Model for Financial Renegotiation in Public-Private Partnership Projects and Its Policy Implications: Game Theoretic View

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Abstract: Today, government is no longer considered the sole provider of public works or services. Public-private partnership (PPP) has been recognized as an important approach to solving problems for governments in providing infrastructure systems. However, the joint ownership or partnership in PPP complicates the administration of PPP projects. Too often, in PPP, many serious problems occur mainly because of bad administration policies. In particular, the fact that government may rescue a distressed project and renegotiate with the developer causes major problems in project procurement and management. This paper aims to study when and how government will rescue a distressed project and what impacts government's rescue behavior has on project procurement and management. A game-theory based model for government rescue dynamics is developed. Propositions, corollaries, and important policy implications are then derived from the model. This pilot study, the writer hopes, may provide theoretic foundations to policy makers for prescribing effective PPP procurement and management policies and for examining the quality of PPP policies. The study can also offer researchers a framework and a methodology to understand the behavioral dynamics of the parties in PPP.

DOI: 10.1061/(ASCE)0733-9364(2006)132:7(678)

CE Database subject headings: Partnerships; Procurement; Contracts; Financial management; Contract management.

Introduction

Private participation in providing public works and services has been recognized as an important approach for governments (Walker and Smith 1995; Henk 1998). The role of government has evolved into the coordinating partner providing infrastructure systems along with the private sector, which has brought both managerial and financial efficiencies into the process. Various terminologies, though sometimes confusing, are used to describe such private participation, such as private finance initiative (PFI) and build-operate-transfer (BOT). A more generic term for such long-term public-private cooperation in providing public works/services is public-private partnership (PPP).

However, the joint ownership or partnership in PPP complicates the project administration, particularly in project procurement and contract management. Here, the "developers" refer to the major shareholding firms, the owners, of a PPP firm. The fact that government may bail out a distressed project and renegotiate with the developer in PPP causes serious opportunism problems in project administration.

- The first problem is the opportunistic bidding behavior during project procurement. In this paper, opportunistic bidding behavior in PPP refers to that the bidders, in their proposals, intentionally understate possible risks involved or overstate the project profitability in order to outperform other bidders. In their pilot study, Ho and Liu (2004) developed a game theoretic claims decision model (CDM) for analyzing the behavioral dynamics of builders and owners in construction claims and the implications on opportunistic bidding. Their model shows that if a builder can easily make an effective construction claim, the builder will have incentives to bid opportunistically. In PPP, a successful request for renegotiation is analogous to an effective claim. In other words, if the request for renegotiation is always granted, the developers would then have incentives to bid optimistically to win the project. The reason that an overly optimistic proposal can have a higher chance of winning is because some crucial and developer-specific information regarding the project is difficult to verify by government and, as a result, can be untruthfully revealed in the development proposal. That is, some important information is *asymmetric* to government. For example, the developer's cost and profit structures, the project's commercial and technical risk, and the risk impacts may not be fully revealed in, or consistent with, the developer's bid proposal. Because of the information asymmetry in PPP, opportunistic bidding may succeed during procurement. Therefore if the developers have incentives to bid opportunistically due to the ex ante ex pectation of ex post renegotiation, the effectiveness of project procurement and management will be influenced significantly. Since this logic between government rescue and project administration effectiveness is not straightforward, the importance of the financial renegotiation problem is underemphasized.
- The second opportunism problem is the principal-agent problem, where the principal is played by government and the agent is played by the developers. This problem is also regarded as moral hazard problem, which happens only after

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Note. Discussion open until December 1, 2006. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on September 17, 2004; approved on January 12, 2006. This paper is part of the ***Journal of Construction Engineering and Management***, Vol. 132, No. 7, July 1, 2006. ©ASCE, ISSN 0733-9364/2006/7-678–688/$25.00.

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the contract is signed. In his repossession game example, Rasmusen (2001) shows that if renegotiation is expected, the agent may choose inefficient actions that will reduce overall or social efficiency, but increase the agent's payoff. In PPP, after signing the concession, moral hazard problems will also occur if renegotiation is expected. For example, given in practice that the developers are often the major contractors or suppliers of the PPP project, the developers may not be concerned too much about project cost overrun because the contractors may benefit from such overspending.

Works by Hart and Moore (1988) and Dewatripont (1988) discuss the essence of renegotiation and its impacts. Hart and Moore studied the effectiveness of building into the contract a mechanism for revising the terms of trade. They found that because of the possibility of renegotiation, the effectiveness of such contract design is limited. Dewatripont investigated how contracts can be designed to achieve renegotiation-proof under particular situations. Their papers gave us the fundamental concept of the renegotiation problems in PPP, although the effort to search a rigorous renegotiation model in PPP was fruitless. There are other complicated and insightful models in renegotiation, such as Maskin and Moore (1999); Segal (1999); Anderlini and Felli (2001); and Ishiguro and Itoh (2001); however, these models involve unnecessarily complicated games for the modeling of PPP renegotiation.

Most PPP contracts go through a regular negotiation process during project procurement. The negotiation process before signing a contract was discussed by Tiong and Alum (1997). Financial renegotiation, in this paper, refers to the rescuing financial subsidy negotiation after the contract is signed, when conditions change unfavorably and significantly. In PPP, financial renegotiation may happen when project cost, market demand, or other market conditions become significantly unfavorable. If government always bails out a financially distressed project, renegotiation will be expected by developers and such expectation can cause opportunism problems. Unfortunately, government is often tempted to bail out distressed projects because of the ex post renegotiation benefits to government and/or the society. The dilemma faced by government is that although financial renegotiation is not considered an option in the contract before project distress, it is often desirable after the distress. Such time inconsistency creates incentives for opportunism. The major objective of this paper is to investigate theoretically when and how government rescues a distressed project and what impacts government's rescue behavior has on the project procurement and management.

This paper presents a game theoretic model to analyze government's procurement and management policies from the perspective of renegotiation. The results may provide theoretic foundations and guidelines for examining the effectiveness of government's procurement and management policies in PPP. This paper begins with a brief introduction of the government rescue problems and the research methodology used in this study. Then I will present an analytical model based on game theory and solve for the game equilibrium solutions. Next I will formally state and prove the propositions and corollaries obtained from the model. Followings are rules, governing principles, and policies implications for PPP derived from the propositions and corollaries.

Research Methodology

The methodology adopted for theoretical study is game theory. Game theory can be defined as "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" (Myerson 1991). Therefore it is critical that readers avoid making any negative ethical implications on "playing games." Among economic theories, game theory has been successfully applied to many important issues, such as negotiations, finance, and imperfect markets. In PPP projects, conflicts among project developer and government are very common, particularly in an adverse situation, and therefore game theory is a natural tool for analyzing the problem. Game theory has been adopted by Ho and Liu (2004) to analyze the behavioral dynamics of construction claims. Ho (2005) also applied game theory to developing a model to evaluate the effectiveness of rewarding bid compensation in project procurement. The following sections describe the basic concepts of game theory that are closely related to the government rescue and renegotiation.

