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FUNDAMENTALS OF THE ANALYTIC NETWORK PROCESS

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Abstract

The Analytic Network Process (ANP) is a general theory of relative measurement used to derive composite priority ratio scales from individual ratio scales that represent relative measurements of the influence of elements that interact with respect to control criteria. Through its supermatrix whose elements are themselves matrices of column priorities, the ANP captures the outcome of dependence and feedback within and between clusters of elements. The Analytic Hierarchy Process (AHP) with its dependence assumptions on clusters and elements is a special case of the ANP. The ANP is a new and an essential phase in decision making, neglected so far because of the linear structures used in traditional approaches and their inability to deal with feedback in order to choose alternatives not simply according to attributes and criteria, but also according to their consequences both positive and negative – an essential and so far a missing consideration in decision making. This paper gives a brief look at the foundation of the ANP together with a simple example.

1. Introduction

The seven pillars of the Analytic Hierarchy Process (AHP) serve as a starting point for the Analytic Network Process (ANP). The ANP provides a general framework to deal with decisions without making assumptions about the independence of higher level elements from lower level elements and about the independence of the elements within a level. In fact the ANP uses a network without the need to specify levels as in a hierarchy. Influence is a central concept in the ANP. The ANP is a useful tool for prediction and for representing a variety of competitors with their surmised interactions and their relative strengths to wield influence in making a decision.

The ANP is a coupling of two parts. The first consists of a control hierarchy or network of criteria and subcriteria that control the interactions. The second is a network of influences among the elements and clusters. The network varies from criterion to criterion and a different supermatrix of limiting influence is computed for each control criterion. Finally, each of these supermatrices is weighted by the priority of its control criterion and the results are synthesized through addition for all the control criteria.

With the ANP a problem is often studied through a control hierarchy or control system of benefits, a second for costs, a third for opportunities, and a fourth for risks each represented in the controlling system. The synthesized results of the four control systems are combined by taking the quotient of the benefits times the opportunities to the costs times the risks for each alternative, then normalizing the results over all the alternatives to determine the best outcome. A rough outline of the steps of the ANP follows.

Outline of the Steps of the ANP Useful for Understanding It by Users of the AHP

1. Determine the control hierarchies including their criteria for comparing the components of the system and their subcriteria for comparing the elements of the system. One hierarchy for benefits, a second for costs, a

third for opportunities, and a fourth for risks. If in some cases, a hierarchy does not apply because its criteria are all unimportant, leave out that hierarchy. For benefits and opportunities, ask what gives the most benefits or presents the greatest opportunity to influence fulfillment of that control criterion. For costs and risks, ask what incurs the most cost or faces the greatest risk. Sometimes, the comparisons are made simply in terms of benefits, opportunities, costs, and risks in the aggregate without using criteria and subcriteria.

2. For each control criterion or subcriterion, determine the clusters of the system with their elements.
3. To better organize the development of the model, as well as you can and roughly, for each control criterion, number and arrange the clusters and their elements in a convenient way (perhaps in a column). Use the identical label to represent the same cluster and the same elements for all the control criteria.
4. Determine the approach you want to follow in the analysis of each cluster or element, being influenced by other clusters and elements, or influencing other clusters and elements with respect to a criterion. The sense (being influenced or influencing) must apply to all the criteria for the four control hierarchies.
5. For each control criterion, construct a three-column table placing each cluster label in the middle column. List in the left column on a line all the clusters that influence the cluster, and in the column on the right those clusters which it influences.
6. Following each entry in the table above, perform paired comparisons on the clusters as they influence each cluster and on those that it influences, with respect to that criterion. The derived weights are used later to weight the elements of the corresponding column clusters of the supermatrix corresponding to the control criterion. Assign a zero when there is no influence.
7. Perform paired comparisons on the elements within the clusters themselves according to their influence on each element in another cluster they are connected to (or elements in their own cluster). The comparisons are made with respect to a criterion or subcriterion of the control hierarchy.
8. For each control criterion, construct the supermatrix by laying out the clusters in the order they are numbered and all the elements in each cluster both vertically on the left and horizontally at the top. Enter in the appropriate position the priorities derived from the paired comparisons as parts (subcolumns) of the corresponding column of the supermatrix.
9. Compute the limiting priorities of each supermatrix according to whether it is irreducible (primitive or imprimitive [cyclic]) or it is reducible with one being a simple or a multiple root and whether the system is cyclic or not.
10. Synthesize the limiting priorities by weighting each limiting supermatrix by the weight of its control criterion and adding the resulting supermatrices.
11. Repeat the synthesis for each of the four control hierarchies: one for benefits, one for costs, a third for opportunities, and a fourth for risks.
12. Synthesize the results from the four control hierarchies by multiplying the benefits by the opportunities and dividing by the costs multiplied by the risks. Then, read off the highest priority alternative or the desired mix of alternatives.

Some fundamental ideas in support of the ANP are:
1) The ANP is built on the AHP;
2) By allowing for dependence, the ANP goes beyond the AHP by including independence and hence also the AHP as a special case;
3) The ANP deals with dependence within a set of elements (inner dependence), and among different sets of elements (outer dependence);
4) The looser network structure of the ANP makes possible the representation of any decision problem without concern for what comes first and what comes next as in a hierarchy;
5) The ANP is a nonlinear structure that deals with sources, cycles, and sinks. A hierarchy is linear, with a goal in the top level, and the alternatives in the bottom level;
6) The ANP prioritizes not just elements but also groups or clusters of elements as is often necessary in the real world;
7) The ANP utilizes the idea of a control hierarchy or a control network to deal with different criteria, eventually leading to the analysis of benefits, opportunities, costs, and risks. By relying on control elements, the ANP parallels what the human brain does in combining different sense data as for example does the thalamus.

2. The Supermatrix of a Feedback System

Assume that we have a system of N clusters or components whereby the elements in each component interact or have an impact on or are influenced by some or all of the elements of that component or of another component with respect to a property governing the interactions of the entire system, such as energy or capital or political influence. Assume that component h, denoted by C_h, $h = 1, ..., N$, has n_h elements, which we denote by $e_{h1}, e_{h2}, \ldots, e_{hn_h}$. The impact of a given set of elements in a component on another element in the system is represented by a ratio scale priority vector derived from paired comparisons in the usual way.

