

Alternative to Government Revenue Guarantees: Dynamic Revenue Insurance Contracts

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Abstract: Public private partnerships (PPPs) are arrangements under which the private sector supplies infrastructure assets and services that traditionally have been provided by the public sector. Public authorities may enhance the marketability of PPP projects by offering revenue guarantees. However, government revenue guarantees can pose significant fiscal risks for the issuing government, particularly during economic crises. This paper presents a new type of revenue risk hedging contract, the dynamic (flexible) revenue insurance contract, which can be offered as an alternative to the conventional government guarantees. This new contract gives PPP stakeholders other than the government the opportunity to participate in the revenue risk coverage. Potential revenue risk insurers include international financial institutions, export credit agencies, and private insurance companies. The key feature of these new contracts is that they facilitate the pooling of project revenue insurers by accommodating insurer financial and risk preferences. These contracts are modeled as multiple exercise options and priced by using two different Monte Carlo methods, the multiple exercise boundary method and the multiple least-squares method. Because of its inherent flexibility, the dynamic revenue insurance contract offers risk coverage similar to the conventional revenue guarantee at much lower cost to the government. A numerical example on a build operate transfer toll road project shows the substantial cost reduction sustained by the government. DOI: 10.1061/(ASCE)IS.1943-555X.0000145. © 2013 American Society of Civil Engineers.

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Introduction

Governments worldwide face constant demand for developing new infrastructure projects and for funding the renewal, maintenance, and operation of existing infrastructure systems. In a public private partnership (PPP), the private sector supplies infrastructure assets and services that are usually provided by the public sector (Esty 2003). The benefits for the public sector to receive additional capital and managerial capabilities are twofold: an increase in operational efficiency and ease of public fiscal constraints. As a result of these benefits, PPP has been proven to be a viable alternative to public infrastructure development (Yescombe 2002). The underlying assumption in adopting a PPP project delivery method is that a successful PPP project can deliver higher quality services at lower costs than the government can provide otherwise; that is, higher value for money (HM Treasury 2006). However, this assumption is only valid if the promised higher private efficiency can offset the higher borrowing costs of the private sector, which is passed to the sponsoring public authority. To secure the required efficiency gains, the public sector (the government) needs to establish a supporting framework that addresses all pertinent legislative, fiscal, technical, and financial aspects. One important feature of this supporting framework is to properly account for and report all fiscal

implications of PPPs. Governments willing to make the PPP project more attractive (marketable) to private investors, may offer financial incentive through fiscal instruments such as output-based cash subsidies, tax breaks, and financial guarantees. Undoubtedly, one of the most recognized advantages of PPP is that it can ease public fiscal constraints. PPP may be dexterously used to bypass public expenditure caps by moving the cost of the public investment off budget, which implies writing off the debt from the government balance sheet. This artificial accounting expedient may allow the government to meet its fiscal targets. However, the idea that a written-off PPP project does not have any other fiscal implications on government budget is incorrect and misleading, because the government may still bear considerable risk and may face potentially large fiscal costs, especially over the medium to long term (H. Polackova Brix, Working Paper, World Bank, Washington, DC; Hemming et al. 2006). In particular, government revenue guarantees are those with potentially disruptive fiscal implications over medium and long-term periods (Hemming et al. 2006). For instance, Mexican government revenue guarantees, provided in the late 80s and early 90s to support PPP toll road projects, had an unanticipated fiscal cost of \$8.9 billion following the 1994 Mexican economic crisis. Moreover, the negative fiscal implications implicit in these types of governmental guarantees are aggravated because they usually occur during an economic crisis that has already been weakening the fiscal capacity of the government (Hemming et al. 2006). The disruptive financial implications for a government may range from the least severe, i.e., incapability to fully support new infrastructure developments, to the most severe, i.e., having the credit rating downgraded. Indeed, correctly pricing governmental revenue guarantees is crucial for governments, which, by mandate, must maintain a sound fiscal policy.

In the recent years, many researchers have presented different methods to price governmental guarantees (Irwin 2003;

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Dailami et al. 1999; Huang and Chou 2006; Wibowo 2006; Cheah and Liu 2006, Chiara and Garvin 2007; Chiara et al. 2007; Brandao and Saraiva 2008). This paper does not present another valuation method for governmental guarantees. Instead, the paper proposes a different, alternative instrument, the dynamic revenue insurance, which can serve the same purpose as a government revenue guarantee without its disruptive financial implications. This new risk hedging product is modeled as a multiple exercise option (Jaillet et al 2004; Meinshausen and Hambly 2004) and its fair value is assessed by using two Monte Carlo methods: the multiple exercise boundary method and the multiple least-squares method (Chiara and Garvin 2007).

The contribution of this paper is dual. First, it provides the conceptual and computational framework for a new type of contract, the dynamic revenue insurance, which is an alternative option to the conventional government guarantee and which places limited economic burden on the government. This new contract gives PPP parties other than the government the opportunity to participate in the revenue risk coverage. Potential revenue risk insurers include international financial institutions [e.g., International Finance Corporation (IFC) or European Investment Bank (EIB)], export credit agencies [e.g., Export-Import Bank of the United States of America (EXIM), Export Development Canada (EDC), and Export Credits Guarantee Department (ECGD)], and private insurance companies [e.g., American International Group (AIG) or Lloyds]. The key characteristic of this contract is that it facilitates the pooling of project revenue insurers by accommodating insurer financial and risk preferences.

Second, this revenue insurance contract has dynamic (flexible) characteristics that make it much less costly than a conventional revenue guarantee, while nearly delivering the same risk coverage. Moreover, the fair premium fees of this type of insurance contract are spread over a long period (up to the concession period), making it a more convenient risk hedging tool from an economic perspective.

The paper is structured as follows. The “Background” section provides the basic notions of government revenue guarantees. The computational framework to value dynamic (flexible) contracts is presented in the “Computational Framework” section. The “Dynamic Revenue Insurance Contract” section illustrates the flexible revenue insurance contracts and their features. The “Numerical Example” section presents a numerical application of the proposed risk mitigation instrument. The “Conclusion” section closes the paper.