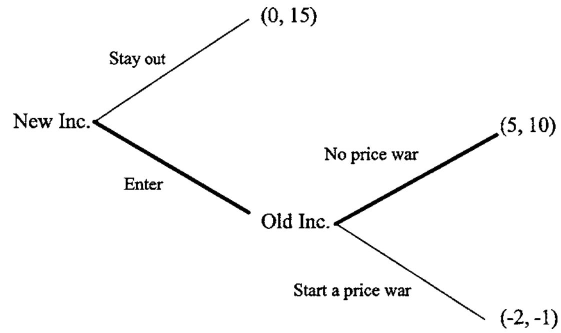

Fig. 1. Simplified market entry game

Types of Games

There are two basic types of games: static games and dynamic games, in terms of the timing of decision making. In a static game, the players act simultaneously. Note that "simultaneously" here means that each player makes a decision without knowing the decisions made by others. On the contrary, in a dynamic game, the players act sequentially. Due to the nature of government rescue, the dynamic game will be used for modeling and analyzing the renegotiation and its associated problems.

Players in a dynamic game move *sequentially* instead of simultaneously. It is more intuitive to represent a dynamic game by a treelike structure, also called the "extensive form" representation. We will use the following simplified *market entry* example to demonstrate the concepts of a game analysis. A new firm, New Inc., wants to enter a market to compete with a monopoly firm, Old Inc. The monopoly firm does not want the new firm to enter the market because new entry will reduce the old firm's profits. Therefore Old Inc. threatens New Inc. with a price war if New Inc. enters the market. Fig. 1 shows the extensive form of the market entry game. If the payoff matrix shown in Fig. 1 is known to all players, the payoff matrix is "common knowledge" to all players and this game is called a game of "complete information." The game tree shows (1) New Inc. chooses to enter the market or not, and then Old Inc. chooses to start a price war or not, and (2) the payoff of each decision combination. Note that the players of a game are assumed to be rational. This is one of the most important assumptions in most economic theories. In other words, it is assumed that the players will always try to maximize their payoffs.

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Game Solution: Nash Equilibrium

As to answer what each player will play/behave in this game, we shall introduce the concept of "*Nash equilibrium*," one of the most important concepts in game theory. The Nash equilibrium is a set of actions that will be chosen by each player. In other words, in a Nash equilibrium, each player's strategy should be the best response to the other player's strategy, and no player wants to deviate from the equilibrium solution. Thus the equilibrium or solution is "strategically stable" or "self-enforcing" (Gibbons 1992).

An intuitive conjecture of the solution of the market entry game is that New Inc. will "stay out" because Old Inc. threatens to "start a price war" if New Inc. plays "enter." However, Fig. 1 shows that the threat to start a price war is *not credible* because Old Inc. can only be worse off by starting a price war if New Inc. does enter. On the other hand, New Inc. knows the pretense of threat, and therefore will maximize the payoff by playing "enter." As a result, the Nash equilibrium of the market entry game is (enter, no price war), a strategically stable solution that does not rely on the player to carry out an incredible threat. Note that this simplified market entry game did not consider that there might be other new companies trying to enter if the old company did not maintain a certain reputation regarding the credibility of threat. The game in Fig. 1 is called a "dynamic game of complete information." A dynamic game can be solved by maximizing each player's payoff *backward recursively* along the game tree (Gibbons 1992). We shall apply this technique in solving the government renegotiation game in PPP.

Financial Renegotiation Game and Equilibria: The Model

As argued in the Introduction, the behavioral dynamics of the renegotiation or government rescue plays a central role in PPP administration when information asymmetry exists. Here, game theory is applied to analyze when government will renegotiate with the developer and the impacts of such renegotiation on the project. While this study is motivated by some real world cases from various countries and the writer's personal consulting experiences, the goal of this model is to provide a framework that is not restricted to a particular environment. In other words, the model is expected to consider various environments characterized by the parameters of the model.

Model Setup

The game theoretic framework for analyzing a PPP investment shown in Fig. 2 is a dynamic game expressed in an extensive form.

Suppose a PPP contract does not specify any government rescue or subsidies in the face of financial crisis. Neither does the law prohibit government from bailing out the PPP project by providing debt guarantee or extending the concession period. Suppose also that government is not encouraged to rescue a project without compelling and justifiable reasons. For example, cost overrun or operation losses caused by inefficient management or normal business risk should not be justified for government rescue, whereas adverse events caused by unexpected or unusual equipment/material price escalation may be justified more easily. Thus it should be reasonable to assume that if

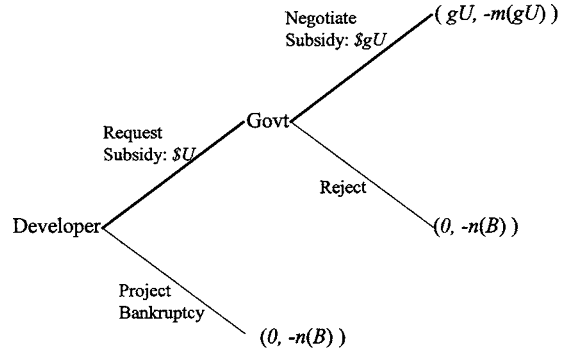

Fig. 2. Renegotiation game's equilibrium path

government grants a subsidy to a project on the basis of unjustifiable reasons, government may suffer from the loss of public trust or the suspicion of corruption.

The dynamic game, as shown in Fig. 2, starts from adverse situations where it is in the developer's (denoted by D in the game tree) or lending bank's best interests to bankrupt the project if government (denoted by G) does not rescue the project. Alternatively, the developer can also request government to rescue and subsidize for the amount of $\$U$, even though the contract clause does not specify any possible future rescue from government. Here U is defined as the present value of the net financial viability change as shown in Eq. (1), and is considered as the maximum possible requested subsidy. Here it is assumed that the maximum possible amount that could be requested as subsidy will not exceed U.

$$U = \mathrm{NPV}_{0,t} - \mathrm{NPV}_t \qquad (1)$$

where the subscripts 0 and t represent the time when the project begins and the current time when the project is in distress, respectively. Here $\mathrm{NPV}_{0,t}$=net present value evaluated at time 0 and converted to time t value; NPV_t=NPV evaluated at time t. Note that U is not the actual subsidy amount. Instead, the actual subsidy is determined in the renegotiation process discussed later.

If the developer chooses project bankruptcy, the payoff will be $-\delta$. Here it is assumed $\delta \to 0$. The main reason is that if the situations call for bankruptcy, the value of the *equity shares* held by the developer should approach zero before project bankruptcy; therefore, the developer, being an equity holder, will lose little if the distressed project is bankrupted. Thus it is assumed that $\delta=0$ in the model. Note that some may argue that δ is significant due to the loss of reputation. However, the loss of reputation occurs when the project is in distress, no matter if the developer chooses to request rescue subsidies or project bankruptcy. Therefore, if δ is defined as bankruptcy payoff, then δ should not be regarded as the loss of reputation. The consideration of reputation loss could be another parallel approach that may discourage opportunistic behaviors. The effect of this parallel strategy, from the game theoretic perspective, is complicated and beyond the scope of the paper.

On the other hand, if a PPP project is bankrupted, the payoff of government is $-n(B)$, where B=government's "budget overspending" when a project is bankrupted and retendered, and n, a function of B, =*political cost due to project retendering*. Generally, from either a financial or political perspective, it is costly for government if a PPP project is bankrupted. Suppose that for a PPP project to proceed beyond procurement stage, the project must have shown to provide the facilities or services that can be justified economically. Then it is reasonable to assume that a

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bankrupted PPP project should be regained by government and retendered to another new developer, unless, in rare occasions, the marginal subsidy for improving project financial viability is greater than the net benefits from the facility/service. Logically, for government to "permanently" terminate a project without retendering, after spending millions or billions of dollars, would only signify that the project was not worth undertaking in the beginning and that a serious mistake was made by government during the project procurement. Therefore, in this game, it is assumed that retendering is desired by government if a project is going bankrupt.

