# Generalized Economic Modeling for Infrastructure and Capital Investment Projects

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**Abstract:** Economic modeling and risk analysis are important processes for the appraisal of infrastructure and revenue-generating projects such as build-operate-transfer (BOT) projects. These processes have been commonly implemented using spreadsheets in which the analyst would build several models to analyze a project under varying conditions and risk assumptions. For better efficiencies in building economic structures and evaluation of projects, the current paper defines "classifications" of estimating and cash flow methods, and develops a generalized model. A classification represents a particular domain—construction, revenues, financing, operation and maintenance, or risk analysis, for example—and holds the estimating methods of that domain. The basic building block behind the model structure is a work package/stream that would have its own properties and estimating methods by direct selection from the relevant classification. By integrating the building blocks together a project economic structure is built and various performance measures are formulated. The model was implemented in a prototype software system called Evaluator. A BOT highway project is used to show an application of the concepts and the generalized model.

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## Introduction

Economic modeling and risk analysis frameworks are generally developed for project appraisal. Spreadsheets are commonly used to build project cash flow models. Few specialized software systems were available to aid in building models; the most common in literature include Computer Aided Simulation for Project Appraisal and Review, or CASPAR (Willmer 1991; Thompson and Perry 1992), Computer Model for Feasibility Analysis and Reporting, or COMFAR III Expert (UNIDO 1994), and INFRISK (Dailami et al. 1999). While these systems have their relative strengths, their underlying economic models represent capital projects at a summarized level of detail that rely mainly on using time- and/or quantity-related costs/revenues. Challenged by the different types of projects, the complexity and number of estimating methods in a project life cycle and how they would be incorporated in a model structure, previous models, and spreadsheet analysis had to be at a summarized level. COMFAR and INFRISK would accept for each period of time the required quantities, prices, and/or loading percentage (e.g., 25% of total cost) so that a final cash flow would be obtained. CASPAR, with time- and cost-related charges, extends the process by using a network where the allocation of an estimate to a specific time would follow the dependencies of work activities. The current work extends the above developments and addresses the complexity of infrastructure by building a generalized model, the concepts of which are explained next, followed by an application on a BOT toll highway.

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## Generalized Model Structure and Concepts

In order to build a model that addresses the large number of variables, estimating, and cash flow methods, the first step was to differentiate between the phases in a project life cycle. The structure of the model, Fig. 1, divides a project life cycle into four cost/revenue components of four domains, namely: capital expenditure (CE), operation and maintenance (OM), revenue (RV), and financing (FN), plus one overall "project" component. Each component is assumed to have its domain's properties and methods. Properties include time and logic properties (e.g., time units, duration, dependencies) while methods are estimating methods (e.g., demand methods).

To account for the methods in a particular domain, "classifications" of methods were built: four cost/revenue classifications, "shape functions," "performance measures," and "risk analysis." The objective is to hold all pertinent methods in a classification such that direct selection from these methods would be available from within the model. As explained below, the methods of the above classifications are the common methods that would be used in a project. Other methods—for example, of specialized projects such as energy projects—would still be integrated without a necessity to restructure the model. That objective of generality was achieved by building the model as a hierarchical network-based continuous function structure, using the principles of continuous

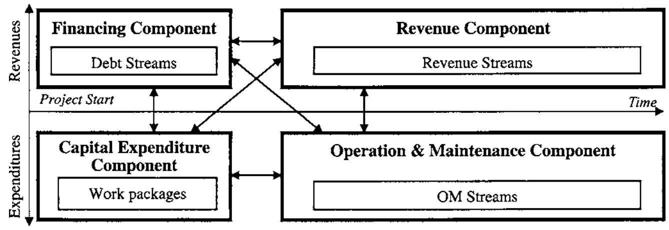


**Fig. 1.** Generalized model: interrelated components and building blocks

modeling (Remer et al. 1984; Tanchoco et al. 1981; Park and Sharp-Bette 1990).

The basic building blocks of the model are called work packages for the CE component and streams for the other components. A building block, Fig. 2, would have its own methods by direct selection from the relevant classification, where the method will be used to obtain an estimate for the building block. An estimate is then distributed over the work package/stream duration, that is, converted into cash flow, by: (1) allowing the variables of the estimating methods to change over time using "shape functions" (time functions, a subset of which is in Appendix I); or (2) using direct loading profiles (shape functions)." By integrating the building blocks together via the model network and continuous modeling, a project economic structure would be formed and cash flows and performance measures would be obtained. This is further explained below.

### *Capital Expenditure Component*

Through work packages, capital expenditures, such as construction costs, would be added to a project economic structure. At a summarized level of detail, the cash flow function is represented by

$$f_c^{CE}(t',\mathbf{x}) = X(t') \cdot e^{\int_0^{t''} \theta_X(\underline{t})\mathrm{d}\underline{t}} \tag{1}$$

where $f_c^{CE}(t',\mathbf{x})$=function name; $c$ defines a single work package; $t'$ and $\mathbf{x}$=arguments where $t'$=time in local time unit and $\mathbf{x}$=vector of variables; $X(t')$=capital expenditure variable representing a lump sum estimate that would be distributed over the work package duration using loading shape function profiles (Appendix I); $\theta_X(t'')$=inflation variable, modeled by shape function, and $t''$=application time referenced to the start of the project or work package. Using this structure, the cash flow could be represented with or without inflation. For example, by giving a value of zero to the inflation rate the inflation term will be equal to unity and the analysis will be in "constant dollar." Alternatively, any value given to the inflation rate will make the analysis in "real money." Governments as well as lending institutions for public–private partnership projects (e.g., BOT projects) tend to explicitly require the inclusion of inflation so that its effects would be considered in project evaluation.

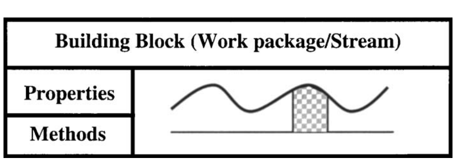


**Fig. 2.** Building blocks with properties and methods

**Table 1.** Highway Project Constant-Dollar Capital Expenditure and Operation and Maintenance

| | |
|---|---|
| Total capital expenditure | |
| Design | \$13 million in 8 months |
| Construction | |
| Road construction | 100% = \$84.75 million |
| Clearing and grubbing | 15%, in 5 months, with 4 months' overlap |
| C&F, rock blasting, compaction | 45%, in 6 months, with 2 months' overlap |
| Road Subbase layer | 15%, in 7 months, with 1 month overlap |
| Road base layer | 15%, in 7 months, with 6 months' overlap |
| Road asphalt pavement | 10%, in 7 months, with 3 months' overlap |
| Road structures | 100% = \$15.25 million |
| Culverts | 13%, in 4 months, with 5 months' overlap |
| Tunnels | 18%, in 4 months, with 5 months' overlap |
| Interchanges | 49%, in 8 months, with 2 months' overlap |
| Bridges | 20%, in 8 months, with 2 months' overlap |
| Operation and maintenance | |
| Maintenance costs | \$0.3 million/yr, and ¢6.5 per vehicle |
| Major maintenance | \$10 million each 10 years after construction |
| Toll operation | \$2.0 million/yr |

At a detailed level of analysis, the cash flow function of a work package is represented by

$$f_c^{CE}(t',\mathbf{x}) = f_{cm}(t',\mathbf{x}) \cdot e^{\int_0^{t''}\theta_m(\underline{t})\mathrm{d}\underline{t}} + f_{cl}(t',\mathbf{x}) \cdot e^{\int_0^{t''}\theta_l(\underline{t})\mathrm{d}\underline{t}} + f_{ce}(t',\mathbf{x}) \cdot e^{\int_0^{t''}\theta_e(\underline{t})\mathrm{d}\underline{t}} + f_{cs}(t',\mathbf{x}) \cdot e^{\int_0^{t''}\theta_s(\underline{t})\mathrm{d}\underline{t}} \tag{2}$$

In this function the cash flow is derived based on estimating methods commonly used in construction material, labor, equipment, and subcontract cost; $\theta_m(t)$, $\theta_l(t)$, $\theta_e(t)$, and $\theta_s(t)$=respective inflation variables. Material cost $f_{cm}(t',\mathbf{x})$ is estimated by three methods in Eq. (3) below: the first has $M(t')$ as a lump sum estimate, while the second and third methods are unit cost methods that have $C_m(t'')$ as unit cost (e.g., \$/m$^3$), $Q(t')$ as quantity (e.g., m$^3$, m$^3$/day), $P_l(t')$ as labor productivity (e.g., m$^3$/mhr), and $U_l(t')$ as labor usage (e.g., mhrs, mhrs/day):

$$f_{cm}(t',\mathbf{x}) = \begin{cases} M(t') & (3a) \\ C_m(t'') \cdot Q(t') & (3b) \\ C_m(t'') \cdot P_l(t') \cdot U_l(t') & (3c) \end{cases}$$

Labor cost $f_{cl}(t',\mathbf{x})$ and equipment cost $f_{ce}(t',\mathbf{x})$ use similar methods, shown in Eq. (4), for the labor cost, including: lump sum estimate $H_l(t')$; "Cost per unit of time," where $W_l(t'')$=wage rate and $U_l(t')$=labor usage (e.g., mhrs); and "Cost per unit of production," where $C_l(t'')$=unit cost

$$f_{cl}(t',\mathbf{x}) = \begin{cases} H_l(t') & (4a) \\ W_l(t'') \cdot U_l(t') & (4b) \\ W_l(t'') \cdot Q(t')/P_l(t') & (4c) \\ C_l(t'') \cdot Q(t') & (4d) \\ C_l(t'') \cdot P_l(t') \cdot U_l(t') & (4e) \end{cases}$$

The subcontracted estimate, $f_{cs}(t',\mathbf{x})$, has $S(t')$ representing lump sum estimates

$$f_{cs}(t',\mathbf{x}) = S(t') \tag{5}$$

Discrete costs are also included; in Eq. (6), $\mathbf{D}_j^{CE}$ represents discrete cost $j$ of a work package, where $\mathbf{Dv}_j^{CE}$ and $\mathbf{Dt}_j^{CE}$=value