Background

PPPs often involve the use of government revenue guarantees, which are government interventions intended to reduce the financial cost of risks faced by the private sector (H. Polackova Bixi, Working Paper, World Bank, Washington, DC; Hemming et al. 2006). Although revenue guarantees and revenue insurance contracts differ from a legal standpoint, their fair values (or expected values) can be assessed by using the same conceptual framework and computational procedures. Thus, it is understood that information about revenue guarantees also applies to revenue insurance contracts. In general terms, a revenue guarantee is a contract in which one party promises the other party to pay the revenue shortfall ($K - X$) relative to a period of time Δt ; that is, the difference between the minimum guaranteed net revenue, K , and the net revenue, X , accumulated in Δt (Chiara and Garvin 2007; Chiara et al. 2007). The contractual period Δt is a project financial auditing interval, which typically occurs on a quarterly, semiannual,

or annual basis. This type of guarantee can be represented as a put option:

$$\Pi(X) = \max(K - X, 0)$$

where the minimum guaranteed revenue K is the strike price and the realized net revenue X is the underlying process.

The features described in the following uniquely define guarantee-type contracts.

- Contractual parties include the guarantor/insurer (G/I), the entity that grants the guarantee/insurance, and the third party guaranteed/insured (TPG/I).
- Rationale of the contract is the contractual tool that permits TPG/I to hedge the risk of having the net revenue, X , relative to a unit time period, Δt , below a minimum value, K .
- Duration of the contract, $[0, T]$: The contract usually starts at the beginning of the project operational period, and it expires at its end. This period is discretized in N unit time periods, i.e., $T = N \cdot \Delta t$. The length of Δt depends on the project audit schedule, which generally occurs on quarterly, semiannual, and annual basis.
- Terms of the contract: G/I grants TPG/I a collection of rights for redeeming the net revenue shortfalls ($K - X$) that may eventually occur at the end of each unit time period, Δt .

The contract is fully specified when the following two elements are set:

- Number of exercise dates, N , representing the dates during which the M exercise rights can be executed. At each date it is possible to execute only one exercise right. The exercise dates recur every unit time step, Δt .
- Number of exercise rights, M , representing the number of times the TPG/I is entitled to redeem the net revenue shortfall.

Once the contractual parties finalize the contract, G/I is bound to refund the net revenue shortfall whenever TPG/I exercises one of their M claims. Thus, the fair value of the contract is the expected amount of money that G/I is supposed to pay TPG/I as a result of TPG/I's exercise decision policy.

Conventional Revenue Guarantee

This type of contract has static or inflexible features because the dates for redeeming the net revenue are fixed beforehand at the closing of the contract. Such a contract can be represented by a strip of European options with times to maturity set at each of the N exercise dates, $\{1\Delta t, \dots, N\Delta t\}$; that is, exercise rights $M = N$.

Dynamic Revenue Risk Hedging Contract

The contract value depends on the decision policy followed by TPG/I when executing their exercise rights and different decision policies return different values of the contract. However, only one exercise policy, the optimal exercise policy, generates the maximum profit for the TPG/I. Accordingly, the fair value of the contract can be defined as the expected amount of dollars to be paid by G/I if TPG/I executes the optimal exercise policy. Such a contract can be modeled through a multiple exercise option. This American-type option can be dynamically exercised M times in N exercise dates with $M \leq N$. There are set no restrictions on when TPG/I has to exercise their rights. Indeed, it is this flexibility that makes these types of contracts a better and more cost-efficient risk managing tool.

Computational Framework

The strip of European options (static contract) and multiple exercise option (dynamic contract) are stochastic multistage decision problems. Although the former is trivial (no optimization is required), the latter results in a more complex problem that requires iterative stochastic optimizations.

Conventional Revenue Guarantee

The valuation method for pricing a strip of European option is straightforward. One can use either a close-form equation such as the Black-Scholes equation, provided that the underlying process X is modeled as geometric Brownian motion, or a Monte Carlo simulation (Hull 2003). According to Dailami et al. (1999), the fair price of such a contract is the value of N European put options with maturity set at the end of each time step, with a payoff of

$$\Pi(t, X_t) = \max(K_t - X_t, 0) \quad (1)$$

where $K_t = \alpha \bar{X}_t$ = guaranteed value at time t ; \bar{X}_t = projected value (i.e., base case value) of the net revenue in the interval $[t - \Delta t, t]$; α = guaranteed percentage (e.g., $\alpha = 90\%$); and X_t = random net revenue collected in the interval $[t - \Delta t, t]$.

Dynamic Revenue Risk Hedging Contract

A four-part scheme, shown in Fig. 1, is employed to assess the fair value of the contract.

Future Scenario Layout

The stochastic evolution of the cumulative net revenue X fully defines the future scenario. The net revenue X is modeled as a discrete-time stochastic process that spans the entire operational period. In this type of setting, the modeler may represent the net revenue either as a continuous one-factor model (Irwin 2003) or as a discrete-time multifactor model (Dailami et al. 1999). When the analyst defines the model for the underlying variable, X , the evolution of the net revenue shortfall over time can be computed as the payoff function

$$\Pi(t, X) = \max(K - X, 0) \quad (2)$$

which is a discrete-time stochastic process depending on the minimum net revenue, K , and the net revenue, X . According to Dailami et al. (1999), the net revenue can be modeled as discrete-time stochastic process:

$$X_t = f(\mathbf{Z}_t, \mathbf{Y}_t)$$

where \mathbf{Z}_t and \mathbf{Y}_t = vector of risk variables and the vector of deterministic parameters relative to the time interval i , respectively.

Decision Making Behavior of TPG/I

The decision-making behavior of TPG/I is characterized by one attribute and one constraint (Chiara and Garvin 2007):

1. TPG/I cannot foresee the future. This reasonable restriction implies that TPG/I's prediction about future revenues is an expectation rather than a certainty.

2. TPG/I is a profit maximizer; that is, given two dollar amounts of P_1 and P_2 , they will always choose the one with the greater value.

Decision Making Process of TPG/I

TPG/I acquires a risk mitigation contract with M exercise rights at time $t = 0$. At the end of the first unit time period, TPG/I must decide between the two following alternatives, which lead to different payoffs:

1. Exercise one of the M exercise rights, with the return as the sum of the payoff due to one exercise and the expected value of the remaining $M-1$ future exercise payoffs, discounted to $t = 1$:

$$P_1 = \Pi_1(X_1) + E \left[\sum_{i=1}^{M-1} \text{disc}_{t=1} \Pi_i(X_t) \right] \quad (3)$$

2. Not exercise, with the return as the expected value of M future exercise payoffs, discounted to $t = 1$:

$$P_2 = E \left[\sum_{i=1}^M \text{disc}_{t=1} \Pi_i(X_t) \right] \quad (4)$$

If TPG/I predicts that $P_1 > P_2$, they will choose P_1 , i.e., they will exercise the right and go to the next time step with $M-1$ rights remaining. Otherwise, if TPG/I supposes that $P_1 < P_2$, they will choose the second option, i.e., they will not exercise the right and go to the next time step with M rights remaining. The TPG/I's decision-making process continues until either the expiration date of the contract or the time step in which the M th right is executed.