Alternatively, as shown in Fig. 2, the developer can negotiate a subsidy starting with the maximum amount $\$U$, where the subsidy can be in various forms such as debt guarantee or concession period extension. Typically, in a financial distress, the bank will not provide extra capital needs without government debt guarantee or other subsidies. Because the debt guarantee is a liability to government, but an asset to the developer, debt guarantee is equivalent to a subsidy from government. Other forms of subsidy may include the extension of concession period, more tax exemption for a certain number of years, or extra loan or equity investment directly from government.

After the developer's request for subsidy, the game proceeds, as shown in Fig. 2, to its subgame: "negotiate subsidy" or "reject." If the government rejects the developer's request, the project will be bankrupted and retendered and the payoff for both parties will be $[0,-n(B)]$. If government decides to negotiate a subsidy, expressed by the *rescuing subsidy ratio* g, a ratio between 0 and 1, the payoff of the developer and government will be $[gU,-m(gU)]$, respectively, where m=*political cost due to the rescuing subsidy to a private party*. Note that although the political cost, m, is also a function of budgeting spending, function m is different from function n, because in the two functions the budget spending goes to different parties. To rescue a PPP project and provide rescuing subsidy to the original PPP firm could bring serious criticism toward government. If government lacks compelling reasons for the subsidy, the criticism will cause significant *political* cost depending on the magnitude of the subsidy. We shall discuss the differences between the two functions in detail later. Also note that here g is not a constant and is used to model the process of "offer" and "counter-offer." More details on the negotiation modeling using g can be found in Ho and Liu (2004).

"Rescue" or "No Rescue:" Nash Equilibria of Rescue Game

As mentioned previously, the financial renegotiation game tree derived above will be solved backward recursively and its Nash equilibrium solutions will be obtained. Since the values for the variables in the game's payoff matrix are undetermined, the payoff comparison and maximization cannot be done to solve for a unique solution. However, we can analyze the conditions for possible Nash equilibria of the game. There are three candidates for the Nash equilibria: (1) developer will "request subsidy," and government will "negotiate subsidy," (2) developer will "request subsidy" and government will "reject," and (3) developer will choose "project bankruptcy."

1. Developer will "request subsidy" and government will "negotiate subsidy." Here, since government chooses to "negotiate subsidy," this equilibrium is called "rescue" equilibrium in this paper. Solving backward from the government's node first, if the payoff from negotiation is greater than that from rejection, i.e., $-m(gU)\geqslant -n(B)$, government will "negotiate subsidy" with the developer. Therefore the condition for negotiation or rescue can be rewritten as

$$m(gU)\leqslant n(B) \tag{2}$$

This condition is straightforward: the political cost of rescue should be less than or equal to the political cost for not rescuing the project. As indicated by the latter bold line in Fig. 2, the payoff for the developer and government will now be $[gU,-m(gU)]$, respectively.

The next step is to solve Fig. 2 backward again, at the developer's node, and obtain the final solution. Now the payoffs for "request subsidy" are $[gU,-m(gU)]$, and the developer will request subsidy if $gU\geqslant 0$. Since g and U will not be negative numbers, the condition for the developer to request subsidy will always be satisfied. In other words, it is always to the developer's benefit to negotiate subsidy if Eq. (2) is satisfied.

Fig. 2 also shows the equilibrium path expressed in bold lines that goes through the game tree. Note that when the developer requests subsidy for U, the final settlement for the subsidy will be a portion of U, gU, which satisfies Eq. (2). The range of the renegotiation offer can be expressed as

$$gU \in \{x{:}x\geqslant 0, m(x)\leqslant n(B)\} \tag{3}$$

From Eq. (3), we know that as long as $n(B)-m(gU)\geqslant 0$, the rescue equilibrium will be the solution of the game, where no one can be better off by deviating from this equilibrium. Note that the condition for this equilibrium needs to be refined due to other concerns, and we will discuss this further in other sections.

2. Developer will "request subsidy" and government will "reject."
If Eq. (2) is not satisfied, "reject" would be a preferable decision to government, and the payoff matrix for both parties is $[0,-n(B)]$. Now turn to the developer's node: it seems that the payoff of either "request subsidy" or "project bankruptcy" is \$0, and the developer is indifferent between the two actions. From the game tree, it is not obvious which action the developer will choose. However, if the developer recognizes the existence of the cost incurred in the process of requesting subsidy, although it may be relatively small compared to other variables in the game tree, the developer should choose "project bankruptcy," instead of requesting subsidy. From this perspective, although the cost of requesting subsidy is suppressed in the game tree for clarity, the cost of requesting subsidy should be recognized whenever there is a tie between "request subsidy" and "project bankruptcy." To summarize, if the developer knows government will "reject" the subsidy request, the developer will choose "project bankruptcy," instead of "request subsidy" in the first place, and this is exactly the third possible equilibrium, "project bankruptcy." Thus the second equilibrium solution cannot exist.
3. Developer chooses "project bankruptcy."
Here, since the developer knows that government will choose to "reject" the subsidy request, the developer will choose project bankruptcy in the first place. We shall term this equilibrium the "no rescue" equilibrium. As argued above, the developer will choose project bankruptcy if and only if it is optimal for government to "reject" the subsidy request. Therefore the condition of this Nash equilibrium would be

$$m(gU) > n(B) \tag{4}$$

In other words, for "project bankruptcy" to be an equilibrium solution, it must be that it is impossible to achieve the "rescue" solution. Eq. (4) can be rewritten as

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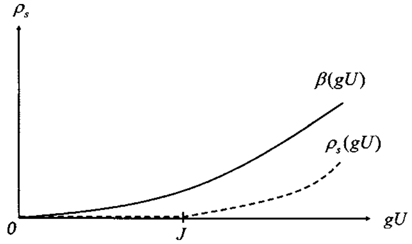

Fig. 3. Political cost function of budget overspending, $\beta(gU)$, and political cost function of oversubsidization, $\rho_S(gU)$

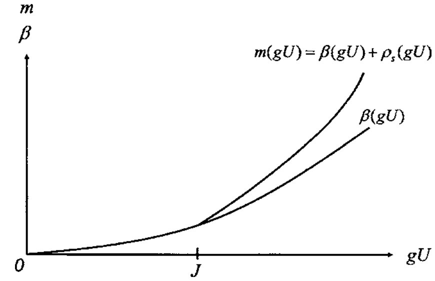

Fig. 4. Political cost function of rescuing a project, $m(gU)$

$$n(B) - m(gU) < 0 \tag{5}$$

To conclude this section, we find Eqs. (2) and (4) for the PPP rescue game's "rescue" and "no rescue" equilibria, respectively. Both equilibria depend solely on the knowledge of government's political cost for rejecting a subsidy and granting a subsidy. We shall assume that the PPP game is a game with *complete information*, where $n(B)$ and $m(gU)$ are common knowledge and both parties know that the other party is equally rational and smart. Note that from the practical perspective, it is not easy for both parties to quantify $n(B)$ and $m(gU)$ because it is difficult to measure political cost in terms of monetary units. Fortunately, the game depicted above can still be analyzed without knowing the exact functions for $n(B)$ and $m(gU)$, and such game theoretic analysis can still lead to important qualitative and quantitative implications on PPP policies and decision making.