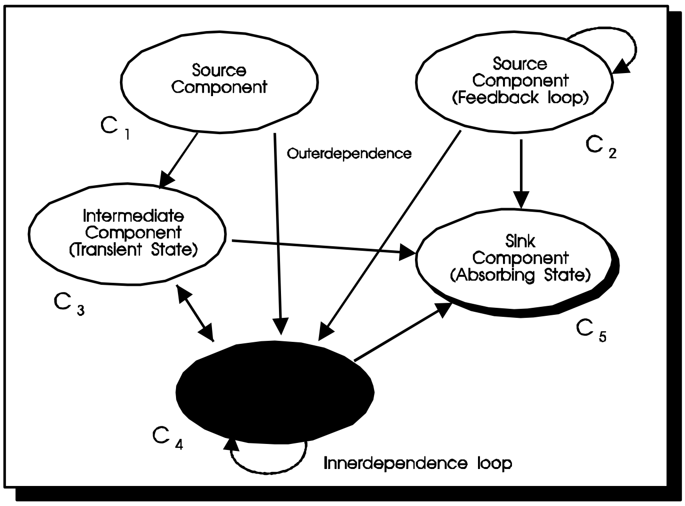


Figure 1. **Connections in a Network**

In Figure 1 no arrow feeds into a source component, no arrow leaves a sink component, and arrows both leave and feed into a transient component. Loops as in C_2 and C_4 feedback into the component itself. Each priority vector is derived and introduced in the appropriate position as a column vector in a supermatrix of impacts (with respect to one control criterion) displayed as shown below.

The Need for the Stochasticity of the Supermatrix

The supermatrix must first be reduced to a matrix, each of whose columns sums to unity, resulting in a *column stochastic* or simply a *stochastic* matrix. *If the matrix is stochastic, the limiting priorities depend on its reducibility, primitivity, and cyclicity, in all leading to four cases to consider* (see Table 1 below). Interaction in the supermatrix may be measured according to several different criteria. To display and relate the criteria, we need a separate *control hierarchy* that includes the criteria and priorities. For each criterion, a different supermatrix of impacts is developed, and in terms of that criterion the components are compared according to their relative impact (or absence of impact) on each other component at the top of the supermatrix, thus developing priorities to weight the block matrices of eigenvector columns under that component in the supermatrix. The resulting stochastic matrix is known as the weighted supermatrix. It needs to be stochastic to derive meaningful limiting priorities.

$$
W = \begin{array}{cc} & \\ & \\ C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots & \vdots \\ C_N & \begin{array}{c} e_{N1} \\ e_{N2} \\ \vdots \\ e_{Nn_N} \end{array} \end{array}
\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_N \\ e_{11}e_{12}\cdots e_{1n_1} & e_{2}e_{22}\cdots e_{2n_2} & & e_{N1}e_{N2}\cdots e_{Nn_N} \end{array} \\ \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \vdots & \vdots & \vdots\;\vdots\;\vdots & \vdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix} \end{array}
$$

where the *i,j* block of this matrix is given by

Table 1. **Characterization of W^{∞} in Terms of Eigenvalue Multiplicity.**

	Acyclic	Cyclic
Irreducible	$\lambda8_{max}$= 1 is a simple root	C other eigenvalues with modulus = 1 (they occur in conjugate pairs)
Reducible	$\lambda8_{max}$= 1 is a multiple root	C other eigenvalues with modulus = 1 (they occur in conjugate pairs)

In general the supermatrix is rarely stochastic because, in each column, it consists of several eigenvectors which each sums to one, and hence the entire column of the matrix may sum to an integer greater than one. The natural thing to do, which we all do in practice, is to determine the influence of the clusters on each cluster with respect to the control criterion. This yields an eigenvector of influence of all the clusters on each cluster. The priority of a component of such an eigenvector is used to weight all the elements in the block of the supermatrix that corresponds to the elements of both the influencing and the influenced cluster. The result is a stochastic supermatrix. This is not a forced way to make the matrix stochastic. It is natural. Why? Because the elements are compared among themselves and one needs information about the importance of the clusters to which they belong, to determine their relative overall weight among all the elements in the other clusters. Normalization would be meaningless, and such weighting does not call for normalization. Here is an example of why it is necessary to weight the priorities of the elements by those of their clusters: If one shouts into a room, Ladies and Gentlemen, the president, everyone is alerted and somewhat awed to expect to see the president of the United States because he is in the news so often. But if the announcement is then followed by, "of the garbage collection association," the priority immediately drops according to the importance of the group to which that president belongs. We cannot avoid such consideration.

3. The Control Hierarchy

Analysis of priorities in a system can be thought of in terms of a control hierarchy with dependence among its bottom-level subsystem arranged as a network (Figure 2). Dependence can occur within the clusters and between them. A control hierarchy at the top may be replaced by a control network with dependence among its clusters. More generally, one can have a cascading set of control networks, the outcome of one used to synthesize the outcomes of what it controls. For obvious reasons relating to the complexity of exposition, apart from a control hierarchy, we will not discuss such complex control structures here. A control hierarchy can also be involved in the network itself with feedback involved from the criteria to the elements of the network and back to the criteria to modify their influence. This kind of closed-circuit interaction between the operating parts and the criteria that drive the parts is likely to be prevalent in the brain.

A component or cluster in the AHP is a collection of elements whose function derives from the synergy of their interaction and hence has a higher-order function not found in any single element. A component is like the audio or visual component of a television set or like an arm or a leg, consisting of muscle and bone, in the human body. A mechanical cluster has no synergy value but is simply an aggregate of elements and is not what we mean by a component or cluster. The clusters of the system should generally be synergistically different from the elements themselves. Otherwise they would be a mechanical collection with no intrinsic meaning.

The criteria in the control hierarchy that are used for comparing the components are usually the major parent criteria whose subcriteria are used to compare the elements in the component. Thus the criteria for comparison of the components need to be more general than those of the elements because of the greater functional complexity of the components. Sometimes for convenience, interactions of both components and elements are examined in terms of the same criteria in the control hierarchy. Although one does that to economize on the effort spent, it is more meaningful to compare the clusters with respect to control criteria and to compare the elements with respect to subcriteria of the control criteria. Otherwise the process can lead to asking difficult questions in making the paired comparisons.

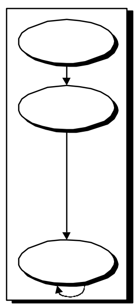
The control hierarchy, critical for ANP analysis, provides overriding criteria for comparing each type of interaction that is intended by the network representation. There are two types of control criteria (subcriteria). A control criterion may be directly connected to the structure as the goal of a hierarchy if the structure is in fact a hierarchy. In this case the control criterion is called a comparison-"linking" criterion. Otherwise a control criterion does not connect directly to the structure but "induces" comparisons in a network. In that case the control criterion is called a comparison-"inducing" criterion.

