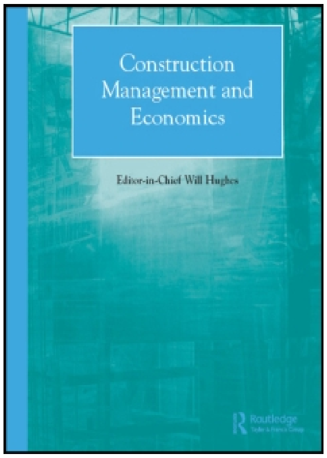

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An option pricing-based model for evaluating the financial viability of privatized infrastructure projects

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Privatized infrastructure projects have to demonstrate their financial and technical viability before they are undertaken. Although it is relatively easy to demonstrate the technical viability of an infrastructure project, the evaluation of the financial viability of a privatized infrastructure project is complex and challenging, mainly because of the uncertainties involved due to the project's scale, long concession period and complexity. Traditional methods, such as net present value (NPV) analysis, fall short in reflecting the characteristics of privatized infrastructure projects and the risks involved. This paper presents an option pricing based model, the BOT option valuation (BOT-OV) model, for evaluating the financial viability of a privatized infrastructure project. This quantitative model considers the project characteristics explicitly and evaluates the project from the perspectives of the project promoter and of the government when the project is under bankruptcy risk. Moreover, the model can evaluate the impact of the government guarantee and the developer negotiation option on the project financial viability.

Keywords: BOT, privatized infrastructure, option pricing theory, financial decision-making, investment evaluation

Introduction

Privatization has been recognized as an approach to solving the difficulty of governments' fiscal shortcomings in funding public works. According to a World Bank report by Roger (1999), from 1990 to 1998 private participation in infrastructure projects grew dramatically from about $16 billion in 1990 to $120 billion in 1997 and $95 billion in 1998. The major technique applied in infrastructure privatization is non-recourse project financing, and the BOT (build-operate-transfer) approach is one of the major non-recourse project financing schemes in practice. BOT could be a win–win solution if it is implemented successfully (Walker and Smith, 1995). A BOT project has to demonstrate its financial and technical viability before it is undertaken. However, although it is relatively easy to demonstrate the technical viability of a BOT project, to evaluate the financial viability is complex and challenging. One major reason is the vast uncertainty involved due to the BOT project's scale, long concession period and complexity.

Tiong and other researchers have conducted several surveys and case studies on BOT projects. Their efforts focused mainly on the BOT project's bidding strategies, such as the critical success factors in winning BOT contracts (Tiong *et al.*, 1992; Tiong, 1995a, 1996; Tiong and Alum, 1997), and the bidder's competitive advantages due to the equity level (Tiong, 1995b). Dias and Ioannou (1995a) studied the BOT project's optimal debt capacity and capital structures. However, the study on the valuation and financial viability measurement of a BOT project is very limited. Ye and Tiong (2000) presented the NPV-at-risk method to analyse the impact of the risks on the value of a BOT project. The basic concept of this method is to simulate the primary variables underlying the net present value of a project, and obtain the distribution and confidence level of the NPV. Myers (1976) pointed out the major limitations of the NPV-at-risk

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method: 'If NPV is calculated using an appropriate risk adjusted discount, any further adjustment for risk is double-counting. If a risk-free rate of interest is used instead, then one obtains a distribution of what the project's value would be tomorrow if all uncertainty about the project's cash flows were resolved between today and tomorrow. But since uncertainty is not resolved in this way, the meaning of the distribution is unclear.'

Trigeorgis (1996) pointed out the major drawbacks of other traditional quantitative capital budgeting techniques, such as payback period method, accounting rate of return (ARR), internal rate of return (IRR), net present value (NPV), and decision tree analysis. The NPV method is considered a superior method among the traditional techniques, since the NPV approach is consistent with the firm's objective of maximizing the shareholders' utilities (Copeland and Weston, 1988). However, when uncertainty is involved in the investment, the discount rate used in NPV calculation can be adjusted for the risks according to the capital asset pricing model (CAPM).[1] Trigeorgis (1996) argued that the basic inadequacy of the NPV method is that NPV ignores or cannot properly capture 'management's flexibility to adapt or revise later decisions when, as uncertainty is resolved, future events turn out differently from what management expected at the outset'. The managerial flexibility creates asymmetric payoffs that coincide with the payoff form of the stock call options as given in

$$F(S_T, T) = Max\,[0, S_T - X] \tag{1}$$

where F is the option value, S_T is the stock price at time T, and X is the exercise price. As a result, modern financial theory considers the option pricing framework as a major approach to pricing such complicated assets or investments that are uncertain and have asymmetric payoffs. In this paper, we examine why the evaluation of BOT investments can be improved by applying option pricing framework, and present an option pricing based model for evaluating the project financial viability.

BOT investment problems

The NPV method falls short in reflecting the characteristics of BOT projects. In this section, we first discuss the characteristics of BOT projects and how traditional evaluation methods fail to consider these characteristics, then we define the financial viability of the BOT project, and third we identify the analogies between BOT projects and financial options and show why the option pricing framework can improve the evaluation of the financial viability.