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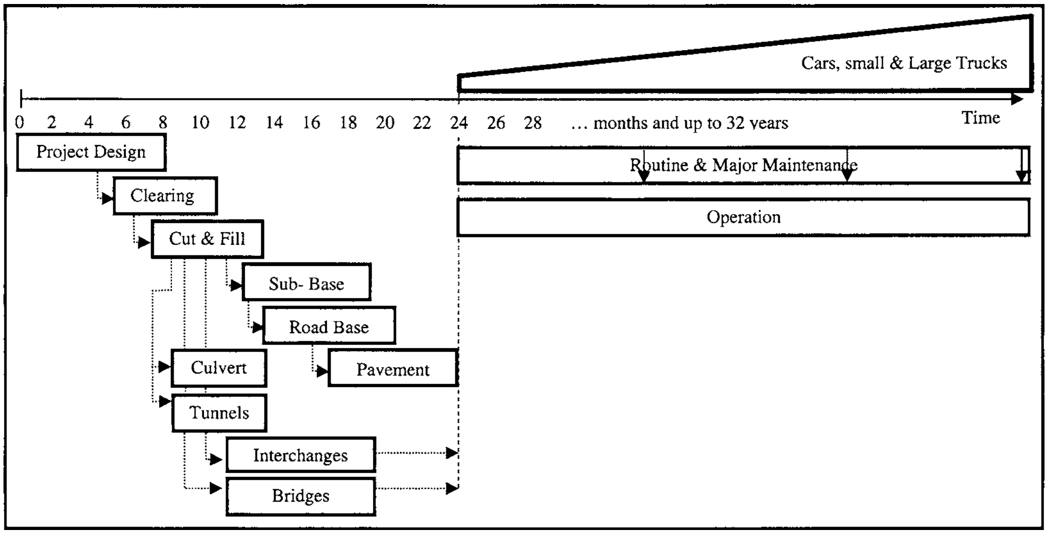


**Fig. 3.** Semidetailed economic structure for the highway project

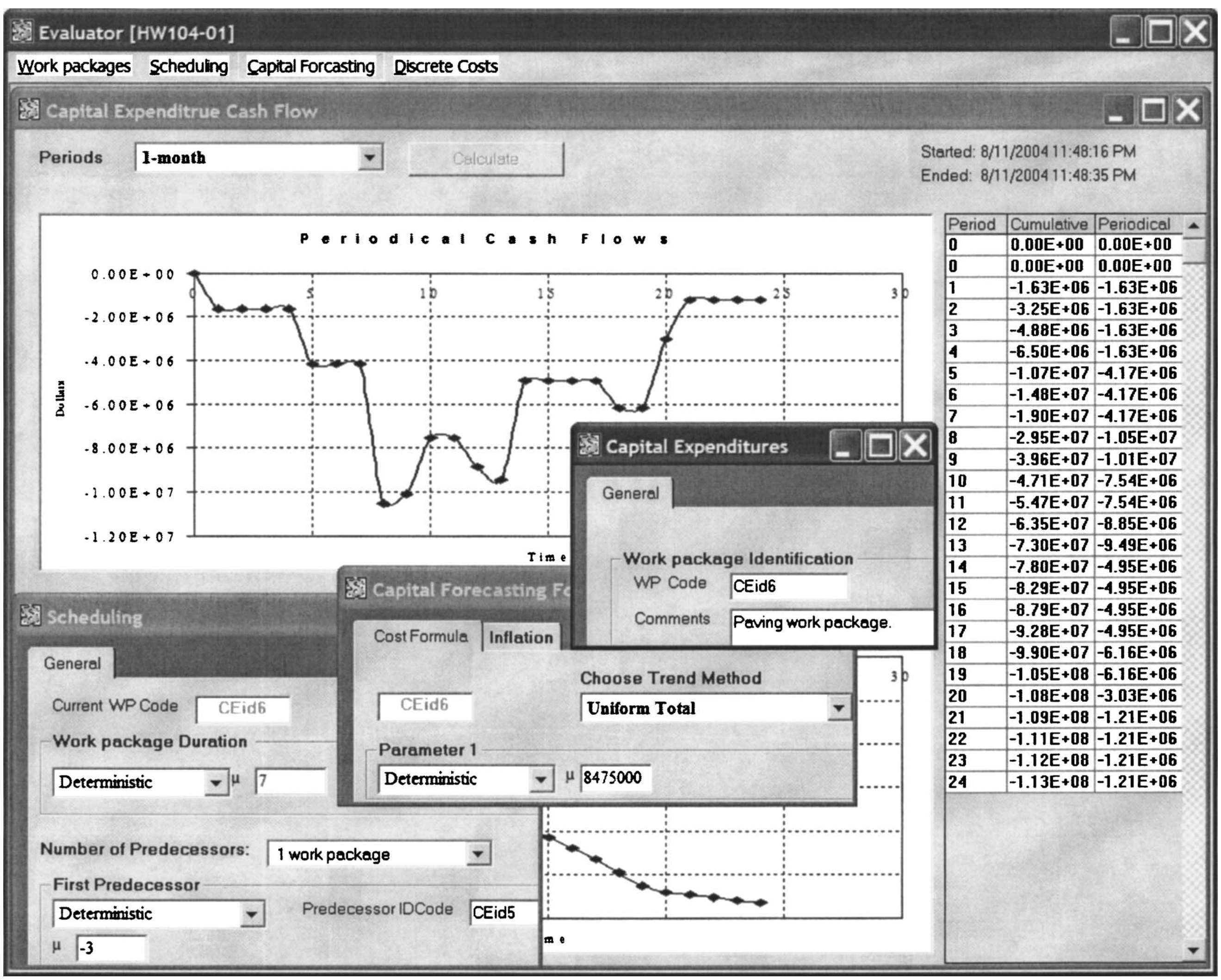


**Fig. 4.** Asphalt paving work package and capital expenditure cash flow, in constant dollars


**Fig. 5.** Detailed estimate of an asphalt paving work package and its cash flow

and time of the discrete cost, and $\theta_d(t)$=inflation variable

$$\mathbf{D}_j^{CE} = \mathbf{Dv}_j^{CE} \cdot e^{\int_0^{t^*} \theta_d(\underline{t})\mathrm{d}\underline{t}} \left| \begin{array}{ll} t^* = \mathbf{Dt}_j^{CE} & \text{if } \theta_d \text{ is in local time} \\ t^* = \mathbf{Dt}_j^{CE} + ESc & \text{if } \theta_d \text{ is in global time} \end{array} \right. \tag{6}$$

The versatility of the work package cash flow function provides flexibility in formulating other cash flow functions. In Eq. (7), $F_c^{CE}(t',\mathbf{x})$ provides the cumulative cash flow of a work package at any local time $t'$. It is in two parts: the first is an integration of the cash flow function [Eq. (1) or (2)] and the second is the summation of $r$ discrete costs of a work package [Eq. (6)]

$$F_c^{CE}(t',\mathbf{x}) = \int_0^{t'} f_c^{CE}(\underline{t},\mathbf{x})\mathrm{d}\underline{t} + \sum_{j=1}^{r} \mathbf{D}_j^{CE}\big|_{\text{for each } \mathbf{Dt}_j^{CE} \leqslant t'} \tag{7}$$

Eq. (7) can be used to derive a cumulative cash flow for the *CE* component, as given in Eq. (8) for $m$ work packages. With work packages having local time units different from that of the project, $\overline{t-ESc_i}$=time conversion in which $ESc_i$=early start of work package $I$ (see Appendix II and its example, and Fig. 15)

$$F_{CE}(t,\mathbf{X}) = \sum_{i=1}^{m} F_c^{CE}(\overline{t-ESc_i},\mathbf{x}_i)_{i|\text{for each } t \geqslant ESc_i} \tag{8}$$

Further, the work package cash flow can be used to determine the discounted cash flow to any reference time $Td$. In Eq. (9), $df_c^{CE}(Td,\mathbf{x})$=discounted work package function where $Td$ and $\mathbf{x}$ are arguments. Where $y$ is a nominal annual discount rate, the first integration part inside the brackets discounts the cash flow [Eq. (1) or (2)] to the start of its work package using its duration $wd$, time elapsed inside the work package $\overline{tn}$ and the converted-to-local-time-units discount rate $\bar{y}$. Similarly, the second part discounts discrete costs $r$. The outside term takes the discounting to the reference time $Td$ using the time elapsed before start of a work package, $\overline{tb}$

$$df_c^{CE}(Td,\mathbf{x}) = e^{-y\cdot\overline{tb}} \cdot \left[ \int_0^{wd-\overline{tn}} f_c^{CE}(\underline{t}+\overline{tn},\mathbf{x}) \cdot e^{-\bar{y}\cdot\underline{t}}\mathrm{d}\underline{t} + \sum_{j=1}^{r} \mathbf{D}_j^{CE} \cdot e^{-\bar{y}\cdot(\mathbf{Dt}_j^{CE}-\overline{tn})}\big|_{\text{for each } \mathbf{Dt}_j^{CE} \geqslant \overline{tn}} \right] \tag{9}$$

in which

$$\overline{tb} = \begin{cases} Esc - Td & \text{if } Td < ESc \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

and

$$\overline{tn} = \begin{cases} Td - Esc & \text{if } Td \geqslant Esc \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

Finally, Eq. (9) is used to derive the discounted cost of the whole *CE* component

**Table 2.** AADT on the Highway Example Project

| Year | Time | AADT |
|---|---|---|
| 1978 | 0 | 2,880.0 |
| 1979 | 1 | 3,020.0 |
| 1980 | 2 | 3,330.0 |
| 1981 | 3 | 3,510.0 |
| 1982 | 4 | 3,800.0 |
| 1983 | 5 | 3,990.0 |
| 1984 | 6 | 3,820.0 |
| 1985 | 7 | 4,190.0 |
| 1986 | 8 | 4,380.0 |
| 1987 | 9 | 4,530.0 |
| 1988 | 10 | 4,620.0 |
| 1989 | 11 | 4,500.0 |
| 1990 | 12 | 4,450.0 |
| 1991 | 13 | 4,890.0 |
| 1992 | 14 | 4,720.0 |
| 1993 | 15 | 5,100.0 |
| 1994 | 16 | 5,410.0 |
| 1995 | 17 | 5,630.0 |
| 1996 | 18 | 6,030.0 |
| 1995 | 19 | 6,400.0 |
| 1996 | 20 | 6,530.0 |
| 1997 | 21 | 6,520.0 |
| 1998 | 22 | 6,600.0 |
| 1999 | 23 | 6,750.0 |
| 2000 | 24 | 6,900.0 |

Note: AADT=Average Annual Daily Traffic

$$df_{CE}(Td,\mathbf{X}) = \sum_{i=1}^{m} df_c^{CE}(Td,\mathbf{x}_i)_i \tag{12}$$

The other components have similar formulations with the RV/OM designations used instead. Unlike previous models, the methods became part of the model, their variables are changing over time (Appendix I), and each estimate and work package has its own inflation variables.

