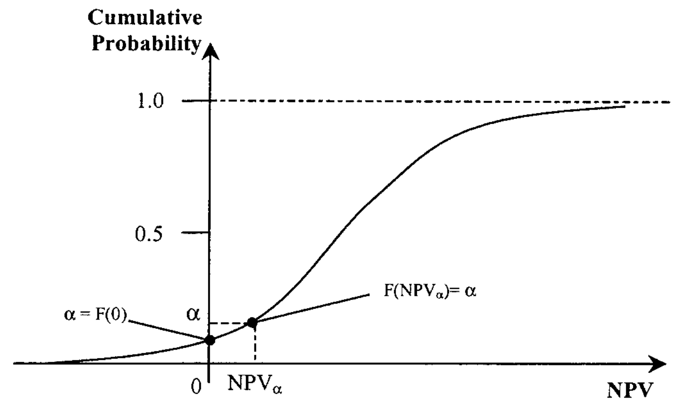

FIG. 3. Calculation of NPV-at-Risk and Confidence Level Based on Cumulative Distribution Function

According to the requirements of decision rules, there are two approaches to investment decision making, the calculation of NPV at a given confidence level and the calculation of a confidence level at the point of zero NPV. Assuming that the probability density function of return is f(NPV), NPV-at-risk at a given confidence α is computed by making the integration between $-\infty$ and NPV_α equal α, and the confidence level at the point of zero NPV is obtained by integration between $-\infty$ and 0 (Fig. 2). When the project's NPVs are normally distributed, the NPV-at-risk can be obtained through the mean-variance method. In this case, NPV-at-risk is the difference of the mean value and a multiple of standard deviation. It can be expressed as deviations from the mean NPV in units of the standard deviation

$$\text{NPV-at-risk} = \text{mean NPV} - Z(\alpha) \cdot \sigma$$

where $Z(\alpha)$ = number of units of standard deviation corresponding to the given confidence level of α; for example, at the 95% confidence level, $Z(\alpha) = 1.65$. This means that 95% of possible outcomes fall within the range from $\mu - 1.65\sigma$ to ∞.

Alternatively, assuming that the cumulative distribution

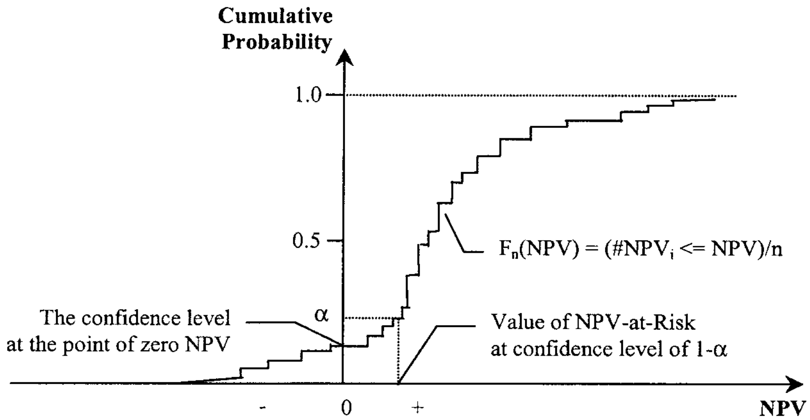

FIG. 4. Calculation of NPV-at-Risk and Confidence Level Based on Simulation-Generated Distribution