Characteristics of BOT projects

Risk characteristics

Risks in BOT projects have been studied and categorized by some researchers (Augenblick and Custer, 1990; Dias and Ioannou, 1995b; Walker and Smith, 1995). In general, the economic risks and the construction risks can be considered as the two major risk categories of a BOT project. The economic or operating risks are the major risks that would affect the future cash flows generated in the operation period. The construction or completion risks, which can be characterized by the construction cost uncertainty, include the risks of cost overrun, schedule delay, and technical difficulties. In the Channel Tunnel project, the construction costs were expected to be less risky due to its technical simplicity, but surprisingly the actual costs were doubled (Finnerty, 1996). Traditional capital budgeting techniques, such as the NPV method, do not take into account the uncertainty in the initial investment or construction cost.

Government guarantees and negotiations

An important BOT practice is that the government may grant loan guarantees to a BOT project when the project is not viable enough or is too risky to be undertaken by private parties. The non-viability will be reflected by the bank's unwillingness to provide loans without government guarantees. The debt guarantee is a liability to the government and an asset to the BOT firm. Therefore, it is essential for the developer, shareholder, and government to evaluate the value of the loan guarantee. For the developers, failing to consider the value of the debt guarantee will underestimate the investment value, and for the government, if the value of the debt guarantee is too large, the government over-subsidizes the BOT firm (Baldwin *et al.*, 1983).

Another important characteristic of BOT projects is that the BOT firm has options to negotiate for government rescue should adverse events occur during the construction. A successful negotiation can prevent a failing project from being bankrupted. As we shall argue later, the NPV method cannot price the value of either the debt guarantees or the negotiation option.

BOT project financing

One of the most essential characteristics of BOT projects is the financing scheme. BOT projects are financed by forming a leveraged BOT firm, which is independent from the project developers or promoters, whereas often other investment projects are financed within an existing firm. As a result, there are two major concerns derived from BOT project financing. First, the developer's investment returns are realized and measured through equity returns, instead of the value of the firm or project, since the developer finances the BOT project by holding a significant portion of the shares of the BOT firm.[2] Second, the BOT firm is subject to bankruptcy before project completion. Existing BOT evaluation methods, which are based on traditional capital budgeting tools, focus on evaluating the asset or project value, instead of the equity value, and therefore fail to measure the payoff of the BOT developer. Although CAPM based discounting or the NPV approach may be widely used for equity pricing, this approach cannot appropriately address various security terms and bankruptcy clauses or conditions that we shall encounter in BOT projects.

Evaluation criteria for financial viability

It is non-trivial to define the financial viability of a BOT project. Moreover, different participants have different perspectives concerning the financial viability of BOT investments.

The developer, from his perspective, needs to consider the equity of the BOT firm and other profit components of a BOT investment, which may include the construction contract profit and other related contract profit (Ho, 2001). In this paper, we assume that BOT firms are levered, since the excessive use of debt is an important characteristic of BOT financing. Because of the developer's role as a shareholder of the BOT firm, the developer should assess the value of the equity, instead of the firm.

From government's viewpoint, because any failure during project development will cause significant political cost, the government will try to avoid any failures in the BOT project. Since the bankruptcy condition of a BOT firm is determined by the asset value and debt value of the firm, by definition, the equity value is closely related to the bankruptcy probability. Thus, to the government, the equity value is the most important evaluation criterion of the financial viability. In some cases, part of the equity of the BOT firms may be publicly placed, and thus project development failures in such cases may significantly increase the political cost.

Therefore, from the perspectives of both the developer and the government, the equity value of a BOT firm can measure the financial viability of a BOT project. Moreover, the values of the government guarantee and negotiation option will be reflected in equity value. Note that, although the developer has to consider other 'profit components' (Ho, 2001) that are not reflected in the equity value, in this paper we focus on the equity value criterion, since it is the most relevant factor that concerns major BOT participants. We argue that if the equity value is greater than the equity investment amount, there will be positive net worth from the equity investment and the project can be considered 'financially viable', and vice versa. Note that this argument is true only when the debt or bank loan is fairly priced. In this paper, we assume that the loan interest rate is fairly determined by lending agencies, and is exogenously given in this paper as a constraint or known variable during the analysis.

BOT project evaluation and option pricing framework

The risk characteristics, managerial options, and the financial arrangements of the BOT project make it a complicated asset that cannot be assessed appropriately by traditional methods. Modern financial theory suggests that the option pricing framework can be applied in the valuation process to consider many complicated asset features, such as financing schemes (Mason and Merton, 1985), security indentures (Black and Cox, 1976; Leland, 1994), and managerial options (Majd and Pindyck, 1987; Trigeorgis, 1993, 1996). The analogies between the issues of the BOT financial viability evaluation problem and of the option pricing problem build the foundation of our option pricing based model for BOT projects.