Modeling of Game Parameters

To perform this analysis, we need to examine the characteristics of the PPP project, especially its bankruptcy conditions and the political costs associated with bankruptcy.

Political Cost of Rescuing a Project by Subsidy

If government negotiates the subsidy with the existing developer and rescues the project, the function of the political cost to government is modeled here as

$$m(gU) = \begin{cases} \beta(gU) & \text{if } gU \leqslant J \\ \beta(gU) + \rho_s(gU) & \text{if } gU > J \end{cases} \tag{6}$$

where J=amount of the subsidy that can be justified without the criticism of oversubsidization, $\beta(gU)$=political cost of budget overspending, and $\rho_s(gU)$=political cost of oversubsidization. The subscript s of $\rho_s(gU)$ denotes subsidy.

The modeling of the political cost of subsidy in Eq. (6) is based on the most fundamental concept in economics that resources are scarce. If government has unlimited funds to spend, there would be no political cost for negotiated subsidy. Since government only has a limited budget to allocate, there will be political cost to government should the funds not be allocated appropriately. The more the subsidy is, the higher the political cost should be. As a result, the political cost of subsidy should be a strictly increasing function of the amount of subsidy, gU. A strictly increasing function is defined here by $\forall x > x'$: $f(x) > f(x')$; i.e., for all $x > x'$, it is true that $f(x) > f(x')$. In Eq. (6), the political cost is further broken down into two elements, namely, $\beta(gU)$ and $\rho_s(gU)$. $\beta(gU)$, as illustrated in Fig. 3, is an increasing function of gU, representing the political cost caused by budget overspending in subsidy, and is considered the "basic" political cost. In addition to the basic political cost, it is argued that for subsidy exceeding a certain justifiable amount, further political cost, $\rho_s(gU)$, would incur so as to reflect a more serious resource misallocation. In the model, J is termed the "justifiable subsidy," which is considered by the public an eligible claim for subsidy. Alternatively, J can be measured by imagining that if the request goes to court, what amount of "claim" by the developer the court will grant. For example, the damages due to force majeur might be considered justifiable. If the subsidy is less than the justifiable claim, government will not be blamed for oversubsidization, and therefore $\rho_s(gU)$ will be considered zero when $gU \leqslant J$. However, when the subsidy is greater than J, government will be criticized for oversubsidization, or be accused of or suspected of corruption, and will suffer further political cost, $\rho_s(gU)$, in addition to the basic political cost, $\beta(gU)$. Fig. 3 also illustrates the function of the political cost of oversubsidization, $\rho_s(gU)$. It is worth noting that the shapes of the functions in Fig. 3 are for illustration purposes. The functions need not to be continuous or convex. The only requirement is that these functions are strictly increasing. Fig. 4 shows the function $m(gU)$ obtained by combing the curves in Fig. 3 as defined in Eq. (6).

Political Cost of Retendering a Project

To analyze the adverse conditions that place a PPP project on the edge of bankruptcy, we need some concepts of the bankruptcy mechanism. A very common bankruptcy condition in debt indenture is the inability of the borrower to meet the repayment schedule. In PPP, the lending bank will also impose certain conditions to trigger bankruptcy and protect the loan should adverse events happen. For example, the lenders could specify the upper limit of cost overrun during the project development or construction. According to financial theory, rational lenders will prevent the *net value of the project up to current progress* from being below the *up-to-date debt outstanding*. Since project value and cost may be volatile from time to time during the project life cycle, to ensure the security of debt, lenders need to evaluate the project viability and debt security periodically in terms of the project's gross value and required debt.

If we assume that the lending bank can effectively monitor the project financial status, we may infer that at the time of bankruptcy, the overall value of the project will be less than but close to the estimated total outstanding debt. As a result, under near bankruptcy conditions, it is not wise for the bank to continue providing additional capital because it is likely that the PPP firm will not be able to repay any further borrowing. Unless government guarantees the repayment of the loan, or secures the additional debt by other means, the lending bank will deny further capital request, even when such capital is still within the project's original loan contract.

When a project is bankrupted, it will be considered "sold" to government and retendered to some other private developer given

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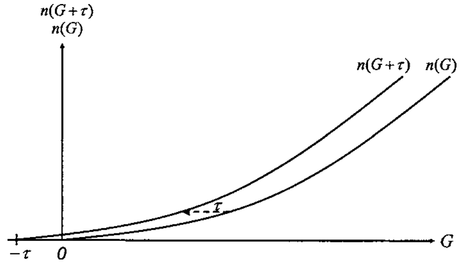

Fig. 5. Function $n(G+\tau)$ with respect to G, given a fixed τ

the assumption made earlier that the project is still worth completing. Government may also want to regain control of the project after previous unsuccessful development because a PPP contract is usually related to public facilities or services and, therefore, cannot be transferred directly to a new developer without a new contract negotiated and signed with government. Thus, to government, the bankruptcy could be considered a costly replacement of the developer. Suppose that under normal situations, the bankrupted project acquired by government will still be financed mainly by debt, and the subsidies for securing the lending bank's new loan are essential in order to complete the project or continue the operation. As a result, when a project is bankrupted, the amount of budgeting overspending can be modeled as

$$B = G + \tau \tag{7}$$

where G=least required subsidy that can persuade the lending bank to support a distressed project, and τ=opportunity cost for replacing developers, which may include the retendering cost and the cost of interruption due to the bankruptcy and retendering process.

Similar to the political cost of rescuing a project, the political cost of project retendering can be modeled by

$$n(B) = \beta(B) \tag{8}$$

Substitute Eq. (7) into Eq. (8), and then Eq. (8) can be rewritten as

$$n(G+\tau) = \beta(G+\tau) \tag{9}$$

Fig. 5 shows functions $n(G)$ and $n(G+\tau)$, defined by Eq. (9), where given τ is fixed, the variable of horizontal axis will be G. Thus function $n(G+\tau)$ is depicted differently from $n(G)$, as shown in Fig. 5, by shifting the original $n(G)$ to the left by τ.

Mathematical Characteristics of Parameters in PPP

- Characteristic 1. As argued previously, by the definition of G, if government intends to rescue a project, the subsidy to the project must be at least equal to G, i.e., $gU \geqslant G$.
- Characteristic 2. Whereas the developer replacing opportunity cost is always positive and significant, i.e., $\tau \gg 0$, in the following analysis, we shall make a weaker assumption that τ is non-negative.
- Characteristic 3. Given Eq. (6) and that β and ρ_s are strictly increasing functions, it is straightforward that $m'(x) > \beta'(x)$ is true for all $x \in (J,\infty)$, formally denoted as $\forall x \in (J,\infty)$: $m'(x) > \beta'(x)$, where $m'(x)$ and $\beta'(x)$=slope of functions m and β, respectively.
- Characteristic 4. Since not all losses due to financial viability change can be justified for subsidy during renegotiation, the range of J can be modeled as

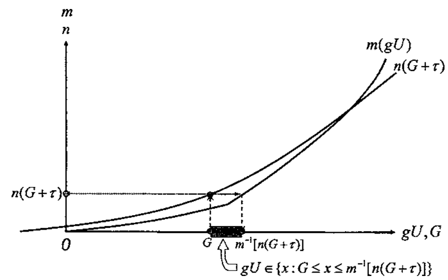

Fig. 6. Renegotiation offer zone in "rescue" equilibrium

$$J \in [0,U] \tag{10}$$

The amount of justifiable subsidy depends on how the public may agree with the subsidy considering the developer's justifiable reasons. Alternatively, J may also be quantitatively determined should the subsidy request be brought to court.