function of return is F(NPV), NPV-at-risk at a given confidence α and the confidence level at the point of zero NPV can be obtained using percentile analysis on the cumulative distribution (Fig. 3). If the distribution functions of return, f(NPV) or F(NPV), are unknown, Monte Carlo simulation can be used to generate the distribution of possible NPVs. It takes samples from the input variable distributions and evaluates the corresponding NPV that is a function of these variables. The process is repeated as many times as desired and the resulting NPV_1, NPV_2, . . . , NPV_n are placed in ascending order to obtain the cumulative distribution of NPV. The distribution function can be estimated by the empirical distribution function $F_n(\text{NPV}) = (\#\text{NPV}_i \le \text{NPV})/n$, which is the relative frequency of NPV, where $\#\text{NPV}_i$ is the number of simulation outputs, NPV_1, NPV_2, . . . , NPV_n, that are no greater than the specified NPV. Thus, NPV-at-risk at a given confidence level can be obtained by calculating the percentile $F_n^{-1}(\alpha) = \text{NPV}_\alpha$, and the confidence level at the point of zero NPV can be obtained by computing the probability of NPV $\le$ 0, that is, $F_n(0) = (\#\text{NPV}_i \le 0)/n$. Fig. 4 shows the calculation of NPV-at-risk at a given confidence level and the confidence level at the point of zero NPV from the empirical cumulative distribution function F_n(NPV).

Obtained in this way NPV-at-risk is subject to estimation error resulting from sampling error, inappropriate discount rate, and inappropriate cash flow model. Therefore, there is a need to validate its reliability. One approach is to use Kolmogorov confidence bands as the confidence bands as the confidence bands for the empirical cumulative distribution. The confidence bands can be obtained by $d_{\alpha,n} = d_\alpha/\sqrt{n}$, which depends upon both the confidence level $1 - \alpha$ and the sample size.

APPLICATION OF NPV-AT-RISK

To demonstrate the application of the NPV-at-risk method, two hypothetical power projects, Plants A and B, are evaluated. Both projects have the same size (2 × 350 MW turbine-generator units), but are located in countries with different degrees of political uncertainty. They are procured under BOT contract with a concession period of 20 years. The base-case cash flows of the projects are derived on the following assumptions. The building of a 2 × 350 MW power plant requires $635,000,000 over 3.5 years. During its operation period, the average demand is 80% of installed capacity at the tariff of $0.07/kW·h. The operation and maintenance cost is 35% of output. The debt-equity ratio is 3, with an annual debt interest rate of 9% and company's hurdle rate of 12%/year. The exchange rate of the local currency for U.S. dollar is 1. The estimated base-case cash flows are shown in Table 1.

The base-case net cash flows before tax are estimated from single values of variables. In fact, the variables, such as completion time, construction cost, market demand, sale price, operation and maintenance (O&M) costs, inflation, foreign exchange rate, and interest rate, are uncertain. The uncertainties of these variables mean that they can be treated as stochastic variables (or ''state variables''). Monte Carlo simulation can be applied to determine the distribution of NPV, given that the probability distribution of each variable is known. The probability distributions of the completion time, construction cost, and O&M costs can be determined from experience of similar projects in approximately similar conditions. Here, they are assumed as lognormal distributions (μ, σ^2). The lognormal assumptions are based on the intuition that the extent of costsaving and timesaving is limited, whereas the extent of cost overrun and time overrun is infinite. The probability distributions of the market demand, sale price, and inflation, foreign exchange, and interest rates can be determined by collecting and analyzing economic data of the host countries. Here, they are assumed as normal distributions (μ, σ^2). The mean μ of distribution function of a variable is assumed to be the estimated value of the variable in base case. The coefficient of variance (CoV) of variables in Plant A is assumed to be 0.1, so the standard deviation σ will be 0.1μ. Assume Plant B has more uncertainty and the CoV of its variables is twice as much as that of Plant A's. The standard deviation of its

TABLE 1. Base-Case Net Cash Flows before Tax ($ million)