The discount factor considered in Eqs. (3) and (4) may be modeled as a discretely compounded discount rate, $1/(1 + \bar{r})^{j \cdot \Delta t}$, where the discount rate \bar{r} takes into account the creditworthiness of the guarantor. Intuitively, because the TPG/I is entitled under the provisions of the contract to receive a specific payment, i.e., the net revenue shortfall [Eq. (1)], the risk associated to the contract lies in the capacity of the G/I to fulfill the contract by paying the payoff of Eq. (1) back to the TPG/I. Accordingly, the cash flow must be discounted by a discount rate that considers the guarantor's risk of default, i.e., its capacity of debt repayment.

Optimal Decision Policy of TPG/I

The TPG/I decision process is a stochastic multistage decision process with a stochastic return associated with each decision. The objective of G/I in analyzing such a process is to determine optimal decision policy of TPG/I, that is, the optimal stopping time set:

$$\{\tau_i\}_{i=1, \dots, M} = \{\tau_1, \tau_2, \dots, \tau_M\} \quad (5)$$

that results in the best total expected return for TPG/I.

Once the optimal stopping time set is determined, the fair value of the contract is given by the expectation of the sum of the payoffs relative to the optimal stopping time set, discounted to time $t = 0$:

$$\psi = E \left[\sum_{i=1}^M \text{disc}_{i=0} \Pi_i(X_{\tau_i}) \right] \quad (6)$$

To compute the optimal stopping time, Eq. (5) uses a stochastic multistage optimization through two Monte Carlo methods, the multiple exercise boundary method and the multiple least-squares method. These methods combine Monte Carlo simulation with approximate dynamic programming techniques. Monte Carlo methods can only yield biased estimates of the value of the multiple exercise option ψ (Meinshausen and Hambly 2004; Andersen and

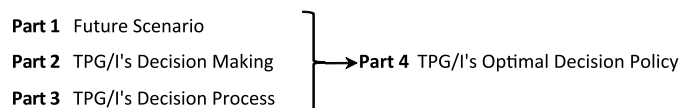


Fig. 1. Contract modeling scheme layout

Broadie 2004; Broadie and Detemple 2004). The usual approach is to find a lower estimator, θ , and an upper estimator, Θ ; in turn, their respective estimates bound ψ , i.e., $\hat{\theta} \leq \psi \leq \hat{\Theta}$. These negative and positive biased estimators can be used to find the $1 - \alpha$ confidence interval for the true contract value:

$$\left[\hat{\theta} - z_{\alpha/2} \sigma_{\theta} / \sqrt{n}, \hat{\Theta} + z_{\alpha/2} \sigma_{\Theta} / \sqrt{n} \right] \quad (7)$$

where $z_{\alpha/2} = 1 - \alpha/2$ quantile of the standard normal distribution; n = number simulations; and σ_{θ} and σ_{Θ} = sample standard deviations of θ and Θ , respectively.

In addition, it is possible to compute the point estimate for the true contract value as the simple average:

$$\hat{\phi} = \frac{\hat{\theta} + \hat{\Theta}}{2} \quad (8)$$

Multiple Exercise Boundary Method

The multiple exercise boundary method (MEB) is obtained extending Andersen's procedure (Andersen 2000) to the multiple exercise case (Chiara and Garvin 2007). The MEB method yields the lower bound, $\hat{\theta}_1$, and the upper bound, $\hat{\Theta}_1$.

The dynamic of the state variable X is simulated by generating n paths $\{\omega(i)\}_{i=1,2,\dots,n}$. The resulting $X_{t_j}^{\omega(i)}$ is the value of the process at time $t_j = j\Delta t$ along the i th simulated path.

The relative Bellman equation that provides the value of the option with M exercise rights for each of the n simulated paths $\{\omega(i)\}_{i=1,2,\dots,n}$ is

$$V_{(M)}^i [t_j, X_{t_j}^{\omega(i)}] = I_X^{(M)} \cdot A_M + [1 - I_X^{(M)}] \cdot B_M \quad (9)$$

where A_M is given by Eq. (3) or

$$A_M = \Pi [t_j, X_j^{\omega(i)}] + E \left[\text{disc}_{t=j} V_{(M-1)}^{j+1} (t_{j+1}, X_{j+1}) \right] \quad (10)$$

B_M is given by Eq. (4) or

$$B_M = E \left[\text{disc}_{t=j} V_M^{i+1} (t_{j+1}, X_{j+1}) \right] \quad (11)$$

and $I_X^{(M)}$ is the exercise indicator, which takes the value of 1 when the path crosses the exercise boundary $\{E_j^{(M)}\}_{j=1,\dots,T}$ and 0 otherwise:

$$I_X^{(M)} = \begin{cases} 1 & \text{if } X_{t_j}^{\omega(i)} < E_j^{(M)} \\ 0 & \text{if } X_{t_j}^{\omega(i)} \geq E_j^{(M)} \end{cases} \quad (12)$$

Accordingly, the exercise boundary point $E_j^{(M)}$ is calculated as the $X_{t_j}^*$ value that maximizes the estimated option value at time j

$$\hat{V}_{(M)} (t_j, X_{t_j}) = \frac{1}{n} \sum_{i=1}^n V_{(M)}^i [t_j, X_{t_j}^{\omega(i)}] \quad (13)$$

that is

$$\hat{V}_{E_j^{(M)}=X_{t_j}^*} (t_j, X_{t_j}) = \sup \left\{ \hat{V}_{E_j^{(M)}=X_{t_j}} \right\} \quad \text{with } X_{t_j} \in \left[\inf \{ X_{t_j}^{\omega(i)} \}, K \right] \quad (14)$$

If the constraint is partially relaxed by permitting the decision maker to foresee the future one time step ahead, i.e., from time t

to time $t + \Delta t$, the expected values in Eqs. (10) and (11) can be approximated by replacing them with their corresponding simulated values:

$$\bar{A}_M = \Pi (t_j, X_j^{\omega(i)}) + \left[\text{disc}_{t=j} V_{(M-1)}^{j+1} (t_{j+1}, X_{j+1}^{\omega(i)}) \right] \quad (15)$$

$$\bar{B}_M = \left[\text{disc}_{t=1} V_M^{j+1} (t_{j+1}, X_{j+1}^{\omega(i)}) \right] \quad (16)$$

Thus, the relative Bellman equation becomes

$$V_{(M)}^i [t_j, X_{t_j}^{\omega(i)}] = I_X^{(M)} \cdot \bar{A}_M + [1 - I_X^{(M)}] \cdot \bar{B}_M \quad (17)$$

Intuitively, by allowing the decision maker to foresee one time step ahead, a positive bias has been created, because now the decision maker is equipped with extra powers. As a result, the decision maker's decision policy will be positively biased, and the corresponding hyper-optimal decision policy will lead to an upper bound of the option value.