### *Revenue Component*

At a summarized level of detail, lump sum revenues $RV(t')$ would be distributed over a revenue stream duration using shape functions where $\eta$=scope parameter and $\theta_{rv}(t)$=revenue inflation variable modeled by shape functions. A revenue stream cash flow function is expressed as

$$f_c^{RV}(t',\mathbf{x}) = \eta \cdot RV(t') \cdot e^{\int_0^{t''}\theta_{rv}(\underline{t})\mathrm{d}\underline{t}} \tag{13}$$

In a detailed analysis, the revenue stream cash flow function is derived as the product of demand $f_{cD}^{RV}(t'',\mathbf{x})$ and service rates $R(t'',d)$, (both expressed as functions)

$$f_c^{RV}(t',\mathbf{x}) = R(t'',d) \cdot f_{cD}^{RV}(t'',\mathbf{x}) \cdot e^{\int_0^{t''}\theta_{rv}(\underline{t})\mathrm{d}\underline{t}} \tag{14}$$

The service rate $R(t'',d)$ would change over time using shape functions. It could also change with the value of demand $d$ as obtained by its stream demand function. The RV classification have several methods for measuring demand for a project: (1) summarized, using trend methods [Eq. (15*a*); and (2) detailed methods, using project market share [Eqs.(15*b*)–(15*e*)], which is a subset of the total demand volume, $TV$. Where $\eta$ is scope parameter, the demand function is

$$f_{cD}^{RV}(t'',\mathbf{x}) = \begin{cases} \eta \cdot TV(t'',\mathbf{x}) & (15a) \\ \eta \cdot TV(t'',\mathbf{x}) \cdot (R(t'',d)/Ro)^{\varepsilon} & (15b) \\ \eta \cdot TV(t'',\mathbf{x}) \cdot [1+\varepsilon \cdot (R(t'',d)-Ro)/Ro] & (15c) \\ \eta \cdot TV(t'',\mathbf{x}) \cdot e^{-\varepsilon \cdot (R(t'',d)-Ro)} & (15d) \\ \eta \cdot TV(t'',\mathbf{x}) \cdot [1-1/(1+e^{Ut(t'')})] & (15e) \end{cases}$$

where

$$Ut(t'') = a_0 + a_1 \cdot (R(t'',d)-Ro) + a_2 \cdot LOS_2 + a_3 \cdot LOS_3 + a_4 \cdot LOS_4 \tag{16}$$

Methods used in Eqs. (15*b*), (15*c*), and (15*d*) are elasticity-based methods (Meyer and Miller 1984), which obtain future demand based on the elasticity $\varepsilon$ of service rate when it changes from its base value $Ro$. Eq. (15*e*) determines demand based on the utility of individuals, $Ut(t'')$, as they choose among a set of alternatives different in the level of service; $a_0$ to $a_4$=coefficients

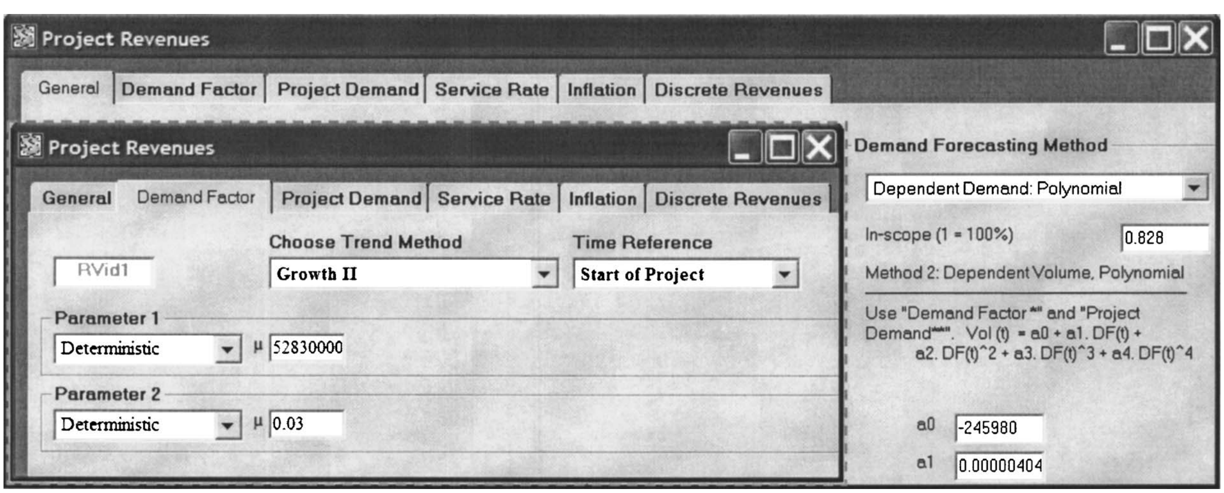


**Fig. 6.** Detailed estimate of cars total volume of demand [see Eq. (17*b*)]



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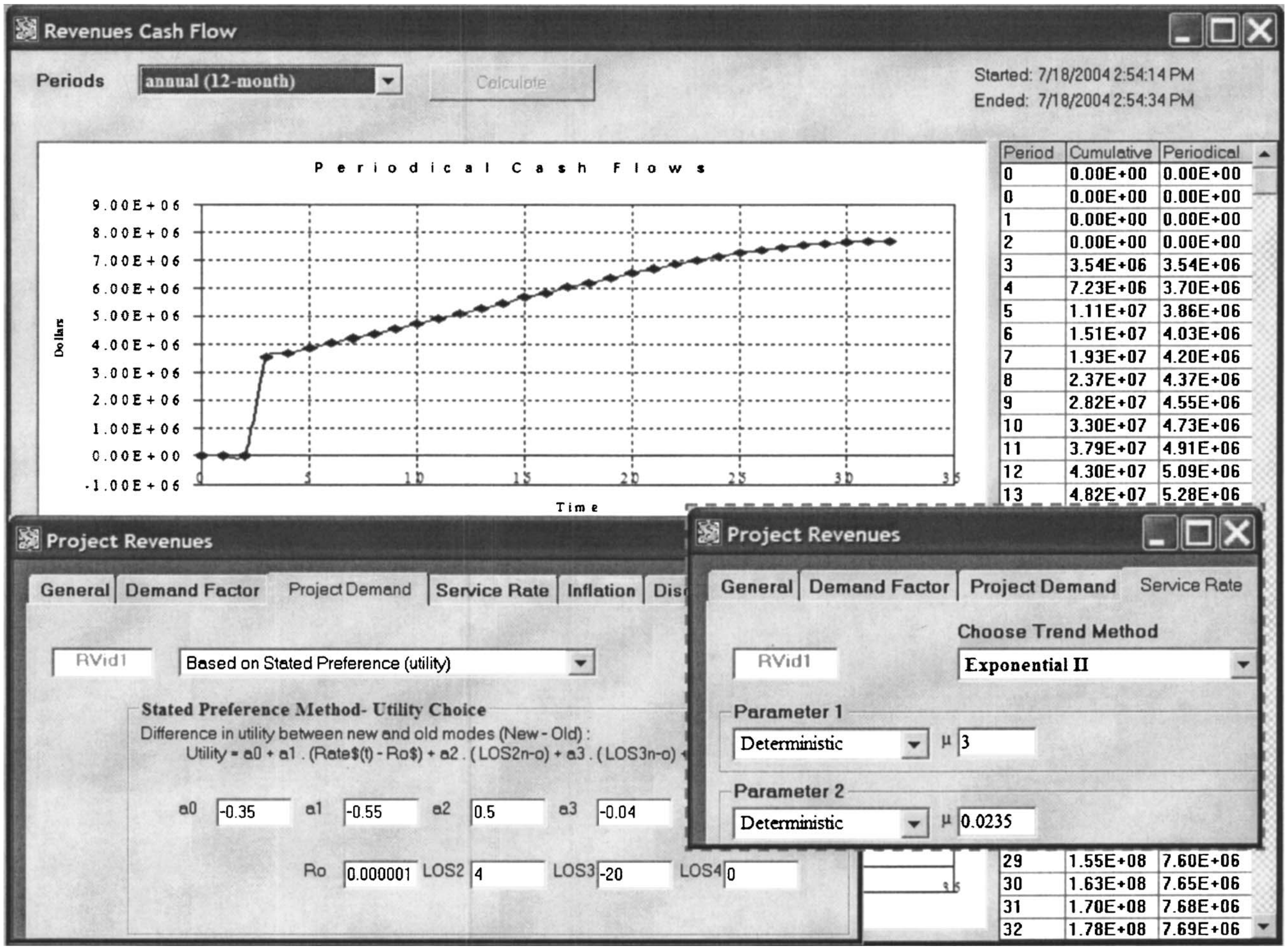


**Fig. 7.** Cars' demand volume based on stated preference technique [see Eqs. (15$e$), (16)]

and $LOS$=difference in values of the level of service/utility (e.g., travel time, toll rate). The method reflects a method of the stated preference techniques (Ortuzar and Willumsen 1994; Pearmain and Swanson 1990). With the model structure and the concept of classification, other demand methods could be added to the RV classification, such as methods of the energy, power, and petrochemical projects.

The total demand used in obtaining project market share could itself be estimated by using several methods, a common subset of which includes

$$TV(t'') = \begin{cases} V(t'') & (17a) \\ b_0 + b_1 \cdot G(t'') + b_2 \cdot G(t'')^2 + b_3 \cdot G(t'')^3 + b_4 \cdot G(t'')^4 & (17b) \\ b_0 + b_1 \cdot \ln(G(t'') + 1) & (17c) \\ b_0 \cdot e^{b_1 \cdot G(t'')} & (17d) \end{cases}$$

in which $b_0$–$b_4$=parameters. Eq. (17$a$) is a summarized general trend method $V(t'')$ (Meyer and Miller 1984). The other polynomial, logarithmic, and exponential dependent-trend methods estimate demand as it depends on demographic/socioeconomic indicators, $G(t'')$ (e.g., population).