An example of dependence between the elements in a component which corresponds to a loop within the component is the input-output of materials among industries. The electric industry supplies electricity to other industries including itself. But it depends more on the coal industry than on its own electricity for operation and also more on the steel industry for its turbines.

To summarize, a control hierarchy is a hierarchy of criteria and subcriteria for which priorities are derived in the usual way with respect to the goal of the system being considered. The criteria are used to compare the components of a system, and the subcriteria are used to compare the elements. The generic question is; Given an element (in the same component or in another component) of the system or given a component of that system, how much more does a given element (component) of a pair influence that element (component) with respect to a control subcriterion (criterion)? The weights of the components are used to weight the blocks of the supermatrix corresponding to the component being influenced. The limiting priorities in each supermatrix are weighted by the priority of the corresponding subcriterion and the results are synthesized for all the subcriteria. If it should happen that an element or a component has no input, a zero is entered in the corresponding priority vector.

In each block of the supermatrix, a column is either a normalized eigenvector with possibly some zero entries, or all of its elements are equal to zero. In either case it is weighted by the priority of the corresponding cluster on the left. If it is zero, that column of the supermatrix must be normalized after weighting by the cluster's weights. This operation is equivalent to assigning a zero value to the cluster on the left when weighting a column of a block with zero entries and then renormalizing the weights of the remaining clusters.

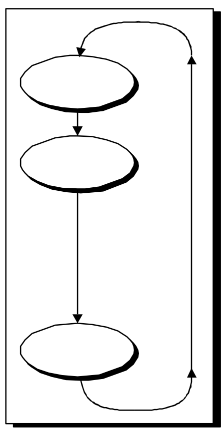
Figures 2 and 3 and their accompanying supermatrices represent a hierarchy and a holarchy respectively whose bottom level is connected to its top level of criteria and has no single element goal as in a hierarchy. Note the difference between the two.



$$W = \begin{bmatrix} 0 & 0 & 0 & \cdots & \bullet & 0 & 0 \\ W_{21} & 0 & 0 & \cdots & \bullet & 0 & 0 \\ 0 & W_{32} & 0 & \cdots & \bullet & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \bullet & \bullet & \bullet & \cdots & W_{n-1,n-2} & \bullet & \bullet \\ 0 & 0 & 0 & \cdots & \bullet & W_{n,n-1} & I \end{bmatrix}$$

Figure 2. The Structure and Supermatrix of a Hierarchy.

The entry in the last row and column of the supermatrix of a hierarchy is the identity matrix *I*. In this case the limiting supermatrix is obtained by raising *W* to powers.



$$W = \begin{bmatrix} 0 & 0 & 0 & \bullet & \bullet & \bullet & 0 & W_{1,n} \\ W_{21} & 0 & 0 & \bullet & \bullet & \bullet & 0 & 0 \\ 0 & W_{32} & 0 & \bullet & \bullet & \bullet & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & W_{n-1,n-2} & \bullet & \bullet \\ 0 & 0 & 0 & \bullet & \bullet & \bullet & W_{n,n-1} & 0 \end{bmatrix}$$

Figure 3. The Structure and Supermatrix of a Holarchy.

A system may be generated from a hierarchy by increasing its connections gradually so that pairs of components are connected as desired and some components have an inner dependence loop.

4. Example - Market Share in the Hamburger Industry

This example applies ANP to the problem of predicting the market share for the big three companies in the hamburger fast food industry: McDonald's, Burger King and Wendy's. These three firms are very competitive and offer a similar menu of hamburgers and other food items. To attract new customers and to retain their own, they have to compete by setting reasonable prices, making quality hamburgers, and promoting support of the community by sponsoring charity events and other public services.

The ANP model consists of clusters of elements connected by their dependence to one another. A cluster therefore allows one to think about grouping elements that share a set of attributes. The marketing mix is an example of a cluster whose elements are: price, product, promotion and location. The basic requirement when identifying clusters and their elements is that the elements be similar.

For this simple network model, we consider a single control criterion: economic influence. Figure 4 below shows the connections between clusters; a cluster is connected to another cluster when at least one element in it is connected to at least one element in another cluster. The elements themselves are not shown in this figure. Except for the customer group cluster, inner dependence exists for all other clusters. In that case, the connections between elements are in the same cluster.

Structure

The structure of the model is described by its clusters and elements, and by the connection between them. These connections indicate the flow of influence between the elements. For example, with respect to promotion, is nutrition more or less important than packaging; and if so, by how much. In other words, given a limited budget, the company has to prioritize spending on promoting one message over others. The importance of this comparison is the basis for connecting Promotion (in the *Marketing Mix* cluster) to elements in the *Contemporary Issues* cluster (packaging, nutrition, waste disposal and recycling). The reverse connection is also important because management is aware of themes in the *contemporary* cluster influence elements in the *marketing mix* differently. For example, using more costly materials that can be recycled, may raise prices more than the promotion of this fact to the public may bring in new business. Through this process of analyzing dependencies, the prevailing understanding of the marketplace is mapped out in the ANP model.

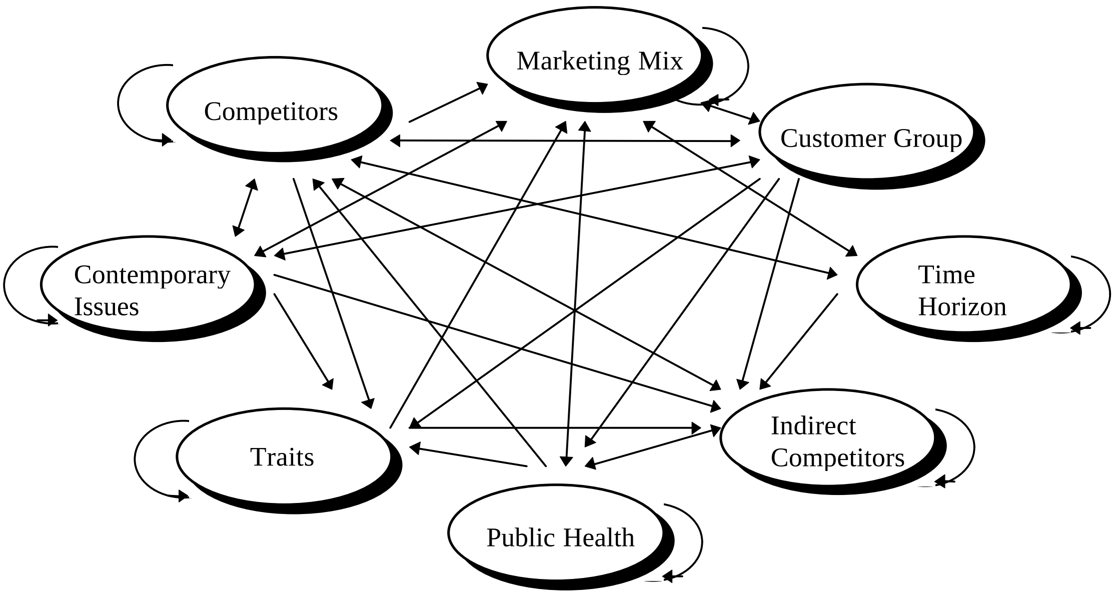


Figure 4. Overall Goal: Market Share of Competitor Group

Direct Competitors Cluster
The big three companies - McDonald's, Burger King and Wendy's - are elements in the *Competitors* cluster. Each is a significant competitor with the other competitors warranting continuous monitoring and quick responses. Pairwise comparisons allow us to evaluate the importance of the two competitors with respect to their influence on each cluster's market share.

