Employing the Net Present Value-Consistent IRR Methods for PFI Contracts

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Abstract: The internal rate of return (IRR) is a common financial indicator for private finance initiative (PFI) projects. Due to the long and complicated cash flow nature of PFI projects, more plausible IRR techniques are necessary for appropriate project evaluation and ranking. However, not all the published articles researching on IRR techniques are reliable. Given the importance of computing the profitability of PFI projects, this paper is intended to introduce three reliable IRR methods, which are proven to be consistent with net present value. Examples are used to illustrate their utility. The paper is of high value as it guides industry's practitioners to use proper IRR methods for selecting PFI projects. It also provides academic researchers a platform to explore more robust methods.

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Introduction

According to the Office of Government Commerce (OGC) (2002) Guidance in the United Kingdom, the internal rate of return (IRR) is used for calculating the expected rate of return (ROR) in private finance initiative (PFI) contracts. This is the case even both the IRR and the net present value (NPV) are popular discounted cash flow methods applied in cash flow forecasting for construction contracts (Hwee and Tiong 2002) including PFI projects [e.g., Housing, Planning and Lands Bureau (HPLB) (2007)]. If we use it correctly, the IRR is a preferred method to the NPV (Hazen 2003; Hartman and Schafrick 2004; Hajdasinski 2004). It is because from the practitioners' perspective, the IRR is the true cost of capital and therefore the real ROR for measuring financial efficiency and project profitability (Hartman and Schafrick 2004). Moreover, business practitioners have found that unlike the NPV, the IRR can be estimated without having to involve the cost of capital in computation (Zhang 2005).

As in standardization of PFI contracts of the U.K.'s HM Treasury (HMT) (2004), PFI contracts were characterized by uneven cash flows in a project profile (or technically the so-called multiple sign changes). Specifically, multiple IRRs may be present when interim cash injections exist in future periods, which would lead to the difficulty of choosing the right IRR for the project (Louderback and McNichols 1986). The IRR method introduced by the HM Treasury (HMT) (2004) is not able to deal with this situation. Apparently, using the wrong IRR method can be extremely harmful (Kelleher and MacCormack 2004). The IRR methods that can address multiple sign changes, just like the NPV method, are named as NPV-consistent IRR methods (Hazen 2003; Hartman and Schafrick 2004). Hence, the present paper introduces three methods that fulfill such requirements. To conform to the scope of this paper, the basics of the IRR would not be introduced here but can be found from standard texts on corporate finance.

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NPV-Consistent IRR Methods

By definition, the IRR is the discount rate that results in a NPV of zero for a sequence of future cash flows in terms of revenues, costs, and initial investment. Mathematically, the IRR can be expressed as

$$\mathrm{NPV}(X) = \sum_i a_i(1+k)^{-i} = 0 \quad \text{where } i = 0,1,2,\ldots,n \qquad (1)$$

where a_i represents the net cash flow that is expected at time t_i; k=IRR; and $(1+k)^{-i}$=discount factor.

If a PFI project is characterized by a single initial investment followed by a string of positive returns, a unique IRR would be created. When there is more than one sign change in a profile of cash flows, multiple IRRs may emerge. It then becomes difficult to choose the correct IRR for the project. In conformity with the NPV wealth maximization (McDaniel et al. 1988), there are three IRR methods, which are able to solve ranking and multiple root problems. These methods are described below, and to justify whether they are NPV compatible, their results will be compared with those of the NPV method.