### Operation and Maintenance Component

Eq. (18) represents the cash flow of OM stream in which estimates for OM are made through the expenditure function $f_{co}(t'', \mathbf{x})$ and converted to real terms using $\theta_{om}(t)$. The methods include summarized method [Eq. (19$a$)] and detailed methods [Eqs. (19$b$), (19$c$)] linked to the demand volume [Eq. (15)]—that is, fixed and variable OM cost; $A(t')$ and $B(t')$=parameter variables

$$f_c^{OM}(t'', \mathbf{x}) = f_{co}(t'', \mathbf{x}) \cdot e^{\int_0^{t''} \theta_{om}(t) \mathrm{d}t} \tag{18}$$

where

$$f_{co}(t'', \mathbf{x}) = \begin{cases} OM(t') & (19a) \\ A(t') \cdot f_{cD}^{RV}(t'', \mathbf{x}) & (19b) \\ A(t') + B(t') \cdot f_{cD}^{RV}(t'', \mathbf{x}) & (19c) \end{cases}$$

### Project Finance Component

Through debt streams financial instruments are added to a project economic structure. The FN classification includes several instruments such as term loans, private placement bonds, and floating rate notes (Brealey et al. 1992; Rhodes 1993; Gelbard 1996; and Ugeux 1981), along with several methods for drawdown, repayments, and floating and fixed interest rates. Discounted formulations for a debt stream and component are expressed as



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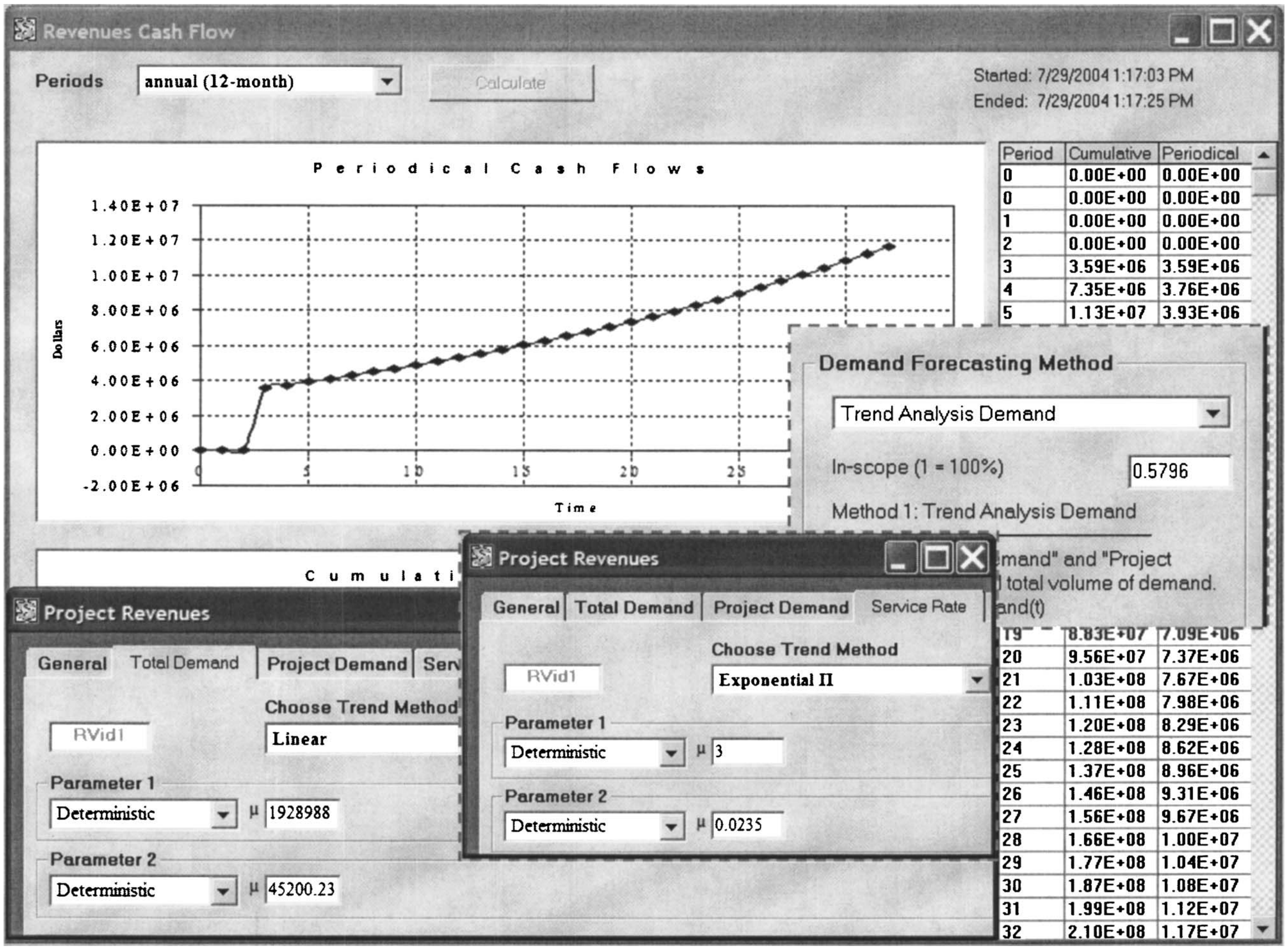


**Fig. 8.** Cars' revenue stream modeled by trend method [see Eq. (13)]

$$
\begin{aligned}
df_c^{FN}(Td,\mathbf{x}) = &\sum_{j=1}^{m} \mathbf{Tv}_j \cdot \; e^{-\bar{\bar{y}}\cdot(\mathbf{Tt}_j - Td)}\big|_{\mathbf{Tt}_j \geq Td} \\
&- \sum_{k=1}^{o} \mathbf{Rv}_k \cdot \; e^{-\bar{\bar{y}}\cdot(\mathbf{Rt}_k - Td)}\big|_{\mathbf{Rt}_k \geq Td} \\
&- \sum_{z=1}^{p} \mathbf{Iv}_z \cdot \; e^{-\bar{\bar{y}}\cdot(\mathbf{It}_z - Td)}\big|_{\mathbf{It}_z \geq Td} \\
&- \sum_{w=1}^{r} \mathbf{Fv}_w \cdot \; e^{-\bar{\bar{y}}\cdot(\mathbf{Ft}_w - Td)}\big|_{\mathbf{Ft}_w \geq Td}
\end{aligned}
\qquad (20)
$$

$$
df_{FN}(Td,X) = \sum_{i=1}^{n} df_c^{FN}(Td,x_i)_i \qquad (21)
$$

Eq. (20) has four parts each discounted to the reference time $Td$ through discount rate $\bar{\bar{y}}$:

1. Tranches: $\mathbf{Tt}_j$ and $\mathbf{Tv}_j$=time and value of each tranche $j$ of $m$ tranches in a stream;
2. Repayments: $\mathbf{Rt}_k$ and $\mathbf{Rv}_k$=time and value of each repayment $k$ of $o$ repayments;
3. Interest: $\mathbf{It}_z$ and $\mathbf{Iv}_z$=time and value of each interest payment $z$ of $p$ payments; and
4. Fees: $\mathbf{Ft}_w$ and $\mathbf{Fv}_w$=time and value of each fee payment $w$ of $r$ fees.

Projects are generally financed by debt and equity capital. A balance between capital expenditure and the required capital must be reached. This is achieved by fixing one of the capitals and deriving the other considering the interest that would be paid during construction.

### Project Component and Risk Analysis

With the network-based continuous model structure of the generalized model, the formulation of cash flows and performance measures becomes an aggregate process that integrates all the above formulations of work packages and streams of revenues, OM, and debts. For example, using the four discounted component cash flows, the net present value (NPV) is expressed as

$$
\begin{aligned}
NPV(Td) = &\, df_{RV}(Td,\mathbf{X}_{RV}) + df_{FN}(Td,\mathbf{X}_{FN}) - df_{CE}(Td,\mathbf{X}_{CE}) \\
&- df_{OM}(Td,\mathbf{X}_{OM})
\end{aligned}
\qquad (22)
$$

Following similar processes, the performance measures classification includes project and component cash flows, NPV, internal rate of return, "aggregate" and "net" benefit-cost ratios, loan-life-cover ratio, and debt-service-cover ratio (Park and Sharp-Bette 1990).

To model the performance measures under uncertainty, spreadsheet models normally use add-in software to carry out risk analysis. CASPAR and INFRISK, as stand-alone packages, have limited risk quantification methods. The risk analysis framework



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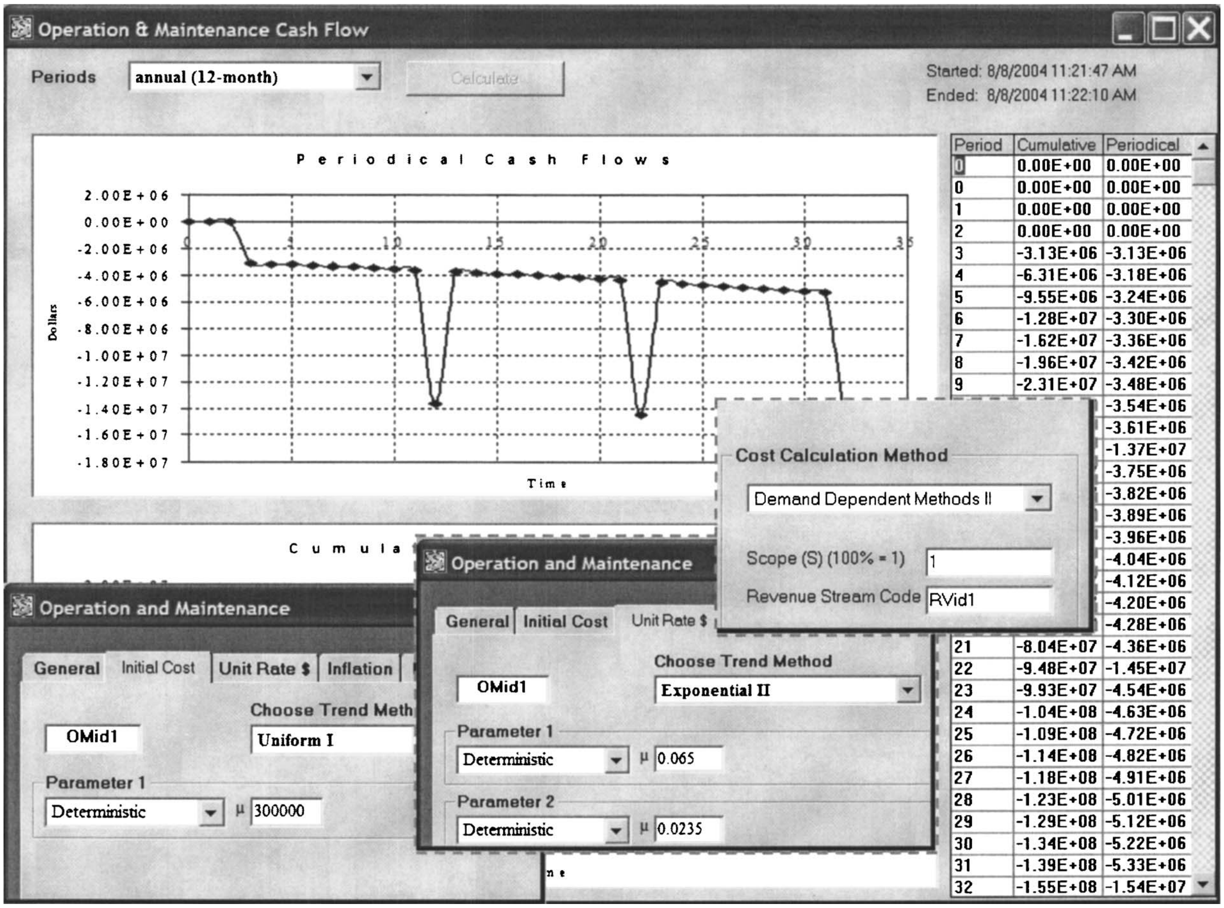