Year (1)	Capital expenditure (2)	Sale revenue (3)	O&M cost (4)	Operating income (5)	Net cash flows before tax (6)
1	123	0	0	0	−123
2	249	0	0	0	−249
3	201	0	0	0	−201
4	62	171.696	60.0936	111.6024	49.6024
5	0	343.392	120.1872	223.2048	223.2048
6	0	343.392	120.1872	223.2048	223.2048
7	0	343.392	120.1872	223.2048	223.2048
8	0	343.392	120.1872	223.2048	223.2048
9	0	343.392	120.1872	223.2048	223.2048
10	0	343.392	120.1872	223.2048	223.2048
11	0	343.392	120.1872	223.2048	223.2048
12	0	343.392	120.1872	223.2048	223.2048
13	0	343.392	120.1872	223.2048	223.2048
14	0	343.392	120.1872	223.2048	223.2048
15	0	343.392	120.1872	223.2048	223.2048
16	0	343.392	120.1872	223.2048	223.2048
17	0	343.392	120.1872	223.2048	223.2048
18	0	343.392	120.1872	223.2048	223.2048
19	0	343.392	120.1872	223.2048	223.2048
20	0	343.392	120.1872	223.2048	223.2048
Total	635	5,665.968	1,983.089	3,682.879	3,047.879

Note: Sale revenue = 80% installed capacity × 365× 24 × selling price = 0.8 × 700,000 × 365 × 24 × 0. 07 = 343.392 million. O&M cost = 35% sale revenue. Operating income = sale revenue − O&M cost.

TABLE 2. Parameters and Types of Distribution of Key Risk Factors

Variable (1)	Distributional assumption (2)	Estimated mean (μ) (3)	Plant A CoV σ/μ (4)	Plant A SD (σ) (5)	Plant B CoV σ/μ (6)	Plant B SD (σ) (7)
Construction cost	lognormal (μ, σ^2)	500,000,000	0.1	50	0.2	100
Completion time	lognormal (μ, σ^2)	3.5 years	0.1	0.4	0.2	0.8
O&M cost	lognormal (μ, σ^2)	35% of sale	0.1	0.035	0.2	0.07
Market demand	normal (μ, σ^2)	560 MW	0.1	56	0.2	112
Sale price	normal (μ, σ^2)	\$0.07/KW·h	0.1	0.006	0.2	0.0123
Inflation rate	normal (μ, σ^2)	2% per annum	0.1	0.002	0.2	0.004
Foreign exchange rate	normal (μ, σ^2)	1.0	0.1	0.1	0.2	0.2
Interest rate	normal (μ, σ^2)	9% per annum	0.1	0.009	0.2	0.018

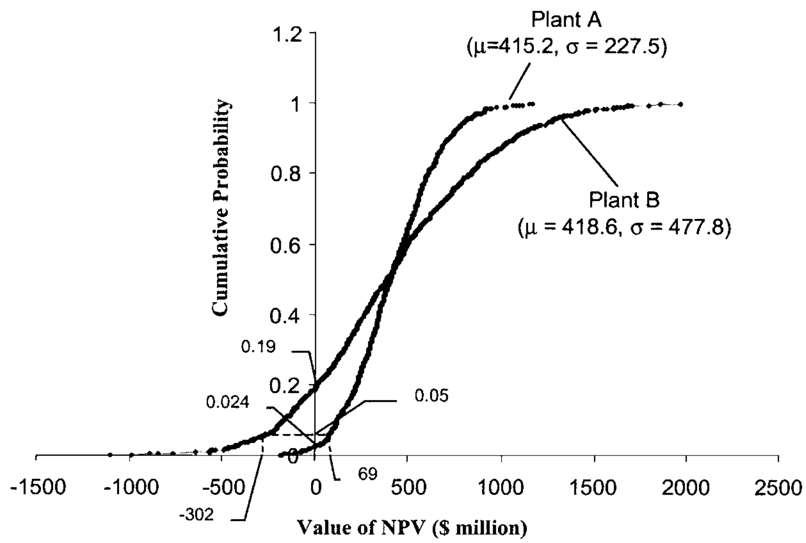

FIG. 5. Comparison of NPV Distributions at Plants A and B

variables will be 0.2μ. Distributions of the variables are given in Table 2.