MRIC

The marginal return on invested capital (MRIC) is a modified IRR (MIRR) method, which assumes that cash flow streams can be reinvested at the marginal cost of capital (MCC) in place of the

Table 1. Three Competing Projects for Relative Comparisons

Project	x_0	x_1	x_2	x_3	NPV@10%
A	−1	6	−11	6	−0.128
B	−1	5	−6	—	−1.413
C	−1	4	−4	—	−0.669

Note: Source from Hazen (2003).

project's own IRR (Kennedy and Plath 1994). The MCC is the cost of capital determined by the market and is regarded as the risk-adjusted ROR of the financial cash flow stream expected by the financier (McDaniel et al. 1988; Hartman and Schafrick 2004). The MRIC only computes unique IRRs and is thus devoid of the conventional IRR problem. This makes the MRIC a better indicator of a project's true profitability. Mathematically, the equation for calculating the MRIC is (Kennedy and Plath 1994)

$$(1+m)^n \sum_t -a_t(1+k)^{-t} = \sum_t b_t(1+r)^{n-t} \quad \text{where } t=0,\ldots,n \tag{2}$$

where m=MRIC; a_t=capital funds required by the project in period t; b_t=operating cash flows generated by the project in period t; k=discounted rate; r=compounded rate; and n=horizon period.

The MRIC method has three steps (McDaniel et al. 1988). The first step is to determine a horizon period for the project (i.e., the concession period of a PFI project). In accordance with the MIRR method by Lin (1976), the second steps are to discount all negative cash flows (i.e., capital funding) to the origin at $t=0$ and to compound all positive cash flows (i.e., operating cash flows) to the terminus at $t=n$. In this paper, the MCC is employed as the discounted rate (k), while the market annual ROR is employed as the compounded rate (r) (or the so-called reinvestment rate). In fact, they are identical because they are the interest rates to the investor and the borrower. The final step compares the MRIC and the MCC. When the MRIC is greater than the MCC, the project is accepted.

We use the example projects from Hazen (2003) to illustrate that the MRIC method is NPV consistent. As shown in Table 1, the MRIC of Project A (m_A) can be computed by substituting the cash flow pattern {−1, 6, −11, 6} in Eq. (2) as follows:

$$(1+m_A)^3[1+11(1.1)^{-2}] = 6(1.1)^2 + 6 \Rightarrow (1+m_A)^3 = 1.313 \Rightarrow m_A = 1.095 - 1 = 0.095 = 9.5\%$$

Since the MRIC is less than the MCC (9.5% < 10%), the proposed project is rejected, which is consistent with the negative NPV (−0.128) as shown in the last column of Table 1.

This method is, however, sensitive to both time parity and size parity. The size parity problem can be solved by the second NPV-consistent IRR method introduced in the next paragraph. To solve the time parity problem, all the compared projects must be based on the same horizon period. In Table 1, Projects B and C had a different horizon period as compared to Project A. To apply the MRIC, the horizon periods of Projects B and C to that of Project A must be extended. The MRIC of Project B with the cash flow pattern {−1, 5, −6, 0} is 0.5% and that of Project C {−1, 4, −4, 0} is 4%. The results are twofold. First, all the proposed projects should be rejected [MRICs < MCC (10%)]. Second, the three projects are ranked as A, C, and B (in descending order of the MRIC). Both results are consistent with the NPV results.

Incremental Approach to the IRR

The concept of the incremental approach to the IRR was probably first introduced by Fisher (1930) more than seven decades ago (Hajdasinski 2004). It was designed to rank mutually exclusive alternatives of the same project or mutually exclusive projects by identifying the generic differences between two competing alternatives/projects (Shull 1992). To apply the approach, we need to have two cash flow profiles (A and B) for a paired comparison (A-B or B-A). For the case of three competing projects, three paired comparisons (i.e., A-B, B-C, and C-A) are needed and so forth.

Mathematically, a new (hypothetical) incremental project (A-B or B-A) is formed where Project A has the cash flow pattern $\{a_0, a_1, a_2, \ldots, a_t\}$ and Project B has $\{b_0, b_1, b_2, \ldots, b_t\}$. Incremental Project (A-B) defines the difference between Project A and Project B in terms of the cash flow sequence $a_0-b_0, a_1-b_1, \ldots, a_t-b_t$. Project A is said to dominate Project B (i.e., drop B in favor of A) when incremental Project (A-B) is acceptable (i.e., the $\text{IRR}_{\text{A-B}}$ > MCC) or vice versa.