Fig. 9. Maintenance cost linked to highway demand [see Eq. (19*c*)]

of the generalized model reaches all the model variables, including the subvariables of the shape functions. Its "risk analysis" classification includes: (1) full probability distributions, including two- and four-parameter beta, two- and three-parameter log-normal, normal, exponential, two- and three-parameter gamma, Gumbel, triangular, chi squared, and uniform distributions (Bury 1999); (2) four statistical moments; and (3) three and five percentile values (Pearson and Tukey 1965; Keefer and Bodily 1983). For quantifying the performance measure's uncertainty, the statistical moments approach is used (Kottas and Lau 1982; Siddall 1972; Elderton and Johnson 1969; Hahn and Shapiro 1994).

## Example Project

The example project shows an application of the generalized model concepts in building and evaluating a highway developed under the BOT delivery system. A request for proposal (RFP) was issued for the development of 45-km four-lane highway under a 30-year concession. Table 1 and Fig. 3 give a summary of project information and a cash flow diagram, respectively. Starting with modeling the capital expenditure estimates in Table 1, the analyst has several options from using summarized to detailed estimating methods. For example, at a summarized level and where no details about the estimates are available, a single work package (WP) could be used to represent the $113 million total cost. In a semidetailed analysis, however, each line estimate in Table 1 would be represented by an individual work package. For example, Fig. 4 shows the asphalt paving modeled with its duration, dependency, and its lump-sum estimate distributed using the "uniform total" profile. This implements Eq. (1) of the model in which the capital expenditure variable $X$ takes the uniform total profile (Appendix I) where the increase/decrease of the WP duration would not change the total estimate of the WP. Note that the cumulative $113 million cash flow in the figure is derived by the model [Eq. (8)] and spread over the 24 months (this is also derived by the model network). The figure is a direct printout of Evaluator, the prototype software that implements the model.

In a detailed analysis, the asphalt paving would be estimated using the detailed methods of Eqs. (2), (3), and (4), of the model. This is shown in Fig. 5, where the asphalt quantity is defined at 169,750 t, a production rate at 24,250 t/month (970 t/day), an equipment rate of $51,970 per month, the labor wage at $65,920 per month, and a material unit cost of $45.065. The total cost of the WP would then be $8,475,000.00. Note that the quantity has a uniform total profile (Appendix I) so that the quantity consumption rate will change with the increase/decrease of the WP duration but will always be uniform, while the "uniform I" profile of the other rates signifies that the rate will not change. These profiles contributed to the derived uniform WP cash flow in the figure. Note that moving from one method to anther was made by direct selection; the analyst would not have to build a new model as would be the case in spreadsheet analysis. Note also that each

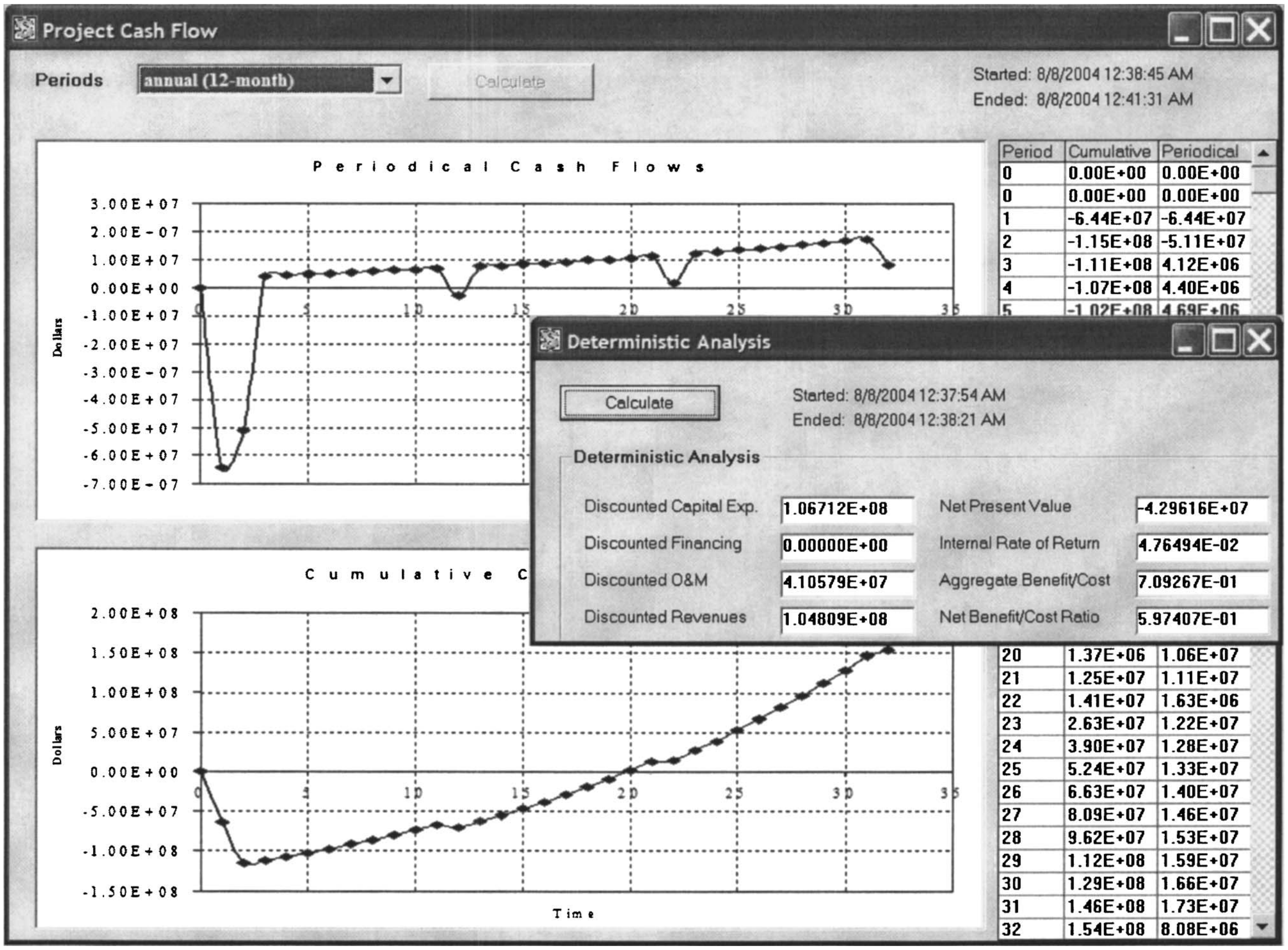


**Fig. 10.** Project cash flow and performance measures

variable in the estimate is defined as "deterministic" and it could have been a risk variable.

For RV modeling, three streams are used to model cars, small trucks, and large trucks with a 30-year duration linked to construction completion. Vehicle classes in the region indicate 75% cars, 6.6% small trucks, and 18.4% large trucks. Also, within the project scope, the car traffic is 82.8%, and truck traffic is 90%. In a detailed analysis, the "project demand" would be based on its market share. Starting at the "total demand" in the region, it is found upon regression analysis that the Average Annual Daily Traffic (AADT) (Table 2) is highly correlated to the gross domestic product (GDP) (in millions)

$$AADT(t) = -898.553 + 0.015 \cdot GDP(t) \tag{23}$$

Forecasting GDP, however, requires considerable analysis. For the project, and using historical data, it is found that future regional GDP is best modeled with 3% growth rate as follows

$$GDP(t) = 5.283 \cdot 10^{11} \cdot (1 + 0.03)^t \tag{24}$$

Using AADT, GDP, 75% cars traffic with 82.8% in-scope, the cars' annual demand volume is

$$\text{Cars Total Demand}(t) = (-2.4598 \cdot 10^5 + 4.0403 \cdot 10^{-6} \cdot GDP(t)) \cdot 0.828 \tag{25}$$

Fig. 6 shows the demand method selected with the parameters of the cars' total demand [Eq. (25)] and the demand factor, GDP, modeled as growth II (Appendix I) [implementing Eq. (17*b*)].

A project market share is the portion of that total demand that will use the new road. Using the "stated preference" (SP) technique (Ortuzar and Willumsen 1994) the propensity of drivers to use the new highway under various toll rates, number of lanes, and time savings is expressed as

$$\text{Utility} = a_0 + a_1 \cdot (\text{number of lanes}) + a_2 \cdot (\text{toll charge}) + a_3 \cdot (\text{time saving}) \tag{26}$$

In this utility function, $a_o$ to $a_3$=parameters derived from SP survey and analysis: $a_0$=−0.35, $a_1$=0.55 (for new toll rates), $a_2$=−0.5 (for using four lanes), and $a_3$=−0.04 (for 20 min time saving). Using total demand [Eq. (25)] and the utility function, the cars' annual demand volume is

$$\text{Cars' Demand} = \text{Cars' Total Demand} \cdot [1 - 1/(1 + e^{\text{Utility}})] \tag{27}$$

Fig. 7 shows the parameters of the utility function [see Eq. (15*e*)] and the derived cash flow. The other small and large truck streams could be defined similarly.