- Characteristic 5. According to the NPV investment rule, we may define G by the equality: $G+\mathrm{NPV}_t=0$, meaning that G will revert the project NPV to zero. This characteristic comes from the requirement that G should improve a project from negative NPV_t to zero NPV. Note that zero NPV indicates that the project has normal profit and is worth continuing for developers.

Refined Nash Equilibrium

Previous sections conclude that Eqs. (2) and (5) are the conditions for "rescue" and "no rescue" equilibria, respectively; however, it is also noted that these conditions need to be refined. By Characteristic 1, to rescue a project the subsidy must be at least equal to G, i.e., $gU \geqslant G$. As a result, the condition for rescue equilibrium becomes

$$m(gU) \leqslant n(B) \quad \text{where } gU \geqslant G \tag{11}$$

Substitute Eq. (7) into Eq. (11), Eq. (11) can be rewritten as

$$m(gU) \leqslant n(G+\tau) \quad \text{where } gU \geqslant G \tag{12}$$

Since $m(gU)$=increasing function, gU must have an upper limit, below which the inequality in Eq. (12) is satisfied. The upper limit of gU can be obtained by solving $n(G+\tau)-m(gU)=0$. Thus the condition for rescue equilibrium can also be reorganized and expressed by the lower and upper limits of the subsidy as shown in Eq. (13)

$$gU \in \{x{:}G \leqslant x \leqslant m^{-1}[n(G+\tau)]\} \tag{13}$$

where $m^{-1}[n(G+\tau)]$=inverse function of m. Here Eq. (13) will be called "Renegotiation offer zone." Fig. 6 shows the rescue equilibrium condition, Eq. (13), and the renegotiation offer zone, indicated by the thick bar in the x axis. Given any G in Fig. 6, $n(G+\tau)$ will be determined first, and then $m^{-1}[n(G+\tau)]$ is obtained so that any gU between G and $m^{-1}[n(G+\tau)]$ will satisfy Eq. (12).

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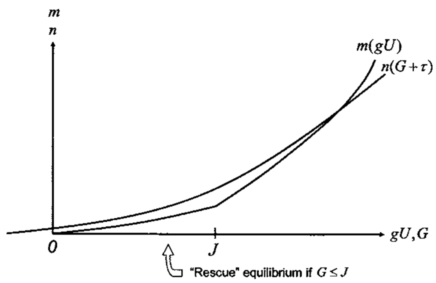

Fig. 7. Illustration of Proposition 2

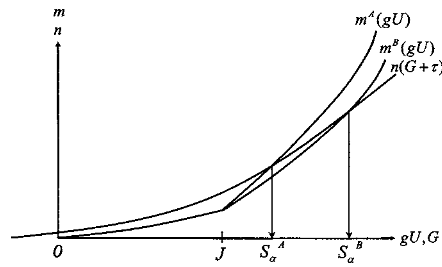

Fig. 9. Illustration of Corollary 5

Propositions, Corollaries, and Rules of Renegotiation

Propositions

Proposition 1

Assume that the rescue renegotiation process follows the game tree in Fig. 2, that g, U, J, G, and τ are non-negative and common knowledge, and that m and n are non-negative increasing political cost functions and common knowledge. Given U, G, τ and functions m and n, if $m(gU) \leqslant n(G+\tau)$, where $gU \geqslant G$, government will "rescue" a distressed PPP project with a negotiated subsidy, and the renegotiation offer zone is $gU \in \{x: G \leqslant x \leqslant m^{-1}[n(G+\tau)]\}$.

Proof. For the smoothness of the reading, the formal proof is placed in the Appendix. Proposition is graphically illustrated in Fig. 6, where the renegotiation offer zone is indicated.

Proposition 2

If all assumptions in Proposition 1 hold, given U, J, τ and functions m and n, the equilibrium must be to "rescue" if $G \leqslant J$.

Proof. See the Appendix.
Proposition 2 is graphically illustrated in Fig. 7.

Corollaries

Note that the proofs of corollaries are also placed in the Appendix.

Corollary 1

Suppose all assumptions in Proposition 1 hold, given U, J, τ, and functions m and n, the equilibrium must be to "rescue" if $G \leqslant J - \tau$ and the renegotiation offer zone is $gU \in \{x: G \leqslant x \leqslant G+\tau\}$

Corollary 2

Suppose all assumptions in Proposition 1 hold. Given U, τ, and functions m and n, when there exists a S_α defined by $S_\alpha = m^{-1}[n(S_\alpha + \tau)]$ and $\forall x \leqslant S_\alpha$: $m(x) \leqslant n(x+\tau)$, the equilibrium must be to "rescue" if $G \leqslant S_\alpha$ and the renegotiation offer zone is $gU \in \{x: G \leqslant x \leqslant m^{-1}[n(G+\tau)]\}$.

Note: Corollary 2 is illustrated in Fig. 8.

Corollary 3

Suppose all assumptions in Proposition 1 hold. Given U, J, τ, and functions m and n, when there exists a "unique" S_α and $\forall J < x < S_\alpha$: $m(x) < n(x+\tau)$, the equilibrium must be "no rescue" if $G > S_\alpha$.

Note: Corollary 3 is also illustrated by Fig. 8.

Corollary 4

Suppose all assumptions in Proposition 1 hold, the equilibrium must be to "rescue" if it is always true that $J = U$.

Corollary 5

Suppose all assumptions in Proposition 1 hold. Let $\rho_s^A > \rho_s^B$ be defined as $\forall x > J$: $\rho_s^A(x) > \rho_s^B(x)$, and ρ_s^A be called the "larger ρ_s." Let S_α^A and S_α^B be the unique S_α for ρ_s^A and ρ_s^B in functions m^A and m^B, respectively. It must be true that the larger ρ_s function will yield a smaller S_α, i.e., $S_\alpha^A < S_\alpha^B$.

Note: Corollary 5 is illustrated by Fig. 9, which shows that the steeper the function m is, the smaller the S_α is.

Corollary 6

Suppose all assumptions in Proposition 1 hold, if ρ_s is sufficiently small such that S_α does not exist, the equilibrium must be to "rescue."

Corollary 7

If all assumptions in Proposition 1 hold except that here we have $\tau = 0$, the equilibrium must be "no rescue" for all $G > J$, and must be to "rescue" at $gU = G$ for all $G \leqslant J$.

Note: Corollary 7 is illustrated by Fig. 10.