Indirect Competitors Cluster
These are companies offering alternatives to hamburgers but competing for the same overall customer base. They include: subs (Miami, Subway, corner sandwich), fried chicken (KFC, BC), Pizza (Pizza Hut), Mexican (Taco Bell), Chinese, Steak (Ponderosa), and Diners (Full service and formal). Companies in this cluster compete indirectly against the big three by offering customers alternate foods and tastes and like the direct competitors, also compete and influence one another for market share.

Customer Cluster
Four basic consumer groups considered include: the white collar professional, the blue collar laborer, the student and the family. These segments help us evaluate the influence other elements may have on each of them. For example, price may influence each segment differently. Students and families on a tight budget may be more concerned with price but the working individual may instead be more concerned with convenience. This is the only cluster without inner dependence as customer segments are perceived not to influence one another.

Marketing Mix Cluster
Price in this model refers to the average price of the typical product, for example, the price of a Big Mac. However, the model could be extended to include other products and their prices. The typical product is that which the company sells the most. For location, we consider the number of established outlets, and for promotion we consider specially packaged lunch deals that would usually be more expensive when bought separately.

Contemporary Concerns Cluster
This cluster includes issues the public is aware of about the fast food industry. For example, CNN raised questions about the nutritional value of fast foods in a news report. Also, environmental groups pressure companies into paying more attention to preserving Nature and the environment by practicing recycling, properly disposing of waste, and by not over-packaging their product. These factors help raise the cost and change the routine of doing business but may also attract more customers.

Public Health Concerns Cluster
Periodic outbreak of meat contamination always serves to create panic as to the safety of the meat supply channels and the adequacy of regulation and inspection. Consumers have also become highly sensitized to other evidence of hygiene related to for example, the site (clean tables, floors) and the personnel (uniforms, hats, gloves, and the handling of money together with food).

Traits Cluster
The elements of this cluster consist of attributes customers may use or recall when judging one eatery over another. They are 1) the speed of service, 2) available seating and parking, 3) whether there is a delivery service and 4) the presence or absence of a drive through facility.

Time Horizon Cluster
This cluster makes managers think about short and medium term measures to improve market share by connecting other elements to this cluster.

Paired Comparisons
In making paired comparisons of homogeneous elements, ratios are estimated by using a 1 to 9 fundamental scale of absolute numbers to compare two alternatives with respect to an attribute, with the smaller or lesser alternative as the unit for that attribute. To estimate the larger one as a multiple of that unit, one assigns to it an absolute number from the fundamental scale. This process is done for every pair. Rather then assigning two numbers w_i and w_j and forming the ratio w_i / w_j, we assign a single number between 1 to 9 to represent the ratio $(w_i / w_j) / 1$. The

absolute number from the scale is an approximation to the ratio w_i / w_j. The derived scale gives us w_i and w_j. This is the central point in the relative measurement approach of the AHP.

Paired comparisons are needed for all the connections in the model. For example, Burger King is connected to elements in the *Customer Group* cluster. There would be a set of numerical judgments and the derived priority weights from these judgments, represented in the reciprocal matrix shown in Table 2 below.

Table 2. Pairwise judgments of the Customer Group for Burger King

Burger King	White Collar	Blue Collar	Student	Family	Priorities
White collar	1	4	5	1 / 3	0.299
Blue collar	1 / 4	1	4	1 / 3	0.138
Student	1 / 5	1 / 4	1	1/ 7	0.051
Family	3	3	7	1	0.512

The judgments in the first row of this reciprocal matrix say that in considering the market share of Burger King, White Collar workers are four times more important than Blue Collar workers, White Collar workers are 5 times more important than Students but only a third as important as Family. The derived priorities in the last column are computed by raising the reciprocal matrix to arbitrarily large powers and then normalizing their row sums. Each priority vector entries sum to unity and are placed in their corresponding location in the supermatrix before raising the supermatrix to powers.

The Supermatrix

A supermatrix is a two-dimensional matrix of elements by elements. The priority vectors from the paired comparisons appear in the appropriate column of the supermatrix. Tables 3, 5 and 6 are given in the appendix. In the supermatrix of Table 3, the sum of each column corresponds to the number of comparison sets. If Burger King only had two comparison sets, then the column would sum to 2 because each priority vector sums to 1. In Table 4 we establish priorities for the clusters as they impact each cluster according to market share. The weights in the column of Table 3 corresponding to the cluster are multiplied by the weight of the cluster. As a result each column of the supermatrix sums to one. The resulting weighted matrix is column stochastic (Table 5). We are concerned with the limiting priorities of the matrix as it represents all possible interactions in the system (Table 6).

Table 4. Cluster Weights with Respect to Economic Impact Control Criterion of the Hamburger Model

	COMPETITORS	CUSTOMER GROUPS	MARKETING MIX	CONTEMPORARY ISSUES	PUBLIC HEALTH	TRAITS	INDIRECT COMPETITORS	TIME HORIZON
Competitors	0.169	0.200	0.151	0.222	0.249	0.252	0.193	
Customer Group	0.186		0.180	0.222	0.175	0.252	0.178	0.391
Marketing Mix	0.139	0.181	0.162	0.201	0.157	0.218	0.112	0.195
Contemporary	0.103	0.113	0.097	0.127			0.092	
Public Health	0.167	0.163	0.170		0.220		0.218	
Traits	0.074	0.113	0.071	0.101	0.088	0.109	0.083	
Indirect Competitors	0.162	0.229	0.106	0.127	0.112	0.169	0.125	0.276
Time Horizon			0.064					0.138