In early appraisal stages, however, projects would only have the AADT data and then only summarized analysis would be used. Hence, future demand would be derived using the general trend methods of Eq. (15*a*). For example, regression analysis for the AADT shows it could be fitted to a linear, second-, or third-order polynomial function; assuming the liner function as a trend method, then



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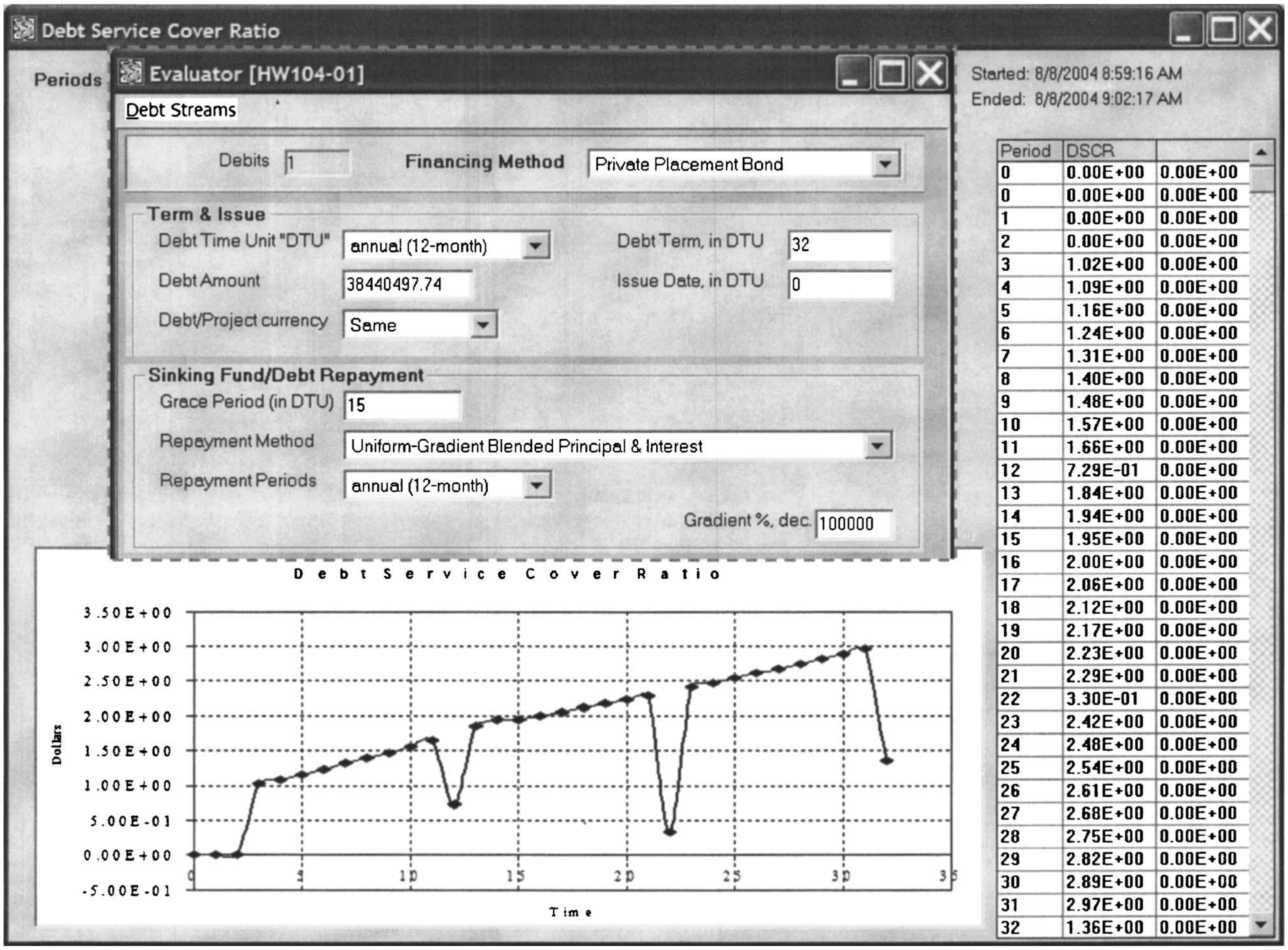


**Fig. 11.** DSCR and project performance measures

$$AADT(t) = 2918.615 + 165.115 \cdot t \tag{28}$$

Using the AADT function and the vehicle class and in-scope data, and assuming 70% traffic likely to use new road, the cars' annual demand volume and cash flow would be as in Fig. 8

$$\text{Cars' Demand}(t) = (1.928988\text{E}6 + 4.520023\text{E}4 \cdot t) \cdot 0.828 \cdot 0.7 \tag{29}$$

Comparing the cash flows of Figs. 7 and 8 shows the effect of increasing tolls in lowering the future demand where project market share was derived based on drivers' choices (i.e., better than trend method). Note that the summarized and detailed methods were available by direct selection; the analyst would not have to build a new model as would be the case in spreadsheet analysis. Note also that even the subvariables of the revenue stream—for example, the toll initial and exponential growth rate—were defined as "deterministic" and could have been defined as risk variables.

For modeling the OM costs in Table 1, two OM streams would be defined, each of 30 years' duration and linked to construction completion. One stream for an annual $2 million operating cost was modeled as "uniform I" profile. The other stream for maintenance is estimated to have fixed and variable cost sums. It is expected that $300,000 per year for annual maintenance will be needed, along with a maintenance that varies with the demand at $0.065 per vehicle and indexed to inflation (2.35%). This is modeled in Fig. 9, which implements Eqs. (18) and (19*c*) of the model in which $A(t)$ is the $300,000 fixed sum per year and $B(t)$ is the variable sum (exponential II), which is linked to the demand function [Eq. (29)]. In the figure, the valleys in the derived cash flow reflect the $10 million discrete overhaul costs of Table 1.

For FN modeling of the project, the analyst would experiment with the model to decide which financial structure is best under

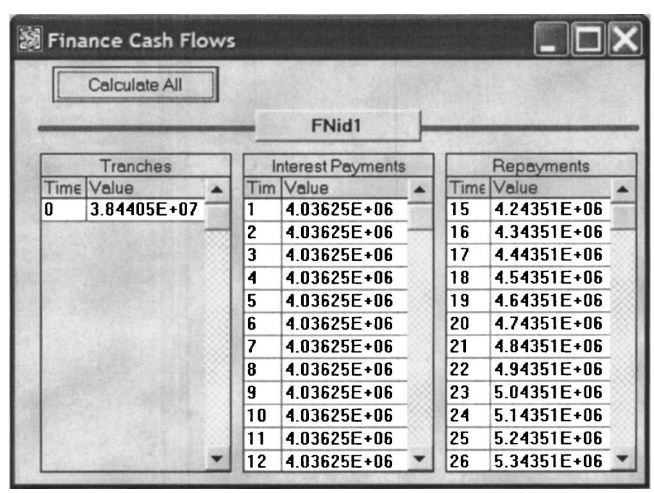


**Fig. 12.** Debt interest and repayments



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**Table 3.** Modeling the Uncertainty of Project Variables

| | |
|---|---|
| Capital expenditure | |
| Cost inflation | Design duration |
| Log N 3 $[\mu=0.026375;\ \mu_2=(0.0035)^2;\ \chi=\$0.0205]$ | Beta 4 $[\mu=8.067;\ \mu_2=(0.25)^2;\ \chi1=7.5;\ \chi2=9]$ |
| C&G cost | C&G duration |
| Log N 3 $[\mu=\$13.557E6;\ \mu_2=(1.50E6)^2;\ \chi=\$10.5E6]$ | Beta 4 $[\mu=5.33;\ \mu_2=(0.50)^2;\ \chi1=4.5;\ \chi2=7]$ |
| C&F cost | C&F duration |
| Log N 3 $[\mu=\$44.020E6;\ \mu_2=(9.00E6)^2;\ \chi=\$32.5E6]$ | Beta 4 $[\mu=6.44;\ \mu_2=(0.6)^2;\ \chi1=5.5;\ \chi2=9]$ |
| Subbase cost | Subbase duration |
| Log N 3 $[\mu=\$13.869E6;\ \mu_2=(2.00E6)^2;\ \chi=\$10.0E6]$ | Log Normal 3 $[\mu=7.12;\ \mu_2=(0.25)^2;\ \chi=6.5]$ |
| Base cost | Best duration |
| Log N 3 $[\mu=\$13.869E6;\ \mu_2=(2.00E6)^2;\ \chi=\$10.0E6]$ | Log Normal 3 $[\mu=7.12;\ \mu_2=(0.25)^2;\ \chi=6.5]$ |
| Asphalt pavement cost | Asphalt duration |
| Log N 3 $[\mu=\$09.122E6;\ \mu_2=(1.00E6)^2;\ \chi=\$7.75E6]$ | Log Normal 3 $[\mu=7.12;\ \mu_2=(0.25)^2;\ \chi=6.5]$ |
| Bridges cost | Bridges duration |
| Log N 3 $[\mu=\$03.345E6;\ \mu_2=(0.45E6)^2;\ \chi=\$2.8E6]$ | Triangle [7.5, 8, 12] |
| | |
| Operation and maintenance | |
| Major maintenance | Operations cost inflation |
| Triangle [$9.5E6, $10.0E6; $13.0E6] | Log N 3 $[\mu=0.026375;\ \mu_2=(0.0035)^2;\ \chi=\$2.0205]$ |
| | |
| Project Revenues | |
| Cars' initial traffic volume | Cars' annual growth |
| Beta 4 $[\mu=1.925E6;\ \mu_2=(3.65E5)^2;\ \chi1=9.13E5;\ \chi2=2.92E6]$ | Beta 4 $[\mu=4.57E4;\ \mu_2=(9.13E3)^2;\ \chi1=1.46E4;\ \chi2=8.21E4]$ |
| Small trucks' initial traffic volume | Small trucks' annual growth |
| Beta 4 $[\mu=1.70E5;\ \mu_2=(1.83E4)^2;\ \chi1=1.24E5;\ \chi2=2.21E5]$ | Beta 4 $[\mu=4.32E3;\ \mu_2=(730.00)^2;\ \chi1=2.92E3;\ \chi2=8.03E3]$ |
| Large trucks' initial traffic volume | Large trucks' annual growth |
| Beta 4 $[\mu=4.71E5;\ \mu_2=(3.10E4)^2;\chi1=\ 3.65E5;\ \chi2=5.65E5]$ | Beta 4 $[\mu=1.21E4;\ \mu_2=(2.19E3)^2;\ \chi1=8.03E4;\ \chi2=2.00E4]$ |
| Toll inflation | |
| Log N 3 $[\mu=0.026375;\ \mu_2=(0.0035)^2;\ \chi=\$0.0205]$ | |

Notes: $\mu$=expected value; $\mu_2$=variance, and $\chi$=location parameter (e.g., lower and/or upper limits)

the existing project conditions. Assume the basic case (Table 1) of 11 work packages (Fig. 4) modeled with a 2.35% inflation rate, three revenue streams (Fig. 8), and two OM streams (Fig. 9). The derived project cash flow, discounted values, and performance measures would be as shown in Fig. 10. Note that in year two the cumulative $115.48 million cost represents $113 million capital expenditure after adding 2.35% inflation. A closer look at the figure shows that by securitizing the future $63.75 million "net" discounted revenues (e.g., by issuing revenue bond) the proceeds would not be sufficient to build the project under its current scope (size), toll rates, and concession period. If the developers would provide the $115.48 million in equity, the investment would not be justified with a negative $42.96 million NPV (8.25% discount rate), a low 4.76% return on equity, and a 20-year payback period (Fig. 10).