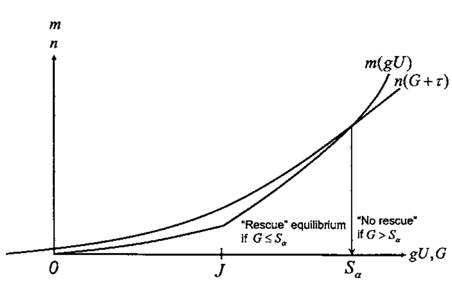

Fig. 8. Illustration of Corollaries 2 and 3

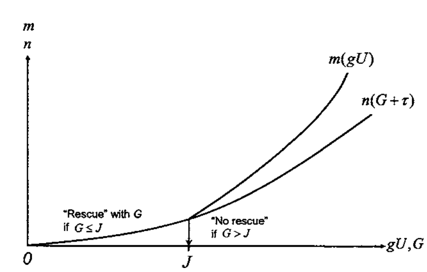

Fig. 10. Illustration of Corollary 7

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Rules from PPP Renegotiation Model

The propositions and corollaries can be transferred into rules to assist policy makers analyzing various renegotiation situations. The following rules are either from the propositions and corollaries directly above, or the logical inference following the propositions and corollaries. Discussions associated with each rule are given after stating each rule. Rigorous proof of these rules is not difficult to obtain and is left to interested readers due to length limitation.

Rules

Rule 1: Equilibrium Determination Rule. Given τ, U, and functions m and n, the equilibrium determination point is S_α. The equilibrium is to "rescue" if $G \leqslant S_\alpha$, and is "no rescue" if $G > S_\alpha$.

Remark: This rule is directly from Corollaries 2 and 3 given that S_α is unique. According to this rule, given all variables fixed but G, the probability of future renegotiation can be calculated by the probability that $G \leqslant S_\alpha$, denoted by $P(\tilde{G} \leqslant S_\alpha)$.

Rule 2: Maximum Subsidy Justification Domination Rule. Given only U and functions m and n, the equilibrium will be to "rescue" if $J=U$. In other words, when the subsidy can always be justified by its maximum amount, U, the equilibrium will be dominated by "rescue."

Remark: Note that this rule is directly from Corollary 4. According to this rule, S_α represents the likelihood that the equilibrium will be to negotiate and "rescue." In practice, if, due to information asymmetry, J could be manipulated so that J would be large up to U, then the "rescue" equilibrium is guaranteed and expected and problems of opportunism will occur.

Rule 3: S_α Determination Rule. Given function n, S_α will depend negatively on ρ_s, and positively on τ and J.

Remark: This rule can be inferred from Corollaries 5 and 6. Furthermore, if ρ_s is small enough to be ignored, then S_α will approach ∞ and the equilibrium will always be to "rescue." A direct inference from this rule is that in a more dictatorial country government will be more inclined to rescue a distressed project, justifiably or not, given that the project is still socially beneficial. Also, given other variables fixed, $\tau=0$ will yield the smallest S_α, which will be J, and functions $m(x)$ and $n(x)$ will collapse for all $x \leqslant S_\alpha = J$.

Rule 4: Renegotiation Offer Zone Rule. Given G, τ, and functions m and n, if the equilibrium is to "rescue," the renegotiation offer zone will be $gU \in \{x : G \leqslant x \leqslant m^{-1}[n(G+\tau)]\}$.

Remark: This rule is directly from Proposition 1. This solution is considered a Pareto optimal solution for both parties since both parties' payoffs will be improved compared to "no rescue" solution. The difference between $m^{-1}[n(G+\tau)]$ and G is the surplus obtained by reaching the settlement. The remaining question is how this surplus will be divided. The division of the surplus may depend on each party's negotiation power and risk attitude (Binmore 1992). For example, the party who is less risk averse may get more surplus than the other party. Detailed discussion is beyond the scope of this paper.

Rule 5: Interval of Renegotiation Offer Zone Rule. Rule 5-1: Given G and functions m and n, if the equilibrium is to "rescue," then the interval of the renegotiation offer zone will depend positively on τ. Particularly, when $\tau=0$ the interval of the zone will be zero, the rescuing subsidy will reach at $gU=G$.

Remark: Literature has attributed the occurrence of renegotiation to the hold-up problem due to the opportunity cost of contract termination, e.g., in our model, the developer replacing cost, τ. This rule confirms that the larger the replacing cost is, the more serious the hold-up problem is, and as a result, the wider the interval of the renegotiation offer zone is. However, surprisingly, Rule 5-1 shows that when there is no replacing cost, i.e., $\tau=0$, the equilibrium still guarantees the occurrence of renegotiation given that the "rescue" condition in Rule 1 is met. The major reason is the existence of the least required retendering subsidy, G. Apparently, G becomes the new basic factor for the hold-up problem when the project is financed through the PPP scheme. By the definition of project distress, G must be positive, and therefore, the hold-up problem must exist.

Rule 5-2: Given J, τ, and functions m and n, if the equilibrium is to "rescue," then the interval of the offer zone will depend negatively on different levels of G. Specifically, (1) When $G \leqslant J-\tau$, the offer zone is $gU \in [G, G+\tau]$ and the interval of the offer zone is τ, which is the maximal possible interval for the offer zone; (2) When $J-\tau < G < S_\alpha$, the offer zone is $gU \in [G, m^{-1}[n(G+\tau)]]$ and the interval of the offer zone is $m^{-1}[n(G+\tau)]-G$, which is less than τ; and (3) When $G=S_\alpha$, the renegotiation offer will be reached at S_α and the interval of the offer zone is zero.

Remark: This rule is inferred from Corollaries 1 and 2. An important insight from this rule is that the interval of the renegotiation offer zone cannot exceed τ. Note that the larger the interval is, the more important the negotiation power is, as the room for negotiation is larger. This means that the negotiation power is more important when G is small. Such a conclusion may contradict our intuition that a project requiring more G may need more efforts to "negotiate." In fact, when G is so large such that $G=S_\alpha$, there is no room for negotiation except to accept that the subsidy $gU=G=S_\alpha$.

Governing Principles and Policy Implications for Project Procurement and Management

Governing principles and administration policy implications can be obtained from the propositions, corollaries, and rules derived from the model. Note that the proposed model does not provide the approaches to quantifying the game parameters; instead, this pilot study focuses on the characteristics of the game parameters/functions and the relationship between these parameters. Particularly, the political cost functions m and n may be the most difficult to be quantitatively determined in this paper. Such tasks are beyond the scope of this paper. Fortunately, useful insights can still be drawn without knowing the approaches to quantifying parameters. The focus of this study will be on what strategies or policies can better handle and reduce the renegotiation problem and enhance the administration in PPP. Suggested governing principles and administration policies for PPP projects are discussed as follows.

Governing Principle 1: Be Well Prepared for Renegotiation Problems, as It Is Impossible to Rule out the Possibility of Renegotiation and the "Rescue" Equilibrium

As shown in Fig. 10, practically, S_α will be greater than 0 as S_α cannot be 0 unless $J=0$ and $\tau=0$. Thus it is always possible that

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$G \leqslant S_\alpha$ given that G is uncertain; i.e., it is impossible to rule out the "rescue" equilibrium. As a result, the government should be well prepared for the opportunism problems induced by the ex ante expectation of renegotiation as discussed previously. Policy implications from this principle include:

- In project procurement, while the developer's financial model is typically included in the proposal for reference, government should recognize the possibility of opportunism problems and always have reasonable doubt on the proposal provided by the developer.
- Government could devise a better mechanism that can enable the developer to reveal true information. For example, government can establish a formal procedure that may disqualify a developer during procurement if the developer is determined to have the history of behaving opportunistically.