The predicted market share is obtained in the row for each firm in the limiting supermatrix. These predictions and the actual market shares as appeared in the Market Share Reporter (Darney and Reddy, 1992) are shown in Table 8:

Table 8. Predicted and Actual Market Shares for Direct Competitors

	Market	Share
Company	Predicted %	Actual %
McDonald's	62.9	58.2
Burger King	23.9	28.6
Wendy's	13.2	13.2

We also found for the indirect competitor the following result in Table 9:

Table 9. Predicted and Actual Market Shares for Indirect Competitors

	Market	Share
Company	Predicted %	Actual %
PIZZA	33.7	37.0
CHICKEN	26.0	28.4
MEXICAN	15.2	22.8
SUBS	25.0	11.7

5. On the Evaluation of Alternatives in Terms of Both Attributes and Consequences

An indication of how the mind limits both our imagination and our ability to cope with complexity is that we are easily trapped by the tools we use. They can prevent us from even considering ideas for which they are not suitable. Our traditional linear structures of decision making derive priorities or utilities for alternatives in terms of attributes and criteria. If we were to consider consequences, whose importance derives from the priorities of the alternatives and also affects these priorities, then we would have to use devious ways to deal with them in our present linear structures. But it can be done easily and far more correctly in a network structure by simply including a cluster of consequences. This cluster would enable one to create a feedback cycle with the alternatives that would modify their priorities as determined by the influence of other clusters in that network.

Most decision making has been concerned with judging alternatives by what goes into them. But considering consequences may be more important than ensuring that every attribute has been included. Linear hierarchic methods of decision making can only project forward. That is not enough. We need to also look into what comes out of the alternatives as consequences or effects and choose the best alternative after such consideration. Willingness to think about consequences means that we not only own the present and dictate our likes and dislikes, but also own and try to control the future by indicating what promising outcomes. The consequences must also need be considered in the frameworks of different benefits, costs, opportunities and risks.

As an example of consequences, for people who seek immediate satisfaction from different foods should they not also consider the consequences that eating the food might have on their health? Should a couple have children if their ability to raise children properly is not taken into consideration? The question then is not what is the best way to have a child, but who should have a child and who should not? Should a nation be punished for the undesirable acts of its leaders to give satisfaction to those who can deal the punishment without consideration of who actually pays the penalty and what effect it has on their future?