The family of M exercise boundaries $\{E_j^{(k)}\}_{j=1,\dots,T}^{k=1,2,\dots,M}$ can be computed by reiteratively applying Eq. (14) along with Eq. (17). Once the exercise boundaries are known, the optimal stopping time set $\{\tau_k^{\omega(i)}\}_{k=1,\dots,M} = \{\tau_1, \tau_2, \dots, \tau_M\}$ for each simulated path $\omega(i)$ is determined by recording the first time the simulated values $\{X_j^{\omega(i)}\}_{j=1,\dots,T}^{i=1,\dots,n}$ cross the M exercise boundaries $\{E_j^{(k)}\}_{j=1,\dots,T}^{k=1,2,\dots,M}$

$$\left\{ \tau_k^{\omega(i)} \right\}_{k=1,\dots,M} = \inf \left\{ \{t_k\}_{k=1,\dots,M} : X_{t_j}^{\omega(i)} < \{E_j^{(k)}\}_{j=1,\dots,T}^{k=1,2,\dots,M} \right\}$$

Thus, the positively biased estimate of the multiple exercise option value is

$$\hat{\Theta}_{(M)} = \frac{1}{n} \sum_{i=1}^n \hat{V}_{(M)}^i [0, X^{\omega(i)}] = \frac{1}{n} \sum_{i=1}^n \sum_{k=1}^M \text{disc}_{t=0} \Pi \left[\tau_k^{\omega(i)}, X_{\tau_k^{\omega(i)}}^{\omega(i)} \right] \quad (18)$$

It holds that the exercise boundaries $\{E_j^{(k)}\}_{j=1,\dots,T}^{k=1,2,\dots,M}$ are sub-optimal exercise boundaries when applied to a new generated set of simulated paths, $\{\bar{X}_j^{\omega(i)}\}_{j=1,\dots,T}^{i=1,\dots,n}$, which are independent from the original simulated paths, $\{X_j^{\omega(i)}\}_{j=1,\dots,T}^{i=1,\dots,n}$. Therefore, if the multiple exercise option value is computed by using the M exercise boundaries $\{E_j^{(k)}\}_{j=1,\dots,T}^{k=1,2,\dots,M}$ along with a new set of independent simulated paths, a negative biased estimate of the option value will be obtained:

$$\begin{aligned} \hat{\theta}_{(M)} &= \hat{V}_{(M)} (0, \bar{X}) = \frac{1}{n} \sum_{i=1}^n \hat{V}_{(M)}^i [0, \bar{X}^{\omega(i)}] \\ &= \frac{1}{n} \sum_{i=1}^n \sum_{k=1}^M \text{disc}_{t=0} \Pi \left[\bar{\tau}_k^{\omega(i)}, \bar{X}_{\bar{\tau}_k^{\omega(i)}}^{\omega(i)} \right] \end{aligned} \quad (19)$$

where the optimal stopping time set

$$\left\{ \bar{\tau}_k^{\omega(i)} \right\}_{k=1,\dots,M} = \inf \left\{ \{t_k\}_{k=1,\dots,M} : \bar{X}_{t_j}^{\omega(i)} < \{E_j^{(k)}\}_{j=1,\dots,T}^{k=1,2,\dots,M} \right\}$$

is the collection of time steps relative to the first time the process enters the M exercise areas bounded by the M exercise boundaries $\{E_j^{(k)}\}_{j=1,\dots,T}^{k=1,2,\dots,M}$.

Multiple Least Squares Monte Carlo Method

The multiple least-squares Monte Carlo method (MLSM) is obtained by extending the procedure proposed by Longstaff and Schwartz (2001) to the multiple exercise case. The MLSM yields the lower estimate $\hat{\theta}_2$ (Chiara et al. 2007).

The Bellman equation of the optimal stopping time is

$$V_{(M)} [t_j, X_j^{\omega(i)}] = \max \left\{ \Pi [t_j, X_j^{\omega(i)}] + E \left[\text{disc}_{t=j} V_{(M-1)} (t_{j+1}, X_{j+1}) \right], E \left[\text{disc}_{t=j} V_{(M)} (t_{j+1}, X_{j+1}) \right] \right\} \quad (20)$$

where the two expected values can be approximated with two continuation values, which are conditional on the information available at time j , $F_j^{\omega(i)}$, obtained by recursively least-squares regressing the $\text{disc}_{t=j} V_{(k)} (t_{j+1}, X_{j+1})$ onto S basis $\{p_s(X)\}_{s=1, \dots, S}$:

$$\left\{ \text{Cont}(j)_{(k)} \right\}_{k=1, \dots, M} = \left\{ E \left[\text{disc}_{t=j} V_{(k)} (t_{j+1}, X_{j+1}) \middle| F_j^{\omega(i)} \right] \right\}_{k=1, \dots, M} \approx \left\{ \sum_{s=0}^S \hat{a}_s^{(k)} p_s [X_j^{\omega(i)}] \right\}_{k=1, \dots, M} \quad (21)$$

Thus, the optimal stopping time associated to the simulated path $\omega(i)$ is given by

$$\left\{ \tau_k^{\omega(i)} \right\}_{k=1, \dots, M} = \inf \left\{ \left\{ t_k \right\}_{k=1, \dots, M} : \Pi [X_{t_j}^{\omega(i)}] + \text{Cont}(j)_{(k-1)} > \text{Cont}(j)_{(k)} \right\}$$

The value of the multiple exercise option is given by discounting to time t_0 the cash flows relative to the optimal stopping time of the n simulated paths:

$$\hat{\theta}_2 = V_0^{(M)} = \frac{1}{n} \sum_{s=1}^n \left[\text{disc}_{t=0} \Pi (X_{\tau_i}^{\omega(i)}) \right] \quad (22)$$