Governing Principle 2: Although Renegotiation Is Always Possible, the Probability of Reaching "Rescue" Equilibrium Should Be Minimized and Could Be Reduced by Strategies That Increase ρ_s and Reduce τ and J

One way to reduce the opportunism problems is to minimize the probability of "rescue" equilibrium and the developer's expectation of the probability. According to Rule 1, the probability of "rescue" equilibrium, $P(\tilde{G} \leqslant S_\alpha)$, can be reduced by having a smaller S_α, which can be achieved by strategies that increase ρ_s and reduce τ and J. Policy implications by this principle may include the following:

- Specific laws may regulate the renegotiation and negotiated subsidy, and such laws will increase ρ_s when the subsidy is not justifiable.
- A good monitoring or early warning system can also give government enough lead time to prepare for replacing a developer with minimal impact, and hence, reduce τ.
- To reduce J, government should pay attention to the quality of the contract in terms of content and implementation, e.g., the scope, risk allocation, documentation, and contract management process.

Governing Principle 3: The Interval of Renegotiation Offer Zone Should Be Minimized and Can Be Reduced by Increasing ρ_s and Reducing τ

The smaller the interval of renegotiation offer zone is, the easier the renegotiation will reach an agreement that is close to the zone's lower bound, G. According to Rule 5, strategies that increase ρ_s and reduce τ can reduce the interval of the offer zone. Policies implied by this principle are the same as those implied by the previous principle. Since these policies are implied by two different governing principles, these policies deserve more attention.

Governing Principle 4: During the Renegotiation Process, the Government Should Try to Settle the Rescuing Subsidy at G, and Spend More Efforts on Determining G Objectively and Conveying Such Information to the Developer, Rather Than on Negotiation Skills

Since the "rescue" equilibrium is a strictly dominant solution for the project developer, the government should try to settle the negotiation at G, the lower bound of the renegotiation offer zone. Policy implications may include: Government could regulate the negotiated subsidy by the laws that explicitly forbid a subsidy being greater than G. According to this policy, government should spend more effort on determining G objectively and conveying such information to the project developer. For example, G can be assessed through the survey toward major bankers in the market on the least required retendering subsidy for a particular project. Therefore government is suggested to build an objective and transparent standard procedure for determining G.

Governing Principle 5: Government Should Make Every Effort on Having More Information for Determining a Fair J That Corresponds to the Developer's Responsibilities and Allocated Risks Specified in the Contract

Holliday et al. (1991) argued that because of the scale and complexity of BOT projects, such as the Channel Tunnel, very often they are *developer-led*, and it is extremely difficult to identify a clear client-contractor relationship at the heart of the project. The "developer-led" phenomenon implies the information asymmetry problem and opportunism problem in PPP projects, where the developer may be more capable of hiding information and may have incentives to behave opportunistically. Another issue is that J is fair only when the allocation of risks and responsibilities is appropriate. As Ho and Liu (2004) proved and Rubin et al. (1983) argued, a harsh contract will only encourage opportunistic behaviors. When the amount of J is brought to court or special committee, the court or committee will consider not only the contract clauses, but also the fairness of the contract. Policy implications may include:

- Government can separate the developer from the builder/contractor in a PPP project in order to have a clearer client-contractor relationship.
- Government can also assign third party experts to serve on the Board of the project company so that proper monitoring and inside information collection are assured.
- Government can have a procedure in forming a special committee consisting of outside experts to determine a fair J for the distressed project.
- Government should devote more efforts on appropriate risk allocation in the contract than on harsh contract clauses.
- Risk assignments between the concessionaire and government should be made explicitly in the agreement. This could help to determine a fair J in the future.
- Government is suggested to carefully specify when and how government can step in.
    1. Government could step in and temporarily take over a project when the project shows certain signs of potential distress according to the monitoring or warning system mentioned above.
    2. Whereas one of the major purposes of the government intervention is to improve the project status so as to prevent an actual distress, other major purposes here could be of information concerns. By temporarily taking over a project, government may have more information regarding how poorly the project has become, who should be responsible, how to minimize the impacts of an actual distress, and how much subsidy could be justified. As a result of stepping in, even if eventually the distress is inevitable, government will obtain more objective information regarding J and G, and will reduce τ due to longer

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lead time to respond and prepare for the retendering.
3. Nevertheless, government should not intervene too hastily or early, since the risk and responsibility may be partly transferred back to government if the step-in itself cannot be justified.
4. Thus the step-in decision should be cautiously made not only by government officials, but also by outside experts, following a standard procedure.

Conclusions

Too often, in PPP, many serious problems occur mainly because of bad administration policies, which are made by either non-engineers who do not know engineering or engineers who do not know management. When we, engineers, have chances to participate in policy making, we should base our decisions on solid economic ground as well as engineering discipline, instead of intuition based superficial reasoning. This paper investigates how government and project developers will behave in various renegotiation situations when a PPP project is in distress, and what impacts government rescue has on project procurement and management. The purpose of this paper is to model renegotiation and then derive policy implications for discouraging opportunistic behaviors in PPP. Although the advance in public project procurement practice has reduced the opportunities for opportunism, opportunism never ceases to exist in the mind of every rational and economic individual. As we have observed in many projects, the exploitation of renegotiation possibility in a complex contract or project, such as that in PPP, is a typical behavior of opportunism that poses many serious problems. The model is expected to help government authorities and policy makers establish more effective polices for PPP projects.

Note that, as in many economics studies, some simplified assumptions in the model setup and game parameter modeling are made in this research, so that useful insights can be drawn from real-life complex situations. These insights could provide decision makers with useful concepts and directional principles, despite that the real situation is more complex. The insights and qualitative implications of an economic model are often more important than the exact solutions obtained in the model. Furthermore, the proposed model can consider various project environments characterized by the parameters of the model. Also note that the validity of this model does not require government and the developer to explicitly "use" game theory; instead, the only requirement is that all players are rational decision makers.

Two propositions and seven corollaries are derived and proved rigorously in the proposed game model. Five rules are further deduced from these propositions and corollaries. The five rules conclude the conditions for possible equilibria and the characteristics of negotiated subsidy. Governing principles and policies inferred from the model for PPP administration are discussed. Based on these principles, important policies for project procurement and management are proposed. Due to the length limitation, detailed real-world case studies are left to a separate paper.

Lastly, although in practice there are some guidelines for various PPP schemes in countries such as the U.K., these guidelines cannot be universal to every country in the world. Guidelines and policies need to be modified to fit the specific environment of a country according to a certain logic. The proposed model may provide such logic for understanding problems and make appropriate modifications. For example, in our model, the possibility of renegotiation depends on the political and legal systems of a country, characterized by the political cost functions. Thus different countries may need different policies for preventing the problems caused by the expectation of financial renegotiation. Rigorous theories and principles regarding government policy in PPP are difficult to find. This pilot study, the writer hopes, may provide a theoretic foundation and analytic logic, from the renegotiation perspective, for prescribing effective PPP administration policies in different countries and for examining the quality of existing PPP guidelines and policies.

Acknowledgment

The work presented in this paper was supported by the National Science Council of TAIWAN, Grant No. NSC93-2211-E-002-058.