References

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APPENDIX

Table 3. The Initial Supermatrix for the Hamburger Model

Initial	MCDO	BURG	WEND	WHIT	BLUE	STUD	FAMI	PRIC	PROD	LOCA	DEAL	NUTR	RECY	WAST	OVER	PERS	FOOD	SITE	SPEE	SEAT	PARK	DELI	DRIV	SUBS	CHIC	PIZZ	MEXI	CHIN	STEA	DINE	SHOR	MEDI
MCDONAL	0.0000	0.9000	0.8330	0.7560	0.7350	0.7350	0.6990	0.6960	0.7050	0.7050	0.6910	0.7170	0.7700	0.7310	0.6910	0.7420	0.7310	0.6670	0.6250	0.0000	0.0000	0.0000	0.5400	0.7050	0.7010	0.6740	0.6250	0.6740	0.6250	0.6140	0.6370	0.6960
BURGER K	0.5000	0.0000	0.1670	0.1880	0.2070	0.2070	0.2370	0.2290	0.2110	0.2110	0.2180	0.2050	0.1620	0.1880	0.2180	0.1830	0.1880	0.2220	0.2380	0.0000	0.0000	0.0000	0.2970	0.2110	0.1930	0.2260	0.2380	0.2260	0.2380	0.2680	0.2580	0.2290
WENDY'S	0.5000	0.1000	0.0000	0.0560	0.0580	0.0580	0.0640	0.0750	0.0840	0.0840	0.0910	0.0780	0.0680	0.0810	0.0910	0.0750	0.0810	0.1110	0.1360	0.0000	0.0000	0.0000	0.1630	0.0840	0.1060	0.1010	0.1360	0.1010	0.1360	0.1170	0.1050	0.0750
WHITE CO	0.3080	0.2990	0.1420	0.0000	0.0000	0.0000	0.0000	0.0840	0.2570	0.1720	0.1090	0.2390	0.1680	0.2000	0.2000	0.2760	0.2660	0.2930	0.4000	0.0000	0.0000	0.2500	0.2980	0.1670	0.2450	0.1110	0.2000	0.2820	0.3070	0.2890	0.0000	0.0000
BLUE COL	0.1010	0.1380	0.2760	0.0000	0.0000	0.0000	0.0000	0.1470	0.1980	0.3170	0.1890	0.2810	0.1980	0.2000	0.2000	0.1180	0.0910	0.0940	0.2000	0.0000	0.0000	0.2500	0.2100	0.3330	0.1760	0.2720	0.2000	0.1490	0.1190	0.1290	0.0000	0.0000
STUDENTS	0.0540	0.0510	0.0600	0.0000	0.0000	0.0000	0.0000	0.4760	0.0940	0.3890	0.3510	0.1400	0.3950	0.4000	0.4000	0.1180	0.0910	0.0940	0.2000	0.0000	0.0000	0.2500	0.2460	0.3330	0.2810	0.3620	0.4000	0.0750	0.0690	0.0710	0.0000	0.0000
FAMILIES	0.5370	0.5120	0.5220	0.0000	0.0000	0.0000	0.0000	0.2930	0.4510	0.1220	0.3510	0.3400	0.2390	0.2000	0.2000	0.4880	0.5520	0.5180	0.2000	0.0000	0.0000	0.2500	0.2460	0.1670	0.2980	0.2550	0.2000	0.4930	0.5050	0.5100	0.0000	0.0000
PRICE	0.3440	0.2600	0.3010	0.1930	0.2370	0.2150	0.1970	0.0000	1.0000	0.0000	0.2500	0.6670	0.7500	0.7500	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1590	0.1250	0.3500	0.2050	0.1680	0.1520	0.2400	0.2120	0.1630	0.2500	0.1960
PRODUCT	0.1920	0.2600	0.3680	0.4520	0.1370	0.1060	0.1580	0.2500	0.0000	0.0000	0.5000	0.3330	0.2500	0.2500	0.5000	1.0000	1.0000	1.0000	1.0000	0.2000	0.2500	0.0750	0.1250	0.2160	0.1880	0.1980	0.1170	0.1160	0.1930	0.4900	0.2500	0.3110
LOCATION	0.3900	0.3810	0.2000	0.1430	0.0790	0.1890	0.0900	0.2500	0.0000	0.0000	0.2500	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8000	0.7500	0.5670	0.7500	0.1120	0.1440	0.2390	0.2900	0.1850	0.1430	0.2310	0.0000	0.0000
DEALS	0.0740	0.0990	0.1310	0.2120	0.5470	0.4910	0.5540	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1990	0.0000	0.3220	0.4630	0.3950	0.4410	0.4580	0.4520	0.1160	0.5000	0.4930
NUTRITIO	0.6260	0.6850	0.6740	0.6380	0.5620	0.4830	0.5990	0.2500	1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5190	0.5000	0.5710	0.5710	0.5000	0.5710	0.5710	0.0000	0.0000
RECYCLIN	0.1990	0.1070	0.1920	0.1720	0.1990	0.2760	0.1800	0.2500	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.3330	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1500	0.1670	0.1430	0.1430	0.1670	0.1430	0.1430	0.0000	0.0000
WASTE DI	0.0640	0.0570	0.0650	0.0640	0.1400	0.1410	0.1330	0.2500	0.0000	0.0000	0.0000	0.0000	0.3330	0.0000	0.6670	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1940	0.1670	0.1430	0.1430	0.1670	0.1430	0.1430	0.0000	0.0000
OVER PAC	0.1110	0.1500	0.0690	0.1260	0.0990	0.1010	0.0880	0.2500	0.0000	0.0000	0.0000	0.0000	0.6670	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1370	0.1670	0.1430	0.1430	0.1670	0.1430	0.1430	0.0000	0.0000
PERSONNE	0.4930	0.5280	0.5280	0.4930	0.4930	0.5280	0.5280	0.3330	0.2680	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3270	0.3330	0.3330	0.3330	0.3330	0.3330	0.3330	0.0000	0.0000
FOOD HYG	0.3110	0.3330	0.3330	0.3110	0.3110	0.3330	0.3330	0.3330	0.6150	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4130	0.3330	0.3330	0.3330	0.3330	0.3330	0.3330	0.0000	0.0000
SITE HYG	0.1960	0.1400	0.1400	0.1960	0.1960	0.1400	0.1400	0.3330	0.1170	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2600	0.3330	0.3330	0.3330	0.3330	0.3330	0.3330	0.0000	0.0000
SPEED OF	0.6740	0.6960	0.5940	0.3490	0.4540	0.4290	0.2110	0.2000	1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.1670	0.0000	0.1270	0.2500	0.2770	0.3620	0.3260	0.3520	0.2840	0.2730	0.2730	0.0000	0.0000
SEATING	0.2260	0.2290	0.2490	0.1640	0.2040	0.2580	0.2160	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.7500	0.0000	0.0000	0.0000	0.2230	0.2500	0.2470	0.2270	0.1180	0.1330	0.2480	0.2730	0.2730	0.0000	0.0000
PARKING	0.1010	0.0750	0.1570	0.1410	0.0910	0.0870	0.1660	0.2000	0.0000	0.2400	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.4040	0.0000	0.4880	0.2500	0.1920	0.1370	0.1520	0.1630	0.1860	0.2730	0.2730	0.0000	0.0000
DELIVERY	0.0000	0.0000	0.0000	0.1070	0.1950	0.1560	0.1170	0.2000	0.0000	0.5500	0.0000	0.0000	0.0000	0.0000	0.2500	0.0000	0.0000	0.1250	0.0000	0.1910	0.5000	0.0000	0.2500	0.1270	0.1370	0.2670	0.1890	0.1810	0.0910	0.0910	0.0000	0.0000
DRIVE TH	0.0000	0.0000	0.0000	0.2400	0.0560	0.0700	0.2890	0.2000	0.0000	0.2100	0.0000	0.0000	0.0000	0.0000	0.2500	0.0000	0.0000	0.1250	0.0000	0.2390	0.5000	0.1620	0.0000	0.1570	0.1370	0.1370	0.1630	0.1010	0.0910	0.0910	0.0000	0.0000
SUBS	0.2560	0.1310	0.2800	0.0570	0.2260	0.2360	0.0680	0.0540	0.1080	0.0950	0.0600	0.0800	0.1430	0.1430	0.1430	0.1430	0.2830	0.1430	0.1430	0.0000	0.0000	0.0000	0.1430	0.2870	0.1670	0.1290	0.1640	0.0880	0.0640	0.0750	0.1430	0.1430
CHICKEN	0.1370	0.2340	0.1800	0.1780	0.1620	0.1790	0.1810	0.1150	0.1790	0.0740	0.0950	0.1420	0.1430	0.1430	0.1430	0.1430	0.1480	0.1430	0.1430	0.0000	0.0000	0.0000	0.1430	0.1560	0.2200	0.2150	0.1610	0.1860	0.1140	0.0750	0.1430	0.1430
PIZZA	0.3720	0.3560	0.2830	0.1060	0.3810	0.2870	0.0690	0.0840	0.1330	0.1270	0.1090	0.0810	0.1430	0.1430	0.1430	0.1430	0.3270	0.1430	0.1430	0.0000	0.0000	0.0000	0.1430	0.1900	0.2020	0.2150	0.1840	0.1630	0.0850	0.0750	0.1430	0.1430
MEXICAN	0.1050	0.1440	0.1000	0.0780	0.0520	0.1140	0.0560	0.0540	0.0640	0.0680	0.0740	0.0860	0.1430	0.1430	0.1430	0.1430	0.0580	0.1430	0.1430	0.0000	0.0000	0.0000	0.1430	0.1030	0.0900	0.0830	0.2250	0.0750	0.0620	0.0750	0.1430	0.1430
CHINESE	0.0460	0.0600	0.0690	0.1970	0.0670	0.0920	0.2300	0.1740	0.2490	0.1580	0.1700	0.2040	0.1430	0.1430	0.1430	0.1430	0.0530	0.1430	0.1430	0.0000	0.0000	0.0000	0.1430	0.1020	0.1250	0.1370	0.0880	0.1620	0.2280	0.2070	0.1430	0.1430
STEAK	0.0440	0.0420	0.0500	0.2300	0.0730	0.0520	0.2210	0.1890	0.2670	0.2390	0.2100	0.2040	0.1430	0.1430	0.1430	0.1430	0.0490	0.1430	0.1430	0.0000	0.0000	0.0000	0.1430	0.0810	0.0980	0.1100	0.0880	0.1630	0.2190	0.2070	0.1430	0.1430
DINERS	0.0390	0.0330	0.0370	0.1540	0.0390	0.0410	0.1750	0.3290	0.0000	0.2390	0.2820	0.2040	0.1430	0.1430	0.1430	0.1430	0.0830	0.1430	0.1430	0.0000	0.0000	0.0000	0.1430	0.0810	0.0980	0.1100	0.0880	0.1630	0.2280	0.2850	0.1430	0.1430
SHORT TE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
MEDIUM T	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000