Remarks

Once the two lower bounds and the upper bound are calculated, the $1 - \alpha$ confidence interval [Eq. (7)] and the point estimate [Eq. (8)] can be represented as

$$\left[\hat{\theta}_1 - z_{\alpha/2} \sigma_{\theta} / \sqrt{n}, \hat{\theta}_1 + z_{\alpha/2} \sigma_{\theta} / \sqrt{n} \right] \quad \text{and} \quad \left[\hat{\theta}_2 - z_{\alpha/2} \sigma_{\theta} / \sqrt{n}, \hat{\theta}_1 + z_{\alpha/2} \sigma_{\theta} / \sqrt{n} \right] \quad (23)$$

$$\hat{\phi} = \frac{\max(\hat{\theta}_1, \hat{\theta}_2) + \hat{\theta}_1}{2} \quad (24)$$

The computational approach used to value the multiple exercise option has the advantage of being far simpler than those presented by other researchers such as Meinshausen and Hambly (2004). These researchers used a relatively complex approach, the duality approach (Haugh 2003), to compute the positive biased estimate. Additionally, a trivial upper bound, $\hat{\theta}^*$, can be computed by allowing TPG/I to completely foresee the future; however, it can be shown that it is always lower bounded by $\hat{\theta}_1$, i.e., $\hat{\theta}_1 < \hat{\theta}^*$.

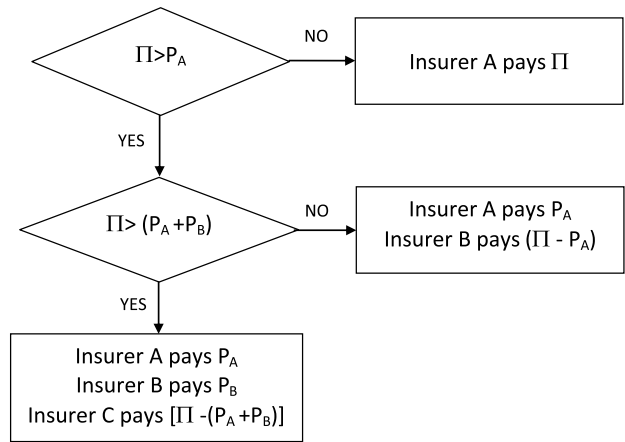


Fig. 2. Flow chart of priority payoff payments

Dynamic Revenue Insurance Contract

This contract provides parties other than the government with the opportunity to participate in the revenue risk coverage. The key characteristic of this contract is that it facilitates the pooling of project revenue insurers by accommodating the financial and risk preferences of different insurers. The pool of insurers can be composed, for instance, of one senior guarantor, Insurer A, and two junior guarantors, Insurers B and C. The seniority among the insurers is established by a predetermined priority payment sequence, such as the one shown in Fig. 2. Furthermore, it is assumed that each insurer is only responsible for its own payment, that is, no insurer is liable if another insurer is in default.

The framework of the dynamic revenue insurance contract is defined in the following list.

- Contractual parties include a pool of insurers (Insurers A, B, and C) that offer the revenue insurance to the sponsor (TPI).
- Duration of the contract $[0, T]$ determines that the contract starts at the beginning of the operational period of the project and expires at the end of the project. The duration is divided into N time units, i.e., $T = N \cdot \Delta t$.
- Rationale of the contract is a contractual instrument that permits the project sponsors to hedge the risk of having the net revenue relative to a unit time period below the minimum net revenue, K .
- Terms of the contract are determined when the pool of insurers provides the sponsors a collection of M rights for redeeming the net revenue shortfalls $(K - X)$ that may eventually occur at the end of each unit time period Δt . The number of exercise dates allowed is N , i.e., an exercise right may be executed at the end of any year over the operational period.

The pool of insurers will pay the net revenue shortfall according to the priority payoff payment shown in Fig. 2. For instance, if the TPI executes an exercise right, each insurer may pay its own share according to the hypothetical payment sequence depicted in Table 1.

The value of the contract can be assessed by using the previously described MLSM and MEB methods. After computing the MLSM low estimate and the MEB low and high estimates, the

Table 1. Hypothetical Priority Payment Sequence

Insurers	Priority payment	Payment limits
A – Senior	1st	$P_1 K - 0.9K$
B – Junior	2nd	$P_2 0.9K - 0.8K$
C – Junior	3rd	P_3 up to $0.8K$

confidence interval [Eq. (23)] and the point estimate [Eq. (24)] can be calculated. The discount rate \bar{r} takes into account the creditworthiness of the insurer. As a result, the higher the insurer's credit rating, the lower the expected discount rate (Brealey and Myers 2003).

Numerical Example

This section presents a numerical application of the previously developed models. This numerical analysis is applied to an infrastructure project with excessive revenue risk. The section is opened with the presentation of a base case analysis on a typical public private transportation project finance arrangement: a toll road developed using a build operate transfer (BOT) scheme. According to the base case analysis, the project results were profitable on the average because the expected net present value of the project is positive. The next section presents a stochastic risk analysis of the project using a Monte Carlo simulation. The results of the risk analysis simulation show that, although the project is expected to be profitable, the probability that the project is not profitable is substantially above the investment risk threshold of the sponsors. Accordingly, sponsors will invest only if the project revenue risk is mitigated down to their investment threshold. The "Conventional Governmental Revenue Guarantee" section examines the conventional (static) revenue mitigation products that the government may grant to the project's sponsors. The "Dynamic Revenue Insurance Contracts" section presents the fair values of dynamic revenue insurance contracts offered by a pool of insurers and highlights their typical benefits.