From these unacceptable results and for the BOT project to be viable, government support might be required, which, in terms of the $65 million contribution, was offered in the first addendum. Therefore, another revenue stream was added for $65 million distributed using uniform total profile over two years' construction. Now, assuming full equity of $50.48 million, the project net present value became $16.95 million, return on equity was 10.68%, and the payback period became 13 years. While the results are reasonable, they might not be satisfactory to the

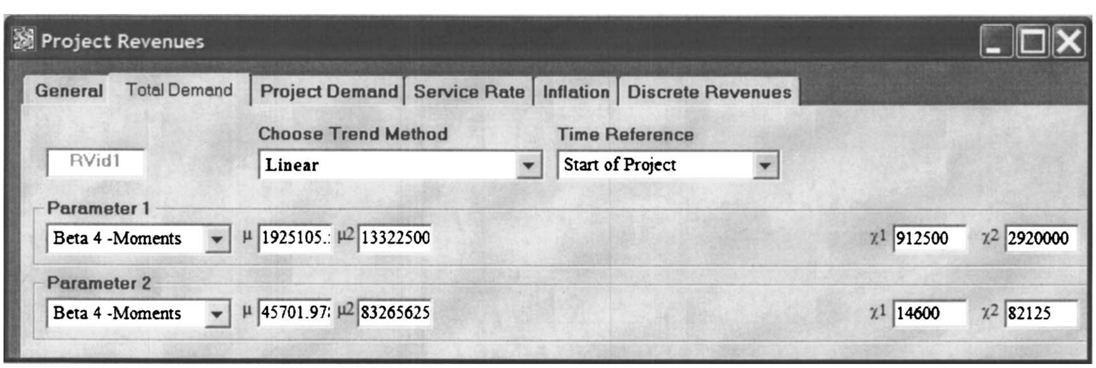


**Fig. 13.** Modeling cars' initial demand and growth rate (see Fig. 8)



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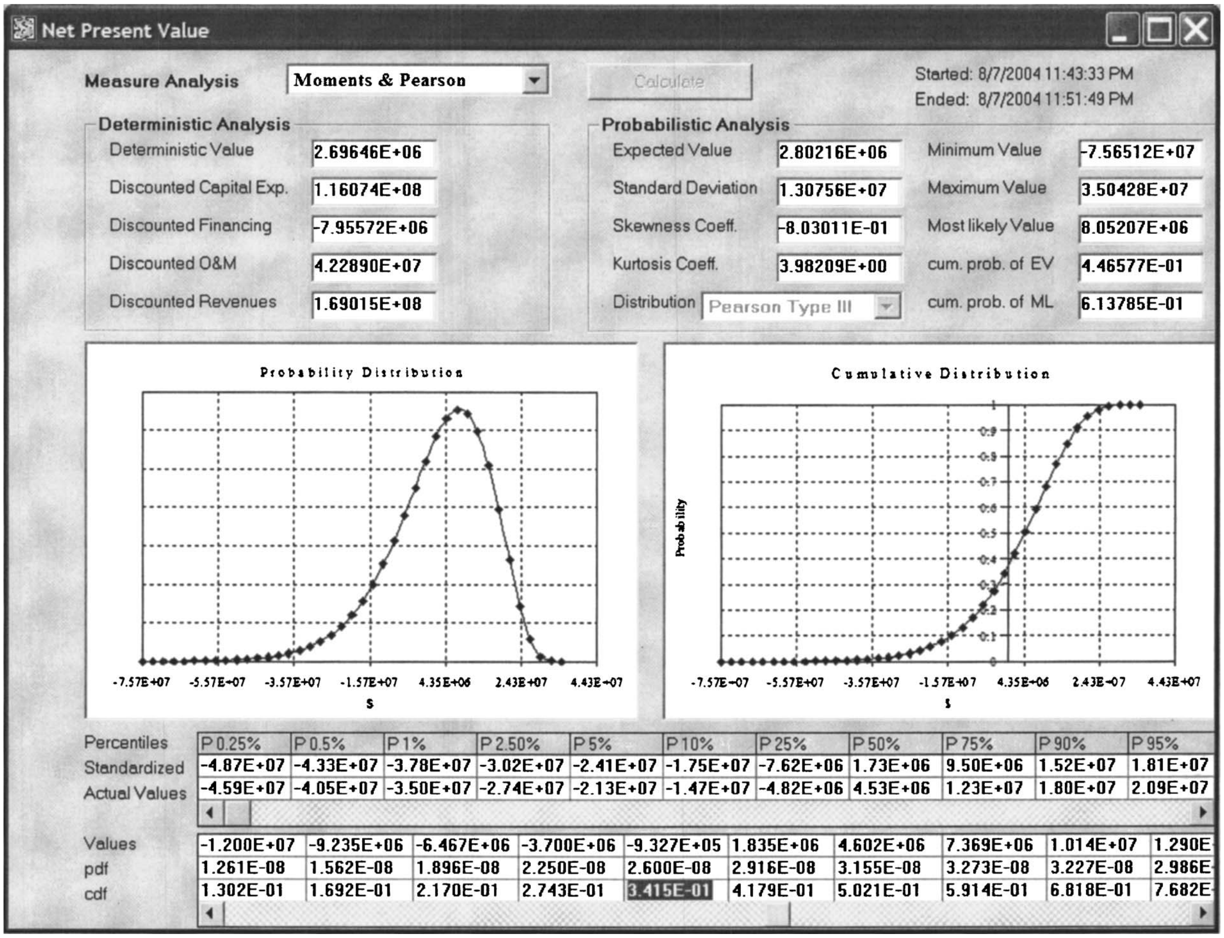


**Fig. 14.** Project NPV Probability distribution

developers, and as such they would investigate other financial structure—a 30:70 debt-to-equity structure.

With $16.5 million fixed equity (30%) and $65 million grants, $38.44 million debt (70%) would be needed, for a total of $119.94 million. The debt is structured to have a 10.5% interest rate, 1.1% fees, a 15-year grace period, a 32-year term, and blended principal and interest repayments that increase annually at $100,000. Such debt characteristics (Figs. 11 and 12), would be offered in the private placement market by institutional investors, such as pension funds. With $115.48 million expenditure, $0.42 million debt fees, and $4.04 million interest during construction, the total becomes $119.94 million, which is balanced by the above credits. With this financial structure, project performance is better with an 11.83% return on equity and $9.00 million NPV. For project developers, the performance still may not be attractive at the low NPV and the return that is only 3.58% higher than an 8.25% yield obtainable on risk-free securities of similar maturities to the project. Also, the debt-service-cover ratio (DSCR) (see Fig. 11), while higher than the 1.2 ratio generally required by lenders for BOT highways, the DSCR in years 12 and 22 will drain previous years' profits; government most probably will require a "major maintenance" fund.

Project developers were in doubt about how the project would behave under uncertainty and a risk analysis was required. Upon a risk analysis process, several risk variables were identified in Table 3 in terms of the probability model, expected value, variance, and limit parameters. For example, for the uncertainty around the cars' initial demand, a 4-parameter beta distribution (see Fig. 13) is used with a lower limit established at 912,500 cars/year, an upper limit at 2,920,000 cars/year, and an average value at 1,925,105 cars/year and variance $(365{,}000)^2$. Note the distribution will give the 1,928,988 cars/year the most likely value used in the deterministic analysis in Fig. 8. With all risk variables defined similarly, risk analysis could be done on any applicable performance measure. Fig. 14 shows the derived, negatively skewed NPV probability distribution and its $2.8 million expected value at a 44.65% probability. What is more troublesome, however, is that the NPV has 36% probability of failure (probability that NPV is less than zero). This high risk justified the developers' doubts and therefore they were dropping their equity and would only seek debt financing. Realizing this, the government decided developers would earn their profits only through construction contracts and that concession period would end once debt is retired.

The above analysis reflects one project scenario; other scenarios could also be established to reflect other toll schemes, demand methods, maintenance plans, and financial structures. Unlike previous models and spreadsheet analysis, however, there will be no need to build new models since the estimating methods are already integrated within the generalized model structure.



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## Summary

The work presented here explained some concepts used in building a generalized economic model for project evaluation and risk analysis. The model has a hierarchical network-based continuous model structure that integrates the properties and estimating methods of the common infrastructure project phases (domains): CE, OM, FN, and RV. The estimating methods are built within classifications (e.g., RV and FN classifications).

The basic building blocks of the model are called work packages and streams. Using the model, a project economic structure could be built by integrating the cash flows of the building blocks together via the model network and continuous modeling. Each building block can have its own time and logic properties, as well as its own estimating and cash flow methods by direct selection from the classifications. A cash flow for a building block estimate is derived through the profiles of the variables of the selected estimating methods, or by direct use of a loading profile.

Based on the above concepts, the structure of the generalized model achieves better efficiencies as it allows for building the economic structures of different types of projects using any number of building blocks, any selected summarized or detailed methods, and adding other methods to the classifications—without rebuilding the model structure.