Table 5. The Weighted Supermatrix for the Hamburger Model

Weighted	MCDO	BURG	WEND	WHIT	BLUE	STUD	FAMI	PRIC	PROD	LOCA	DEAL	NUTR	RECY	WAST	OVER	PERS	FOOD	SITE	SPEE	SEAT	PARK	DELI	DRIV	SUBS	CHIC	PIZZ	MEXI	CHIN	STEA	DINE	SHOR	MEDI
MCDONAL	0.0000	0.1521	0.1408	0.1512	0.1470	0.1470	0.1398	0.1051	0.1065	0.1065	0.1043	0.1592	0.1709	0.1623	0.1534	0.1848	0.1820	0.1661	0.1575	0.0000	0.0000	0.0000	0.1361	0.1361	0.1353	0.1301	0.1206	0.1301	0.1206	0.1185	0.2491	0.2721
BURGER K	0.0845	0.0000	0.0282	0.0376	0.0414	0.0414	0.0474	0.0346	0.0319	0.0319	0.0329	0.0455	0.0360	0.0417	0.0484	0.0456	0.0468	0.0553	0.0600	0.0000	0.0000	0.0000	0.0748	0.0407	0.0372	0.0436	0.0459	0.0436	0.0459	0.0517	0.1009	0.0895
WENDY'S	0.0845	0.0169	0.0000	0.0112	0.0116	0.0116	0.0128	0.0113	0.0127	0.0127	0.0137	0.0173	0.0151	0.0180	0.0202	0.0187	0.0202	0.0276	0.0343	0.0000	0.0000	0.0000	0.0411	0.0162	0.0205	0.0195	0.0262	0.0195	0.0262	0.0226	0.0411	0.0293
WHITE CO	0.0573	0.0556	0.0264	0.0000	0.0000	0.0000	0.0000	0.0151	0.0463	0.0310	0.0196	0.0531	0.0373	0.0444	0.0444	0.0483	0.0466	0.0513	0.1008	0.0000	0.0000	0.0630	0.0751	0.0297	0.0436	0.0198	0.0356	0.0502	0.0546	0.0514	0.0000	0.0000
BLUE COL	0.0188	0.0257	0.0513	0.0000	0.0000	0.0000	0.0000	0.0265	0.0356	0.0571	0.0340	0.0624	0.0440	0.0444	0.0444	0.0207	0.0159	0.0165	0.0504	0.0000	0.0000	0.0630	0.0529	0.0593	0.0313	0.0484	0.0356	0.0265	0.0212	0.0230	0.0000	0.0000
STUDENTS	0.0100	0.0095	0.0112	0.0000	0.0000	0.0000	0.0000	0.0857	0.0169	0.0700	0.0632	0.0311	0.0877	0.0888	0.0888	0.0207	0.0159	0.0165	0.0504	0.0000	0.0000	0.0630	0.0620	0.0593	0.0500	0.0644	0.0712	0.0134	0.0123	0.0126	0.0000	0.0000
FAMILIES	0.0999	0.0952	0.0971	0.0000	0.0000	0.0000	0.0000	0.0527	0.0812	0.0220	0.0632	0.0755	0.0531	0.0444	0.0444	0.0854	0.0966	0.0906	0.0504	0.0000	0.0000	0.0630	0.0620	0.0297	0.0530	0.0454	0.0356	0.0878	0.0899	0.0908	0.0000	0.0000
PRICE	0.0478	0.0361	0.0418	0.0349	0.0429	0.0389	0.0357	0.0000	0.1620	0.0000	0.0405	0.1341	0.1508	0.1508	0.1005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0347	0.0273	0.0392	0.0230	0.0188	0.0170	0.0269	0.0237	0.0183	0.0488	0.0382
PRODUCT	0.0267	0.0361	0.0512	0.0818	0.0248	0.0192	0.0286	0.0405	0.0000	0.0000	0.0810	0.0669	0.0503	0.0503	0.1005	0.1570	0.1570	0.1570	0.2180	0.0436	0.0545	0.0164	0.0273	0.0242	0.0211	0.0222	0.0131	0.0130	0.0216	0.0549	0.0488	0.0606
LOCATION	0.0542	0.0530	0.0278	0.0259	0.0143	0.0342	0.0163	0.0405	0.0000	0.0000	0.0405	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1744	0.1635	0.1236	0.1635	0.0125	0.0161	0.0268	0.0325	0.0207	0.0160	0.0259	0.0000	0.0000
DEALS	0.0103	0.0138	0.0182	0.0384	0.0990	0.0889	0.1003	0.0810	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0434	0.0000	0.0361	0.0519	0.0442	0.0494	0.0513	0.0506	0.0130	0.0975	0.0961
NUTRITIO	0.0645	0.0706	0.0694	0.0721	0.0635	0.0546	0.0677	0.0243	0.0970	0.0000	0.0970	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0477	0.0460	0.0525	0.0525	0.0460	0.0525	0.0525	0.0000	0.0000
RECYCLIN	0.0205	0.0110	0.0198	0.0194	0.0225	0.0312	0.0203	0.0243	0.0000	0.0000	0.0000	0.0000	0.0000	0.0635	0.0423	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0138	0.0154	0.0132	0.0132	0.0154	0.0132	0.0132	0.0000	0.0000
WASTE DI	0.0066	0.0059	0.0067	0.0072	0.0158	0.0159	0.0150	0.0243	0.0000	0.0000	0.0000	0.0000	0.0423	0.0000	0.0847	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0178	0.0154	0.0132	0.0132	0.0154	0.0132	0.0132	0.0000	0.0000
OVER PAC	0.0114	0.0155	0.0071	0.0142	0.0112	0.0114	0.0099	0.0243	0.0000	0.0000	0.0000	0.0000	0.0847	0.0635	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0126	0.0154	0.0132	0.0132	0.0154	0.0132	0.0132	0.0000	0.0000
PERSONNE	0.0823	0.0882	0.0882	0.0804	0.0804	0.0861	0.0861	0.0566	0.0456	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0713	0.0726	0.0726	0.0726	0.0726	0.0726	0.0726	0.0000	0.0000
FOOD HYG	0.0519	0.0556	0.0556	0.0507	0.0507	0.0543	0.0543	0.0566	0.1046	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0900	0.0726	0.0726	0.0726	0.0726	0.0726	0.0726	0.0000	0.0000
SITE HYG	0.0327	0.0234	0.0234	0.0319	0.0319	0.0228	0.0228	0.0566	0.0199	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1100	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0567	0.0726	0.0726	0.0726	0.0726	0.0726	0.0726	0.0000	0.0000
SPEED OF	0.0499	0.0515	0.0440	0.0394	0.0513	0.0485	0.0238	0.0142	0.0710	0.0000	0.0710	0.0000	0.0000	0.0000	0.0505	0.0000	0.0000	0.0000	0.0000	0.0182	0.0000	0.0138	0.0273	0.0230	0.0300	0.0271	0.0292	0.0236	0.0227	0.0227	0.0000	0.0000
SEATING	0.0167	0.0169	0.0184	0.0185	0.0231	0.0292	0.0244	0.0142	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0660	0.0000	0.0000	0.0000	0.0243	0.0273	0.0205	0.0188	0.0098	0.0110	0.0206	0.0227	0.0227	0.0000	0.0000
PARKING	0.0075	0.0055	0.0116	0.0159	0.0103	0.0098	0.0188	0.0142	0.0000	0.0170	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0440	0.0000	0.0532	0.0273	0.0159	0.0114	0.0126	0.0135	0.0154	0.0227	0.0227	0.0000	0.0000
DELIVERY	0.0000	0.0000	0.0000	0.0121	0.0220	0.0176	0.0132	0.0142	0.0000	0.0391	0.0000	0.0000	0.0000	0.0000	0.0253	0.0000	0.0000	0.0110	0.0000	0.0208	0.0545	0.0000	0.0273	0.0105	0.0114	0.0222	0.0157	0.0150	0.0076	0.0076	0.0000	0.0000
DRIVE TH	0.0000	0.0000	0.0000	0.0271	0.0063	0.0079	0.0327	0.0142	0.0000	0.0149	0.0000	0.0000	0.0000	0.0000	0.0253	0.0000	0.0000	0.0110	0.0000	0.0261	0.0545	0.0177	0.0000	0.0130	0.0114	0.0114	0.0135	0.0084	0.0076	0.0076	0.0000	0.0000
SUBS	0.0415	0.0212	0.0454	0.0131	0.0518	0.0540	0.0156	0.0057	0.0114	0.0101	0.0064	0.0102	0.0182	0.0182	0.0182	0.0160	0.0317	0.0160	0.0242	0.0000	0.0000	0.0000	0.0242	0.0359	0.0209	0.0161	0.0205	0.0110	0.0080	0.0094	0.0395	0.0395
CHICKEN	0.0222	0.0379	0.0292	0.0408	0.0371	0.0410	0.0414	0.0122	0.0190	0.0078	0.0101	0.0180	0.0182	0.0182	0.0182	0.0160	0.0166	0.0160	0.0242	0.0000	0.0000	0.0000	0.0242	0.0195	0.0275	0.0269	0.0201	0.0233	0.0143	0.0094	0.0395	0.0395
PIZZA	0.0603	0.0577	0.0458	0.0243	0.0872	0.0657	0.0158	0.0089	0.0141	0.0135	0.0116	0.0103	0.0182	0.0182	0.0182	0.0160	0.0366	0.0160	0.0242	0.0000	0.0000	0.0000	0.0242	0.0238	0.0253	0.0269	0.0230	0.0204	0.0106	0.0094	0.0395	0.0395
MEXICAN	0.0170	0.0233	0.0162	0.0179	0.0119	0.0261	0.0128	0.0057	0.0068	0.0072	0.0078	0.0109	0.0182	0.0182	0.0182	0.0160	0.0065	0.0160	0.0242	0.0000	0.0000	0.0000	0.0242	0.0129	0.0113	0.0104	0.0281	0.0094	0.0078	0.0094	0.0395	0.0395
CHINESE	0.0075	0.0097	0.0112	0.0451	0.0153	0.0211	0.0527	0.0184	0.0264	0.0167	0.0180	0.0259	0.0182	0.0182	0.0182	0.0160	0.0059	0.0160	0.0242	0.0000	0.0000	0.0000	0.0242	0.0128	0.0156	0.0171	0.0110	0.0203	0.0285	0.0259	0.0395	0.0395
STEAK	0.0071	0.0068	0.0081	0.0527	0.0167	0.0119	0.0506	0.0200	0.0283	0.0253	0.0223	0.0259	0.0182	0.0182	0.0182	0.0160	0.0055	0.0160	0.0242	0.0000	0.0000	0.0000	0.0242	0.0101	0.0123	0.0138	0.0110	0.0204	0.0274	0.0259	0.0395	0.0395
DINERS	0.0063	0.0053	0.0060	0.0353	0.0089	0.0094	0.0401	0.0349	0.0000	0.0253	0.0299	0.0259	0.0182	0.0182	0.0182	0.0160	0.0093	0.0160	0.0242	0.0000	0.0000	0.0000	0.0242	0.0101	0.0123	0.0138	0.0110	0.0204	0.0285	0.0356	0.0395	0.0395
SHORT TE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0320	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1380
MEDIUM T	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0320	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1380	0.0000