Referring back to the three projects in Table 1, we first compare between Project A and Project B. As Project B has no cash flow at x_3, we therefore can assume that it is zero. The incremental cash flows of Project (A-B) are {0, 1, −5, 6}. Since there are sign changes in this example, we need to apply the MRIC method. By substituting the cash flows into Eq. (2), we get (at the MCC=10%) the $\text{IRR}_{\text{A-B}}$ as 20.4% (>MCC). Therefore, Project A is said to be dominant over Project B. Employ the same method to produce new combined cash flows of (B-C), which are {0, 1, −2, 0}. The $\text{IRR}_{\text{B-C}}$ is −9.9% (<MCC). That is, Project C is dominant over Project B. Now, we need to compare A to C as they both are higher than B. The $\text{IRR}_{\text{A-C}}$ of (A-C) with new cash flows of {0, 2, −7, 6} is 13.3% (>MCC). Then, we have a rank order of A, C, and B. The results are consistent with the MRIC results and are NPV compatible.

The incremental approach has two drawbacks. First, the MRIC method must be applied for cash flows with more than one sign change. Second, given a set of n projects, $[n(n-1)/2]$ times separate incremental IRR calculations are required, which inevitably complicate the analysis (Kennedy and Plath 1994). Nevertheless, the incremental approach can handle the parity of project sizes.

We use the data of Kennedy and Plath (1994) to illustrate how this approach can solve the size parity problem. As shown in Table 2, the large project is more favorable than the small one although they both have positive NPVs. Nevertheless, the con-

Table 2. Small Project versus Large Project

Project size	Year 0	Year 1	NPV@10%	IRR (%)
Small (S)	−$100	$1,000	$809	900
Large (L)	−$100,000	$200,000	$81,818	100

Note: Source from Kennedy and Plath (1994).

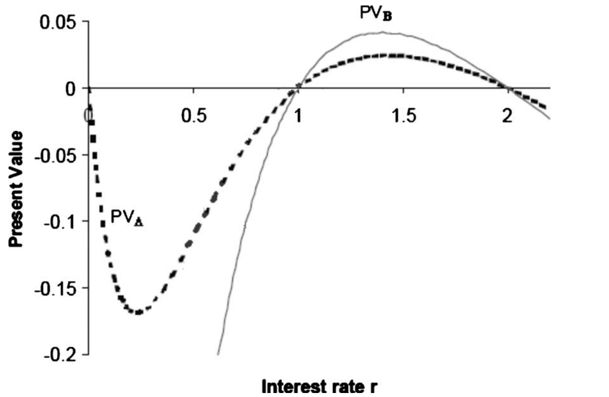

Fig. 1. NPVs of the two projects as functions of the interest rate r (adapted from Hazen 2003)

ventional IRR method gives an opposite outcome since the relative distance between the two years (in terms of the absolute value of the ratio of Year 1 to Year 0) of the small project $(1{,}000 \times 100\%/100 = 1{,}000\%)$ is much larger than that of the large project $(200{,}000 \times 100\%/100{,}000 = 200\%)$. They suggested the use of the incremental approach but did not perform the calculation in their paper. Now, we compare the two projects with the incremental cash flow pattern (L-S) as {−99,900, 199,000}. The $IRR_{L\text{-}S}$ is 99.2%, which implies that the large project is more favorable as compared to the small one, and this is consistent with the finding from the NPV.