A Monte Carlo simulation of 1,000 iterations was carried out on a simulation model developed by the writers to obtain the distribution of NPV for Plants A and B, respectively. Based on the generated cumulative distribution of Plant A's NPV, it can be computed that NPV-at-risk of Plant A at 95% confidence level is \$69,000,000. Because NPV-at-risk of Plant A is greater than zero, the project is investable. Alternatively, it can be computed that the probability at the point of zero NPV is 0.024. In other words, there is 97.6% confidence that NPV is greater than zero. This confidence level is greater than the predetermined confidence level of 95%, so this project is investable. These two approaches have the same conclusion. Similarly, it can be computed that NPV-at-risk of Plant B at 95% confidence level is −\$302,000,000 and 0.19 probability at the point of zero NPV. Thus, Plant B is not investable. The simulation results are shown in Fig. 5. If the debt-equity ratio of Plant B decreases from 3 to 0.25, its NPV-at-risk will increase from −\$302,000,000 to \$51,000,000. Plant B becomes investable. This is because the sponsor bears the major part of risk in the form of equity investment. That is why a riskier project often requires more equity investment.

COMPARISON OF EVALUATION METHODS

The methods indicated in Fig. 1 have their own data requirements such as base-case cash flows, statistical data, and simulations.

Based on the base-case cash flows, the payback period for both projects is 6.4 years. This method thus cannot distinguish which project is preferable. Similarly, the accounting rate of return and IRR methods cannot distinguish between the two projects, although the returns are greater than the hurdle rate of 12%. Both NPV at the minimum annual rate of return and NPV at the discount rate of WACC cannot distinguish between the two projects, but show that both projects are investable because their NPVs are greater than zero. The hypothetical projects were not evaluated by the APV method and the methods calculating NPV at discount rates determined by CAPM, APT, and the management, because they require more information besides base-case cash flows.

Based on the same simulation data, the expected return method concludes that Plant B is preferable to Plant A and that both plants are investable because their expected NPVs are greater than zero. The mean-variance method provides only two numbers, mean NPV and standard deviation. Although mean NPV is greater than zero, the method cannot decide whether the projects are investable, because the decision depends on the trade-off between risk and return. It also failed to distinguish which is preferable, because both the mean and variance of Plant B are greater than their counterparts for Plant A. The mean CoV method is better than the mean-variance method, because it can judge the preferred project (i.e., Plant A), but it failed to make a decision. The cumulative distribution analysis cannot make a decision. It also experienced difficulty in distinguishing the preferable project. Table 3 tabulates the results of different evaluation methods. Compared with the NPV-at-risk method, the methods based on base-case cash flows failed either to take risk into account (e.g., payback period) or to handle random variables if risk is considered (e.g., NPV at a discount rate determined by