Table 6. The Synthesized or Limiting Global Supermatrix for the Hamburger Model

Global	MCDO	BURG	WEND	WHIT	BLUE	STUD	FAMI	PRIC	PROD	LOCA	DEAL	NUTR	RECY	WAST	OVER	PERS	FOOD	SITE	SPEE	SEAT	PARK	DELI	DRIV	SUBS	CHIC	PIZZ	MEXI	CHIN	STEA	DINE	SHOR	MEDI
MCDONAL	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298	0.1298
BURGER K	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478	0.0478
WENDY'S	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266	0.0266
WHITE CO	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414	0.0414
BLUE COL	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312	0.0312
STUDENTS	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323	0.0323
FAMILIES	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654	0.0654
PRICE	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447	0.0447
PRODUCT	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647	0.0647
LOCATION	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386	0.0386
DEALS	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281	0.0281
NUTRITIO	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435	0.0435
RECYCLIN	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118	0.0118
WASTE DI	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083	0.0083
OVER PAC	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093
PERSONNE	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601	0.0601
FOOD HYG	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485	0.0485
SITE HYG	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326	0.0326
SPEED OF	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302	0.0302
SEATING	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145	0.0145
PARKING	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119	0.0119
DELIVERY	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122	0.0122
DRIVE TH	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113	0.0113
SUBS	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230	0.0230
CHICKEN	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237	0.0237
PIZZA	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306	0.0306
MEXICAN	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144	0.0144
CHINESE	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209	0.0209
STEAK	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213	0.0213
DINERS	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187	0.0187
SHORT TE	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013
MEDIUM T	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013	0.0013