Base Case Analysis

The numerical analysis example concerns a hypothetical \$235 million toll road project connecting White Plains to New

Rochelle, two suburban areas of New York City. This toll road will route the traffic generated by White Plains commuters working in Manhattan toward interstate I-95 while decongesting interstate I-87. This 12 km (7.5-mi) roadway will be developed by using a BOT project delivery method in which the private investors, the sponsors, will design, build, finance, and operate the toll road within the pre-fixed concession period. The planned construction and operation phases are two and 35 years, respectively. To build the toll road, the project requires capital expenditures of \$200 million during the first two years; these expenditures will be depreciated over 14 years. The sponsors will fund 59.6% of the project's total cost with two rounds of nonrecourse bank debt: \$40 million in the first year and \$100 million in the second year. The debt will carry a 10% coupon and will be fully amortized over 14 years. The sponsors will fund the remaining 40.4% (\$95 million) with equity investment in Years 1 and 2. The total investment of \$235 million exceeds the total capital expenditures of \$200 million by \$35 million. This amount includes funds for start-up administrative expenses, interest expenses during construction, and net working capital. No reserve funds for debt service are considered. Beginning in Year 3, the toll road will be open to traffic with an initial predicted annual traffic volume of 6.9 million vehicles (Column 1 of Fig. 3). For the sake of simplicity, it is assumed that all vehicles are paying the same toll (Column 2), i.e., no toll differentiation among vehicle classes. Operating expenses start from Year 3 (Column 4) and the tax rate is constant at 35%. Sponsors usually use two methods to value their equity investments (Esty 1999). Either they value equity directly by discounting equity cash flows (ECFs) by using the cost of equity (K_E), or they value equity indirectly by discounting free cash flows (FCFs) by using the weighted average cost of capital (WACC). The FCF/WACC and ECF/ K_E are theoretically equivalent and prone to similar errors. However, it is easier to correctly implement the ECF approach

Year	Predicted Annual Traffic Volume (1,000)	Toll (1,000)	Total Revenue (1,000)	Operating Expenses (1,000)	Total Debt Service (1,000)	Debt Service Coverage Ratio (DSCR) (1,000)	Equity Cash Flow (ECF) (1,000)	Discounted Equity Cash Flow (DECF) (1,000)
	1	2	3	4	18	19	20	21
1			\$0	\$0	(36,000)		(\$64,000)	(55,652)
2			0	0	(86,000)		(\$30,849)	(23,326)
3	6,935	\$5.00	34,675	(8,186)	19,000	1.34	\$6,449	4,240
4	7,074	5.00	35,369	(8,739)	18,500	1.37	\$6,864	3,925
5	7,215	5.50	39,683	(9,275)	18,000	1.54	\$9,644	4,795
6	7,359	5.50	40,477	(9,802)	17,500	1.58	\$10,140	4,384
7	7,507	6.00	45,040	(10,329)	22,000	1.37	\$8,087	3,040
8	7,657	6.00	45,941	(10,864)	21,000	1.27	\$5,723	1,871
9	7,810	6.50	50,765	(11,419)	20,000	1.71	\$14,146	4,021
10	7,966	6.50	51,780	(12,005)	19,000	1.62	\$11,823	2,922
30	10,414	14.00	145,799	(32,636)	0		\$73,336	1,108
31	10,518	14.00	147,257	(34,350)	0		\$73,167	961
32	10,624	14.00	148,730	(36,181)	0		\$72,932	833
33	10,730	14.00	150,217	(38,134)	0		\$72,627	721
34	10,837	16.50	178,812	(40,216)	0		\$89,858	776
35	10,945	16.50	180,600	(42,435)	0		\$89,576	673
36	11,055	16.50	182,406	(44,798)	0		\$89,211	583
37	11,165	16.50	184,230	(47,313)	0		\$88,760	504

Fig. 3. Financial base case projections

(Esty 1999). The first step in the ECF valuation is to compute the equity cash flows, assuming that all residual cash flows are distributed to sponsors as dividends (Fig. 3: Column 20). The equation of the equity cash flows is

$$\text{ECF} = \text{CADS} - \text{PP} - \text{IP} - \text{ED} \quad (25)$$

where CADS = cash available for debt service; PP = principal payments; IP = interest payments; ED = equity drawdown.

The financial projection data are shown in Fig. 3. The expected cost of equity (K_E) is estimated by using the capital asset pricing model (CAPM). According to CAPM, the cost of equity (the expected return on equity from the sponsors' perspective) is a function of the risk-free rate (R_f), the equity or levered beta (β_E), and the market risk premium ($R_M - R_f$):

$$K_E = R_f + \beta_E(R_M - R_f) \quad (26)$$

The long-term risk-free rate, R_f , is normally determined from yields on national debt, and in this case, a value of 5% was selected. The US market risk premium, $R_M - R_f$, historically falls between 6 and 8%, and a value of 8% was selected. The equity beta, β_E , is a function of the project's asset or unlevered beta, β_A , and the project's leverage, V/E :

$$\beta_E = \beta_A(V/E)$$

The project's leverage is the ratio between the firm value (V) and equity value (E), where the firm value is the sum of the debt value and equity value ($V = D + E$). In the current example, $\beta_E = 3.47$ from considering $V/E = (1/0.404) = 2.47$ and $\beta_A = 1.4$. Plugging all of these values into the CAPM equation [Eq. (26)] yields a cost of equity $K_E = 15\%$. Finally, the project's net present value of the equity cash flows is given by

$$\text{NPV}(\text{ECF}) = \sum_{i=1}^{35} \frac{\text{ECF}_{F_i}^{\text{BaseCase}}}{(1 + K_E)^i} \quad (27)$$

Because the investment yields a positive expected net present value (NPV=\$3.4 million), the project is considered profitable from the perspective of sponsors.

Risk Analysis under Uncertainty

The fundamental assumption underlying the ECF base case analysis is that the project's sponsors are adequately diversified investors acting in a complete market. The market completeness assumption ensures that sponsors can hedge away project private risks (project specific or nonsystematic risks) by building well-diversified investment portfolios. As a result, expected project returns should compensate sponsors only for systematic risks associated to the market as a whole (market risk). Under this assumption, it is correct to discount the predicted ECF with the cost of equity K_E , a discount rate that considers only the market risk premium. However, infrastructure assets (e.g., toll roads, water-treatment facilities, and airports) are characterized by market incompleteness and lack of liquidity that do not allow complete removal of the nonsystematic risk. Therefore, project private risks that cannot be diversified or hedged become residual risks. Such risks should be included in the project risk analysis. To model the risks of the project in Eq. (27), the approach was to incorporate the residual private risks in the numerator and the market risk in the denominator of Eq. (27). Accordingly, the ECF was modeled as a stochastic process to account for the residual private risks and the CAPM cost of equity, $K_E = 15\%$, was considered to account for the project market risk. This approach guarantees consistency with the base case analysis and avoids the double-counting of risks noted by Myers (1976).