IRR Parity Technique

The IRR parity technique is developed by Zhang (2005), which focuses on the parity (either even or odd) of the number of real IRRs that are greater than the cost of capital. The technique is convenient to address multiple IRR problems as it helps to dictate the accept/reject decision with the following stepwise process (Zhang 2005):

1. Solve for all the real IRRs using Eq. (1). If there is only one real root, it is the single IRR. If multiple real roots exist, go to Step 2.
2. In the situations where the initial cash inflow is zero, the time axis needs to be recalibrated to assign time 0 as the location of the first nonzero cash flow. The truncated project's IRRs will be identical to the original project's IRRs.
3. Count the number of real IRRs that are greater than the cost of capital r. An even number (including zero) leads to the $\text{NPV}(x) < 0$, while an odd number leads to the $\text{NPV}(x) > 0$. Note that this is the investment decision (opposite criteria are for borrowing decisions).
4. Make the decision based on the parity (odd or even) of real IRRs and pick the "relevant IRR" that is one of the IRRs adjacent to r. For an acceptance decision, pick the IRR that is adjacent to and greater than r. For a rejection decision, pick the one adjacent to and less than r.

Using Projects A and B in Table 1 as an example, we can illustrate how the above steps can be followed. Multiple IRRs of Project A turn out to be 0, 100, and 200%, while those of Project B are 100 and 200% (see Fig. 1). Then, we can apply the technique for both projects at $r = 10\%$. Projects A and B should be rejected as they both have an even number (=2) of IRRs greater than r. The relevant IRR of project A is 0%. For Project B, it is impossible to pick a relevant IRR that is smaller than r, and thus we can simply choose the one adjacent to r (i.e., 100%). The results here are consistent with the previous two methods, and the technique is also NPV compatible.

There are a number of drawbacks of this technique. First, it does not compute a true IRR for the project. Second, real projects may have many real IRRs and need to solve the complicated polynomial equations. Third, this technique is not capable of ranking projects directly. For ranking projects, the incremental approach to forming an incremental project (X-Y) should be applied prior to employing the IRR parity technique to make accept/reject decisions.

Comparisons among the Three IRR Methods

So far we have discussed about the three most convincing NPV-consistent IRR methods that can be used to make proper project evaluation and ranking. Although IRR methods have been studied for several decades, misconceptions are still present in recent research publications (Hajdasinski 2004). To make productive project evaluation and ranking, we should rely on plausible IRR techniques. Table 3 lists the pros and cons of the three presented methods. To apply these methods, several guidelines are also listed in Table 4.

Conclusions

PFI contracts often involve the evaluating and ranking of projects by a set of financial criteria. As it outweighs the NPV by provid-

Table 3. Pros and Cons of the Three NPV-Consistent IRR Methods

Method	Pros	Cons
The MRIC	• Evaluate one or more than one project • No need to compute multiple real IRRs • Calculate true IRR for each project	• Need calculation of the discounted and compounded cash flows • Not suitable for dealing with size parity
The incremental approach to the IRR	• Especially good for ranking two mutually exclusive projects • Can solve problems of size parity • No need to compute multiple real IRRs	• Use paired comparison, which may be complicated when ranking many projects • Not able to calculate true IRR • Need to apply the MRIC when there is more than one sign change
The IRR parity technique	• Evaluate one project • Very simple to use if multiple real IRRs have been provided	• Not able to calculate true IRR • Not able to rank projects directly

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Table 4. Guidelines for Choosing the IRR Methods

Decision problem	A	B	C
One project with a unique IRR	+	0	+
One project with multiple IRRs	+	0	+
Want for true IRRs	+	0	0
Rank two mutually exclusive projects	+	+	0
Rank two mutually exclusive options of a project	+	+	0
Rank a large number of projects	+	0	0
Rank a large number of options of a project	+	0	0
Projects with parity of project sizes	0	+	0
If real IRRs are already provided	0	0	+

Note: A=MRIC; B=incremental approach; C=IRR parity technique; +=recommended; and 0=not recommended.

ing a ROR, the IRR has been widely used to evaluate the acceptability of a PFI project. In order to address the misconception of IRR techniques in PFI contracts, we have introduced three reliable NPV-consistent IRR methods. Previous fallacies on the IRR have disrupted the development of proper PFI investment analysis methods. All future research on PFI financing should be built on true and credible investment theories.

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