TABLE 3. Comparison of Different Evaluation Methods

Evaluation method (1)	Analysis Results: Plant A (2)	Analysis Results: Plant B (3)	Decision (4)	Remarks (4)
Payback period	6.4 years	6.4 years	Plant A is same as Plant B. Investability depends on experience.	Base-case cash flows are required. Calculation is simple.
Accounting rate of return	23.5%	23.5%	Plant A is same as Plant B. Investability depends on predetermined hurdle rate.	Base-case cash flows are required.
Internal rate of return	26%	26%	Plant A is same as Plant B. Investability depends on predetermined hurdle rate.	Base-case cash flows are required. Calculation is complex.
NPV	$569,000,000	$569,000,000	Plant A is same as Plant B. Both projects are investable (NPV > 0).	Base-case cash flows and discount rate are required (Discount rate = 0.12, i.e., company's hurdle rate).
NPV (WACC discount rate)	$785,000,000	$785,000,000	Plant A is same as Plant B because they have same capital structure and interest rate. Both are investable (NPV > 0).	Base-case cash flows, and information on capital structure, debt interest rate and equity return are required. (equity return = company's hurdle rate)
NPV-at-risk	$69,000,000	−$302,000,000	Plant A is investable, but Plant B is not, based on 95% of confidence.	Distribution of NPV is required. Difficulty in evaluating probability distributions of variables.
Cumulative distribution analysis	Fig. 4	Fig. 5	Plant A is preferable to Plant B for risk-averse investors. But this does not mean that Plant A is investable.	Distribution of NPV is required. Difficulty in interpreting their results when two cumulative distributions intersect.
Decision tree/expected return	$415,200,000	$418,200,000	Plant B's NPV is greater than Plant A's. Both are investable because NPV > 0.	All possible outcomes and their probabilities are required. Difficulty in evaluating probability of outcome.
Mean return-variance	μ = 415.2 σ = 227.5	μ = 418.6 σ = 477.8	Both plants have positive expected NPV, but is risk acceptable? Decisions depend on risk-return trade-off.	Distribution of NPV is required. Difficulty in evaluating probability distributions of variables and trade-off between risk and return.
Mean return-CoV	μ = 415.2 σ/μ = 0.55	μ = 418.6 σ/μ = 1.14	Plant A is preferable to Plant B. Decisions depend on risk-return trade-off.	Distribution of NPV is required. Difficulty in evaluating probability distributions of variables and trade-off between risk and return.
NPV (CAPM discount rate)	—	—	N/A	Base-case cash flows are required. Statistical data is required to estimate risk factor β. It is difficult to obtain statistical data for stand alone projects.
NPV (APT discount rate)	—	—	N/A	Base-case cash flows are required. Statistical data is required to estimate β_i. This method is designed for determining the rate of return on securities.
NPV (subjective discount rate)	—	—	N/A	Base-case cash flows are required. Discount rates are determined by management subjectively.
Value-at-risk	—	—	N/A	This method is designed for determining the rate of return on securities. Statistical data is required.
APV	—	—	N/A	Base-case cash flows and components of value are required. Each component of value must be calculated individually.
Utility theory	—	—	N/A	Values of utility vary with decision-makers. Very subjective.
Risk-rating systems	—	—	N/A	Ratings require specialized teamwork. Very complicated process.

Note: N/A = not applicable.

WACC). The methods calculating NPV at a risk-adjusted discount rate provide NPV without indication of reliability. The decision-tree (or expected return) method provides the expected NPV, but fails to measure risk. The other existing methods based on probability analysis produce the expected NPV from a range of possible outcomes. However, they failed to provide a criterion for an accept/reject decision relating to risk. The NPV-at-risk method produces a single NPV value from a range of outcomes at a given confidence level. It therefore overcomes the problems in the other methods. In addition, unlike risk-rating methods that require a specialized team to perform, NPV-at-risk method can be carried out by promoters using commercially available software.

The NPV-at-risk method is different from the value-at-risk method. The value-at-risk is a measure of maximum potential change in value of a portfolio of financial instruments with a given probability over a preset horizon (J. P. Morgan and Reuters 1996), whereas NPV-at-risk is the measure of minimum expected return from a project at a given confidence level. Value-at-risk mainly focuses on market risk and other risks such as credit, liquidity, and cash flow, whereas the NPV-at-risk method takes the following factors into account: (1) All the possible returns resulting from uncertainty; (2) the time value of money; (3) the impact of financing methods; and (4) various risks associated with BOT projects. However, both of them use probabilistic and statistical analyses techniques, and Monte Carlo simulation.

A major requirement in using the NPV-at-risk method is the availability of data for statistical analysis. Although Monte Carlo simulation offers a powerful means to generate data, some reasonable statistical distributions of risk variables should be specified because Monte Carlo simulation requires the distributions of the variables. Another problem is to determine correlation between risk variables in the cash flow model. As a result, the reliability of NPV-at-risk depends on the simulation results derived from the specified distributions and cash flow models. Its ability to deal with risk also depends on the quality of the simulation model.

CONCLUSIONS

Compared with other types of capital investments, BOT projects are exposed to more risks. They require a more vigorous investment decision method. A systematic review of various investment decision-making methods shows that WACC and mean-variance methods can be combined to form the NPV-at-risk method. It incorporates the time value of money into the mean-variance method using NPV concept and takes financing methods into account using WACC as the discount rate. The comparison of different methods for two hypothetical projects shows that this combination can overcome some problems inherent in other methods, and the method can be used in decision making for privately financed infrastructure projects.

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