Appendix: Proofs of Propositions and Corollaries

Proof of Proposition 1

According to the game analysis associated with Eq. (2) and the equilibrium refining analysis associated with Eq. (12), the game equilibrium is to rescue and negotiate if $m(gU) \leqslant n(G+\tau)$, where $gU \geqslant G$. The renegotiation offer zone, according to the renegotiation offer analysis associated with Eq. (13), will be $gU \in \{x: G \leqslant x \leqslant m^{-1}[n(G+\tau)]\}$. (End of Proof)

Proof of Proposition 2

The proposition can be proved by showing that $\forall G \leqslant J$: $m^{-1}[n(G+\tau)] \geqslant G$ so that there must exist $gU \in \{x: G \leqslant x \leqslant m^{-1}[n(G+\tau)]\}$. By definitions of m, n, and β, we have $\forall G \leqslant J$: $n(G+\tau)-m(G)=\beta(G+\tau)-\beta(G) \geqslant 0$, i.e., $\forall G \leqslant J$: $n(G+\tau) \geqslant m(G)$. Then, since m is an increasing function, it must be true that $m^{-1}[n(G+\tau)] \geqslant m^{-1}[m(G)]=G$. Therefore, for all $G \leqslant J$, the renegotiation offer zone $gU \in \{x: G \leqslant x \leqslant m^{-1}[n(G+\tau)]\}$ must exist and the equilibrium must be to rescue. (End of Proof)

Proof of Corollary 1

$G \leqslant J-\tau$ implies $G \leqslant J$ as $\tau \geqslant 0$. According to Proposition 2, the equilibrium must be to rescue and negotiate. Furthermore, by definition of m and n in Eqs. (6) and (8), we have $\forall x \leqslant J$: $m(x)=\beta(x)=n(x)$, i.e., $\forall x \leqslant J$: $m^{-1}[n(x)]=x$. Therefore if we let $G+\tau=x$, since $G \leqslant J-\tau$ implies $G+\tau \leqslant J$, i.e., $x \leqslant J$, it must be true that the upper limit of renegotiation offer equals $m^{-1}[n(G+\tau)]=m^{-1}[n(x)]=x=G+\tau$, and that the renegotiation offer zone is $gU \in \{x: G \leqslant x \leqslant G+\tau\}$. (End of Proof)

Proof of Corollary 2

Given that the existence of S_α implies $\forall x \leqslant S_\alpha$: $m(x) \leqslant n(x+\tau)$, $G \leqslant S_\alpha$ implies the existence of gU, where $G \leqslant gU \leqslant S_\alpha$ and $m(gU) \leqslant n(gU+\tau)$. So, according to Proposition 1, $\forall G \leqslant S_\alpha$, the equilibrium must be to rescue, and there must exist $gU \geqslant G$ so that $gU \in \{x: G \leqslant x \leqslant m^{-1}[n(G+\tau)]\}$; i.e., the renegotiation offer zone is $gU \in \{x: G \leqslant x \leqslant m^{-1}[n(G+\tau)]\}$. (End of Proof)

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Proof of Corollary 3

The uniqueness of S_α implies that $\forall x > S_\alpha$: $m(x) > n(x+\tau)$. By Proposition 1, the "rescue" solution will imply $gU \geq G$. When $G > S_\alpha$, $gU \geq G$ implies $gU > S_\alpha$, which further implies that $m(gU) > n(gU+\tau)$. Therefore, according to Proposition 1, "rescue" cannot be the solution. So, the equilibrium must be to reject the subsidy and "no rescue." (End of Proof)

Proof of Corollary 4

According to parameter Characteristic 5, we have $G = -\mathrm{NPV}_t$. From Eq. (1), we have $U = \mathrm{NPV}_{0,t} - \mathrm{NPV}_t$. Given that $\mathrm{NPV}_{0,t} \geq 0$, the two equalities above imply $G \leq U$. Note that $\mathrm{NPV}_{0,t} \geq 0$ is a plausible assumption, since a rational developer will invest in a project only when its original estimated NPV is greater or equal to zero. Combing $J = U$ and $G \leq U$, we have $G \leq J$, and thus, by Proposition 2, the equilibrium must be to rescue.

Proof of Corollary 5

Let S_α^A and S_α^B be the unique S_α for ρ_s^A and ρ_s^B, respectively. If $\rho_s^A > \rho_s^B$, then $\rho_s^A(S_\alpha^A) > \rho_s^B(S_\alpha^A)$. By definition of m, we have $m^A(S_\alpha^A) > m^B(S_\alpha^A)$. By definition of S_α^A, we have $n(S_\alpha^A+\tau) = m^A(S_\alpha^A) > m^B(S_\alpha^A)$, i.e., $n(S_\alpha^A+\tau) > m^B(S_\alpha^A)$. By characteristic of S_α, $\forall J < x < S_\alpha^B$: $n(x+\tau) > m^B(x)$. Because S_α is unique, $n(x+\tau) > m^B(x)$ also implies that $J < x < S_\alpha^B$. Because $n(S_\alpha^A+\tau) > m^B(S_\alpha^A)$, it must be true that $J < S_\alpha^A < S_\alpha^B$; i.e., larger ρ_s function will yield a smaller S_α. (End of Proof)

Proof of Corollary 6

According to Corollary 5, if ρ_s is small such that S_α does not exist, it must be that S_α is so large that $S_\alpha \to \infty$. According to Corollary 2, when $S_\alpha \to \infty$, the equilibrium must be to rescue.

Proof of Corollary 7

When $\tau = 0$, by definitions of game parameters, we may obtain that $\forall G > J$: $m(G) - n(G+\tau) = \beta(G) + \rho_s(G) - \beta(G+\tau) = \beta(G) + \rho_s(G) - \beta(G) = \rho_s(G) > 0$. Further considering that $\tau = 0$, we obtain $m(G) - n(G+\tau) = \beta(G) + \rho_s(G) - \beta(G) = \rho_s(G) > 0$ for all $G > J$. As a result, according to Proposition I, "rescue" cannot be the solution. Thus the equilibrium must be to reject a subsidy and "no rescue" for all $G > J$.

When $G \leq J$, according to Proposition 2, the equilibrium must be to rescue and the upper limit of the rescuing offer is $m^{-1}[n(G+\tau)] = m^{-1}[n(G)]$ if $\tau = 0$. When $\tau = 0$, we have $\forall G \leq J$: $m(G) - n(G) = \beta(G) + \rho_s(G) - \beta(G) = \rho_s(G) = 0$, i.e., $m(G) = n(G)$. Therefore the upper limit of the renegotiation offer is $m^{-1}[n(G+\tau)] = m^{-1}[n(G)] = G$, which is the same as the lower limit of the offer. Thus, when $\tau = 0$, the equilibrium must be to rescue and negotiate at $gU = G$ for all $G \leq J$. (End of Proof)

Notation

The following symbols are used in this paper:

B = amount of budget overspending;
G = least required subsidy that can persuade the lending bank to support a distressed project;
g = rescuing subsidy ratio;
J = justifiable subsidy;
m = political cost function of rescuing a project;
m^{-1} = inverse function of m;
n = political cost function of retendering a project;
U = present value of net financial viability change, and also the maximum possible requested subsidy;
β = political cost function of budget overspending;
ρ_s = political cost function of oversubsidization; and
τ = developer replacing opportunity cost.

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