The risk modeling of the residual private risks in the ECFs was conducted in two steps. First, the identified project-specific risks that could have been diversified or hedged through available financial instruments (macroeconomic risk hedging), insurance products (environmental and force major risk hedging), legal agreements with governmental institutions (political risk hedging), and legal agreements with contractors (construction and supply risk hedging) were considered. It was realized that some factors such as the local transportation network and user behavior would have driven some private risks (i.e., residual private risks) that could not be hedged. Second, a sensitivity analysis was performed on the key parameters that might negatively affect the project profitability. In the current example, the equity cash flow equation [Eq. (25)] contained six key parameters, $\text{ECF} = f(p_1, p_2, p_3, \dots, p_6)$: the annual traffic volume, operating expenses, toll rate, tax rate, debt interest rate, and inflation rate. Their contributions to the residual project private

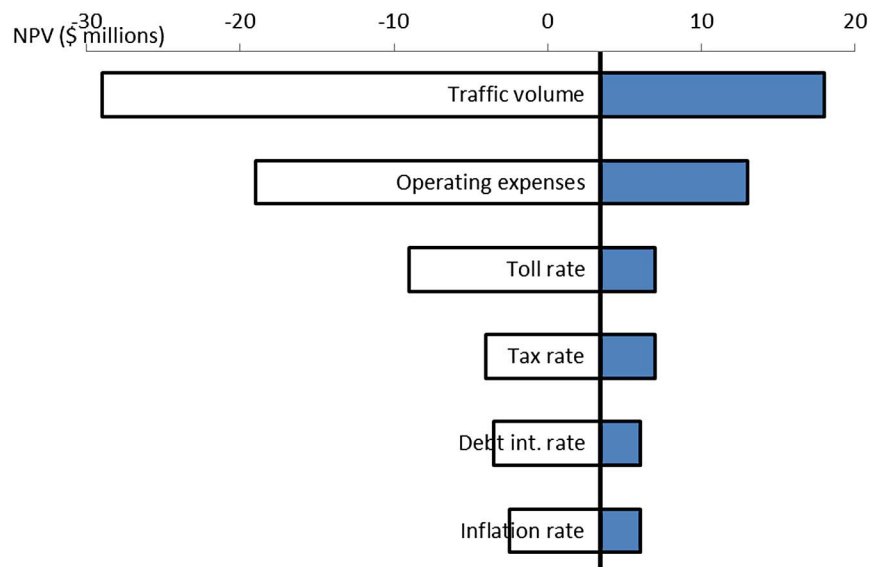


Fig. 4. Tornado diagram of the project

STOCHASTIC VARIABLE	DISTRIBUTION	
Annual Traffic Volume *	Triangular	
	Min	70% Projected value
	Mean	Projected value (column 1 in Fig. 3)
	Max	115% projected value
Operating Expenses*	Triangular	
	Min	90% Projected value
	Mean	Projected value (column 4 in Fig. 3)
	Max	120% projected value

*Correlation coefficient $\rho = 0.6$

Fig. 5. Distributions of annual traffic volume and operating expenses

risks were considered by evaluating their influence to the ECF uncertainty of the project. A tornado graph showed that annual traffic volume and operating expenses were the most sensitive parameters in terms of residual private risks (Fig. 4). Accordingly, in the succeeding risk analysis, only these two parameters were modeled as risk variables, while the remaining four parameters were kept deterministic:

$$ECF = f(X_1, X_2, p_3, \dots, p_6) \quad (28)$$

Risk modeling literature provides a wide choice of stochastic models that can be used to represent the annual traffic volume and operating expenses (Chiara and Garvin 2008; Dailami et al. 1999). The selection of the most appropriate stochastic model depends on several factors: whether historical data are available (e.g., direct data, proxy data, or subjective data), quality of data, anticipated characteristics of the risk variable evolution (e.g., seasonality or jumps), and tradeoff between simplicity and completeness. For the sake of simplicity in the exposition, it was decided to model the annual traffic volume and operating expenses by using two correlated triangular distributions with variances that increase with time. The mean values of the triangular distributions are the predicted values shown in Columns 1 and 4 of Fig. 3, and the lower and upper bounds of the distributions are constant percentages of the projected values that are shown in Fig. 5. More complex stochastic models could have been selected (Dailami et al. 1999; Chiara and Garvin 2008). Operation expenses are usually split into fixed and variable expenses, where the latter refers to expenses that are proportional to the road traffic volume. Therefore, it was reasonable to assume that traffic volume and operation expenses are correlated. Their correlation coefficients were estimated by using historical data of similar road projects. A correlation coefficient of 0.6 was selected. In the final risk analysis step, a Monte Carlo simulation of Eq. (27) was performed by using 10,000 ECF paths of Eq. (28). The correlated traffic volume and operation expenses were simulated by using a Gaussian copula, which was obtained using Iman and Conover's Latin hypercube method (Iman and Conover 1982). The selection of 10,000 paths provided a good balance between robustness of results and total computation time. The ECF risk analysis yielded a probability that the net present value is negative of more than 35%. Such a high probability that the project is not profitable may discourage the sponsors to invest in the project.

Conventional Governmental Revenue Guarantee

In the conventional framework, public entities grant revenue guarantees to make BOT projects more attractive to private investors.

The fair value of a government revenue guarantee covering the revenue risk in each year of the operational period is given by a stream of 35 European put options:

$$G_{Eur, Gov} = \sum_{i=1}^{35} \frac{1}{N} \sum_{k=1}^N \frac{\max(K_i - ECF_i^k, 0)}{(1 + \bar{r})^i} = \sum_{i=1}^{35} \frac{1}{N} \sum_{k=1}^N \frac{\max(\alpha ECF_i^{BaseCase} - ECF_i^k, 0)}{(1 + \bar{r})^i} \quad (29)$$

where $i = ith$ operational year; $N =$ number of simulated paths; $K_i = \alpha ECF_i^{BaseCase} =$ strike price of the European put option, which represents the percentage α of the annual base case equity cash flow that is guaranteed by the government; $ECF_i^k = kth$ simulated path obtained from Eq. (28); and $\bar{r} =$ discount rate, which is given by adding the credit risk premium to the risk free rate. The credit risk premium reflects the creditworthiness of the government pledging the guarantee.

The values of the government revenue guarantees for protections of 90, 80, and 70% are shown in Table 2. The government costs for full coverage (i.e., 35 years) are \$14, \$6.9, and \$3.5 million for 90, 80, and 70% protection, respectively.