A highway project example was used to show some applications for the generalized model. Conclusions from the example show the model could be used: (1) to build and evaluate several scenarios for infrastructure project development; (2) to define the approach to debt structuring for large infrastructure; (3) to evaluate whether government subsidies are needed and how large they should be; (4) to identify key risk variables so that they could be better managed; and (5) to forecast economic impact of large-scale projects.

## Appendix I. Shape Functions

| Pattern | Shape function: $f_s(t,y)$ | Remarks |
|---|---|---|
| Uniform I | $Qu$ | $Qu$=value per time unit |
| Uniform (total) | $Qt/b$ | $Qt$=total value |
| Linear | $Qs+Qr\cdot t$ | $Qs$, $Qr$=start value, rate |
| Linear (total) | $(2\cdot Qt-Qr\cdot b^2)/2\cdot b+Qr\cdot t$ | $Qt$, $Qr$=total value, rate |
| Exponential I | $Qs\cdot Qr^t$ | $Qs$, $Qr$=start value, rate |
| Exponential I (total) | $Qt\cdot \ln(Qr)\cdot Qr^t/(Qr^b-1)$ | $Qt$, $Qr$=total value, rate |
| Exponential II | $Qs\cdot e^{Qr\cdot t}$ | $Qs$, $Qr$=start value, rate |
| Exponential II (total) | $Qt\cdot Qr\cdot e^{Qr\cdot t}/(e^{Qr\cdot b}-1)$ | $Qt$, $Qr$=total value, rate |
| Exponential III | $Qs\cdot t\cdot e^{-Qr\cdot t}$ | $Qs$, $Qr$=magnifier, rate |
| Exponential III (total) | $\dfrac{-Qt\cdot Qr^2}{\exp(-b\cdot Qr)\cdot b\cdot Qr+\exp(-b\cdot Qr)-1}\cdot t\cdot \exp(-Qr\cdot t)$ | $Qt$=total value<br>$Qr$=growth rate |
| Logarithmic | $Qs+Qr\cdot \ln(t+1)$ | $Qs$, $Qr$=start value, rate |
| Logarithmic (total) | $Qt\cdot \ln(t+1)/[\ln(b+1)\cdot b-b+\ln(b+1)]$ | $Qt$=total value |
| Growth I | $Qs+Qa\cdot(1-e^{-Qr\cdot t})$ | $Qs$=start value<br>$Qa$, $Qr$=amplitude, rate |
| Growth I (total) | $\dfrac{Qt\cdot Qr}{b\cdot Qr+\exp(-b\cdot Qr)-1}\cdot[1-\exp(-Qr\cdot t)]$ | $Qt$=total value<br>$Qr$=growth rate |
| Growth II | $Qs\cdot(1+Qr)^t$ | $Qs$, $Qr$=start value, rate |
| Growth II (total) | $\dfrac{Qt\cdot \ln(1+Qr)}{\exp(b\cdot \ln(1+Qr))-1}\cdot(1+Qr)^t$ | $Qt$=total value<br>$Qr$=growth rate |
| Polynomial | $Q0+Q1\cdot t+Q2\cdot t^2+Q3\cdot t^3+Q4\cdot t^4$ | $Q0$ to $Q4$=parameters |
| Normal I | $Qs+(Qm-Qs)\cdot \exp\left[\dfrac{(t-\mu)^2}{\sigma^2}\right]$ | $Qm$=maximum value<br>$Qs$=start value<br>$\mu$=time at max. value<br>$\sigma$=shape factor |
| Normal I (total) | $\dfrac{Qb}{b}+(Qt-Qb)\cdot\left\{\dfrac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\cdot \exp\left[\dfrac{-1}{2}\cdot\left(\dfrac{t-\mu}{\sigma}\right)^2\right]\right\}$<br>where: $\mu:=0.5\cdot b$ and $\sigma:=0.5\cdot b/3.9$ | $Qt$=total value<br>$Qb$=total base value |
| Sinusoidal | $Qs+Qr\cdot t+Qa\cdot \sin\left(\dfrac{2\cdot\pi}{Qc}\cdot t\right)$ | $Qs$, $Qr$=start value, rate<br>$Qa$=amplitude, cycle length |

Note: Shape functions: (1) "rate" functions describe time-related functions; and (2) total functions describe rate functions with constrained total value; $b$ represents total duration.



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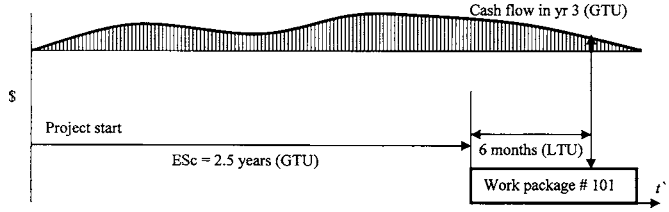


**Fig. 15.** Conversion of global to local time units

## Appendix II. Local and Global Time Units

To allow flexibility in modeling a project economic structure, the building blocks (work packages and streams) of the generalized model could have their own properties and estimating methods. Among these properties is the time unit, where a work package could have its time units in month units and a revenue stream could have its time units in year units. These time units of the building blocks are called local time units (LTU).

A global time unit (GTU) is a time unit that belongs to the project as a whole and could be used for discounted analysis and for calculating periodical and cumulative cash flows for a project component or for the whole project.

To derive a project cash flow in global time units, the local time units of the work packages and streams will need to be converted to the global time units [$\overline{t-ESc_i}$ as in Eq. (8)]. For example, as shown in Fig. 15, a construction work package has duration of nine months (i.e., the LTU is in month units), and it is required to calculate a project cumulative cash flow in the third year [$t=3$ years] (i.e., GTU is in year units).

With the work package having an early start *Esc* of 2.5 years, we need to include 0.5 years (GTU) of the cumulative cash flow of the work package. Since the work package is defined in month LTU, we need to convert the 0.5 years GTU into the work package LTU. The conversion factor is defined as

Global to local time conversion

$$(\text{GtoL}) = \text{GTU (in months)/LTU (in months)} \tag{30}$$

In this example, $\text{GtoL}=12/1=12$

The work package required time, $t'$, is then

$$t' = (t - Esc)^{*}\text{GtoL}; \tag{31}$$

$$\text{then } t' = (3 - 2.5)^{*}12 = 6(\text{months})$$

If the work package was defined in terms of quarter time units instead of month units, then

$$\text{GtoL} = 12/3 = 4$$

$$t' = (3 - 2.5)^{*}4 = 2(\text{quarters})$$

## Notation

*The following symbols are used in this paper:*

$A(t), B(t)$ = two parameter variables in the OM component;
$a_0$ to $a_4$ = coefficients of a utility function;
$BC$ = benefit-cost ratio, can be aggregated ($g$) or netted ($n$);
$b_0$ to $b_4$ = parameters of a polynomial regression function;
$CE, OM, RV, FN$ = subscript or superscript to the capital expenditure, operation and maintenance, revenue, and financing components, respectively;
$C_l(t'')$ = labor (equipment if subscripted to $e$) cost per unit of $Q(t')$ in a CE work package;
$C_m(t)$ = unit cost of material (e.g., $\$/\text{m}^3$) in a CE work package;
$\mathbf{D}$ = vector of current dollar discrete costs, can be superscripted to CE, RM, and OM denoting a specific component;
$\mathbf{Dv}, \mathbf{Dt}$ = vectors of constant dollar discrete costs ant times in a construct, can be superscripted to CE, RV, and OM denoting a component;
$Esc$ = early start of a construct;
$G(t)$ = variable representing demographic and socioeconomic trends/indicators;
$H_l(t)$ = gross labor (equipment if subscripted to $e$) cost per unit of time in a CE work package;
$LOS$ = vector of four attributes in a utility function;
$M(t)$ = gross material cost per unit of time in a CE work package;
$OM(t)$ = variable representing future recurring OM costs;
$P_l(t)$ = labor productivity in placing a unit of quantity (e.g., $\text{m}^3/\text{mhr}$) in a CE work package;
$Q(t)$ = work package scope/quantity placed per unit of time in a CE work package;
$R(t,d)$ = service charge variable (e.g., dollar tolls), $d$ is demand level at time $t$;
$Ro$ = a base service charge;
$RV(t)$ = constant dollar revenue variable;
$S(t)$ = subcontract/indirect cost per unit of time in a CE work package;
$\mathbf{Tt}, \mathbf{Rt}, \mathbf{It}, \mathbf{Ft}$ = time in GTU of single tranche, repayment, interest, and fee payment, respectively;
$\mathbf{Tv}, \mathbf{Rv}, \mathbf{Iv}, \mathbf{Fv}$ = dollar value of single tranche, repayment, interest, and fee payment, respectively;
$t$ = time defined in GTU and referenced to project start;
$\overline{t-ESc_i}$ = time elapsed in a construct and converted from GTU to LTU;
$t$ = dummy variable used in integration calculations;
$t'$ = time defined in LTU and referenced to construct start;
$t''$ = time defined in either of $t$ or $t'$;
$\overline{tb}$ = time before start of a construct, converted from GTU to annual time unit;
$Td$ = time defined in GTU and referenced to project start;
$\overline{tn}$ = time elapsed in a construct, converted from GTU to LTU;
$U_l(t)$ = labor usage/input per unit of time (e.g., mhrs/day) in a CE work package;
$W_l(t'')$ = labor (equipment if subscripted to $e$) cost per unit of time in a CE work package;
$wd$ = duration of a work package/stream in LTU;
$\mathbf{X}$ = matrix of variables made of all $\mathbf{x}$ vectors in a component, can be subscripted by CE, OM, RV, or FN, denoting the component it represents;

$X(t')$ = constant dollar capital expenditure variable in the CE component;

$\mathbf{x}$ = vector of variables, can be subscripted by CE, OM, RV, or FN, denoting the construct type to which it belongs;

$y$ = vector of variables representing a shape function, Appendix II;

$y$ = annual discount rate representing minimum acceptable rate of return (MARR);

$\bar{y}$ = $y$ converted from annual to LTU;

$\bar{\bar{y}}$ = $y$ converted from annual to GTU;

$\varepsilon$ = elasticity variable;

$\eta$ = scope variable;

$\theta_m(t), \theta_l(t), \theta_e(t), \theta_s(t), \theta_X(t), \theta_d(t)$ = inflation variables of the material, labor, equipment, subcontract, capital expenditure, and discrete costs in the CE component; and

$\theta_{rv}(t), \theta_{om}(t)$ = variables of revenues and OM costs respectively.

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