Dynamic Revenue Insurance Contracts

In this setting, it was conceived that, instead of providing a direct revenue guarantee, the government will purchase revenue insurance

Table 2. Government Guarantee Covering Various Percentages of the Annual Base Case Equity Cash Flow

Exercise rights M	90% base case ECF (\$ million)	80% base case ECF (\$ million)	70% base case ECF (\$ million)
1	3.1415	2.1629	1.3337
2	6.1672	3.9058	2.1681
3	7.0616	4.5855	2.6709
4	7.5589	4.9065	2.8674
5	8.0050	5.1820	3.0273
6	8.4261	5.4037	3.1344
7	8.8264	5.6118	3.2321
8	9.2223	5.8024	3.3121
9	9.5453	5.9457	3.3649
10	9.8607	6.0561	3.3955
30	13.6033	6.8744	3.4947
31	13.7616	6.9046	3.4970
32	13.9191	6.9342	3.4991
33	14.0642	6.9634	3.5009
34	14.0800	6.9722	3.5026
35	14.0944	6.9763	3.5034

Table 3. Contract Parameters for Insurers A, B, and C

Parameter	Insurer A	Insurer B	Insurer C
Coverage range	$K-0.9K$	$0.9K-0.8K$	$<0.8K$
Discount rate (%)	6	7	8

Note: $K = \alpha(\%) \times \text{ECF}$.

Table 4. Dynamic Revenue Insurance Covering 90% of the Base Case Equity Cash Flow

Exercise rights M	Insurer A 6% (\$ million)	Insurer B 7% (\$ million)	Insurer C 8% (\$ million)	Point estimate (\$ million)	95% confidence interval (MLSM, MEB) (\$ million)	
Case 1: $K_1 = 0.9\text{ECF}$						
1	0.297	0.056	0.000	0.353	0.347	0.360
2	0.576	0.133	0.000	0.709	0.699	0.718
3	0.712	0.213	0.000	0.925	0.910	0.939
4	0.839	0.306	0.000	1.146	1.126	1.166
5	0.947	0.412	0.001	1.360	1.336	1.383
Case 2: $K_2 = 0.8\text{ECF}$						
1	0.233	0.034	0.000	0.267	0.263	0.271
2	0.430	0.077	0.000	0.507	0.499	0.515
3	0.529	0.141	0.000	0.671	0.656	0.685
4	0.611	0.215	0.001	0.827	0.809	0.844
5	0.679	0.303	0.003	0.984	0.961	1.007
Case 3: $K_3 = 0.7\text{ECF}$						
1	0.159	0.018	0.000	0.177	0.174	0.181
2	0.269	0.049	0.000	0.319	0.311	0.326
3	0.335	0.102	0.001	0.439	0.427	0.451
4	0.388	0.153	0.002	0.543	0.527	0.559
5	0.429	0.201	0.004	0.634	0.614	0.653

from a pool of three insurers. It is assumed that these insurers (Insurers A, B, and C) agreed to set the priority payoff payment, as shown in Table 3. Insurer A (the senior insurer) covers any payment between K and $0.9K$, ($P_A = 0.1K$); Insurer B covers any payment between $0.9K$ and $0.8K$ ($P_B = 0.1K$); and Insurer C covers any payment below $0.8K$ ($P_C = 0.8K$). It is also assumed that these insurers have different creditworthiness. Their creditworthiness can be expressed in terms of discount rate (the lower the better): 6% for Insurer A, 7% for Insurer B and 8% for Insurer C. The fair cost of the dynamic contracts covering five annual exercise rights within the 35-year concession period for $K = 0.9 \times \text{ECF}$, $K = 0.8 \times \text{ECF}$, and $K = 0.7 \times \text{ECF}$ are presented in Table 4. The point estimates and 95% confidence intervals shown in Table 4 are calculated through Eqs. (23) and (24) by using the MLSM and MEB methods detailed in the previous sections. Table 4 also shows the disaggregated expected losses incurred by each of the

Table 5. Cost Comparison between Contracts

Protection	Government guarantee	Dynamic contract	Fee installments ^a		
	35 years (\$ million)	5 exercise rights/ 35 years (\$ million)	10 years (\$ million)	15 years (\$ million)	20 years (\$ million)
90% ECF	14.094	1.3599	0.1761	0.1310	0.1091
80% ECF	6.9763	0.9839	0.1274	0.0948	0.0790
70% ECF	3.5034	0.6335	0.0820	0.0610	0.0508

^aInterest rate of 5%.

three insurers. After analyzing the results, the following conclusions can be drawn:

1. Insurer A takes the majority of the payoff payments, whereas Insurer C takes almost nothing. This occurs, of course, because of the chosen coverage range and the priority payment order. Should the cover range be different, the financial exposure of each insurer will differ.
2. The lower the insurer's creditworthiness, the higher the discount rate used to discount \$1 coverage. Accordingly, the insured party should pay less for a \$1 coverage offered by a low creditworthy insurer.
3. Studies conducted by Guasch (2004) show that, on average, if a transportation project experiences more than three years of negative financial results, sponsors will ask for a renegotiation of the concession agreement and the governmental financial support. Under this assumption, the five-year dynamic revenue insurance contract has similar coverage effect to a 35-year governmental guarantee. However, the fiscal implications for a government's perspective are different. Table 5 shows what would be the fair cost of these contracts to the local government. For instance, a 90% ECF coverage would burden the government balance sheet by approximately \$14 million if provided with a conventional revenue guarantee, or would cost \$1.36 million if provided through a five-annual-exercise-rights insurance contract (Table 5). Furthermore, the insurance cost is not a lumped cost, but will be spread over a predetermined period. For instance, Table 5 shows that the annual fair costs with 10, 15, and 20 installments assuming an interest rate of 5% are \$0.176, \$0.131, and \$0.109 million, respectively.
4. The flexibility of the computational framework of the methodology can easily be extended to quantify the risk exposure of an insurer covering a portfolio of statistically correlated projects.

Conclusion

Government revenue guarantees are usually provided in connection with PPP projects to increase the marketability of the projects and make them more attractive to private investors. However, revenue guarantees can potentially pose significant fiscal risks, particularly during an economic crisis. Because government budget and accounting systems are biased in favor of guarantees over other forms of financial support, it is important to value their impacts on the annual fiscal budget with a certain approximation. The conventional government guarantees are static (inflexible) contracts that require full coverage of the operational period of the project to effectively mitigate the project revenue risk. This setting may be costly, and under certain conditions, they may unnecessarily offer over-redundant risk hedging protection. This study presented a new family of contracts, dynamic revenue insurance contracts, which allow the government to provide revenue risk coverage that is equivalent to the conventional government revenue guarantee, but

at much lower cost. The providers of these insurance contracts can be other PPP stakeholders such as financial institutions, expert credit agencies, and private insurance companies. This set of new contracts can help governments to achieve their institutional goals with a very limited fiscal risk exposure.

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