- *Asymmetric payoff due to the limited liability of equity*. The equity payoff is contingent on the asset value of the firm V, and the debt value D. When the asset value is greater than the debt, the payoff of the equity can be considered as $V - D$, and when the firm value is less than the debt, the firm will be bankrupted and the payoff of the equity is zero because of the limited liability of the equity holder. As a result, the equity payoff is asymmetric and can be expressed as max $[0, V - D]$, which is identical to the payoff form of a call option in Eq. 1. Note that the symmetric form of the payoff is $V - D$. Equation 1 gives the payoff of a 'European call option'. European style options can be exercised only on a specific future date. An 'American call option' allows the holder to exercise the option *before* its maturity date. Since a BOT firm can be bankrupted before project completion, the equity of a BOT firm is analogous to the American option, except that in an American option the conditions of early exercise are solved differently from those of the BOT equity. An option pricing framework can more accurately assess the asymmetric payoff under various bankruptcy conditions.
- *Asymmetric payoff due to the government debt guarantee and the developer negotiation option*. As argued previously, government debt guarantees

and various managerial options are valuable to the developers. Generally, the debt guarantee will take effect only when the developer is unable to repay the debt or when the project is bankrupted. Such 'contingent' cash flows create an asymmetric payoff, which cannot be addressed properly by an NPV or discounting approach. Similarly, the negotiation for government rescue will occur only when adverse events occur and therefore will generate an asymmetric payoff. These asymmetric payoffs can be assessed more accurately by option pricing framework.

- *Financial impacts due to the construction risks.* In BOT investments, because of the large project scale and the long construction period of an infrastructure project, it is critical to consider the uncertainties of both construction cost and future operating profit. Traditional capital budgeting methods typically assume fixed investment outlays and uncertain future cash inflow streams, and fail to consider the risks of the initial costs. In the NPV method, there is no rule to determine an appropriate discount rate for risky investment costs. An option pricing framework can consider explicitly the construction risks and the consequent financial impacts on the project viability.

Option pricing theory

The option pricing theory by Black and Scholes (1973) and Merton (1973) is the building block of our model. Option pricing theory was based on the assumption that stock prices follow diffusion processes. It was assumed that the stock price follows a geometric Brownian motion process:

$$\frac{dS}{S} = \mu dt + \sigma dz \qquad (2)$$

where S is the stock price, μ is the instantaneous rate of return, σ^2 is the instantaneous variance of the rate of return, and dz is a random increment to a standard Wiener process.[3] It can be proved that the geometric Brownian motion process is distributed as a lognormal distribution, which is appropriate for modelling the price of a limited liability security, such as a common stock (Luenberger, 1998). The geometric Brownian motion process assumption is considered to reasonably represent the stochastic behaviour in modelling the values of risky physical assets, such as an oil reserve or a start-up venture (Brennan and Schwartz, 1984; Dixit and Pindyck, 1994; Leland, 1994; Schwartz and Moon, 2000).

The value of a European call option is obtained by solving the parabolic partial differential equation derived by Black and Scholes (1973), subject to one terminal condition and two boundary conditions. For a European call option, the terminal condition is given by the option's 'payoff function' as given in Eq. 1. Black and Scholes obtained an analytical solution, the so-called 'Black and Scholes formula'. The mathematical treatment of the option pricing framework is quite complicated, and the analytical solutions are limited to some special cases only. Thus, numerical methods are needed to solve for solutions in most cases, including our model.

Recently, option pricing theory was used in the evaluation of non-financial or real assets. Researchers sometimes use the term 'real options'. Therefore, the model proposed in this paper may also be regarded as a real option based model.

Evaluation model for the financial viability of BOT projects

On the basis of the option pricing theories, we have developed a five-step financial viability evaluation model for BOT projects, the BOT-OV model. The model computes the equity value of a BOT firm and explicitly considers the project risks and bankruptcy possibility, and also evaluates the project financial viability from the perspective of the shareholder and government. The option pricing based model presented here was verified and validated by both real cases in Kemna (1993) and the Channel Tunnel project in Ho (2001). A thorough case study was conducted by Ho (2001) on the application of the proposed model, using real data to form various scenarios to verify and validate the proposed model. The results indicated that the BOT-OV model is able to represent real world situations more closely, and provide a more accurate assessment of the BOT projects. We shall discuss the model, which comprises five steps, and give an illustrative example in the next section.

Step one: select the risk variables for the BOT project and determine their dynamics and current values

BOT risk variables and their dynamics

A risk variable is defined herein as an uncertain variable upon which the payoff of a BOT project is contingent. In other words, risk variables are the major risk factors or categories that determine the payoff. In our model, two risk variables are identified for the BOT project: the project value due to the net operating cash flows V, and the construction cost K. Each risk variable has its dynamics and current estimated value, and it is critical to consider specifically the characteristics of a BOT investment and to determine the dynamics and current value of each risk variable.

In the BOT-OV model, the dynamics of the first risk variable, project value, are given by the geometric Brownian motion:

$$\frac{dV_t}{V_t} = (\mu_V - \delta_V)dt + \sigma_V dz_V \tag{3}$$

where V_t represents the market value of a completed project at time t if the project is completed at t, μ_V is the market equilibrium rate of return of the BOT project, σ_V represents the volatility of the operating cash flows, δ_V is the rate of return shortfall of a non-traded project, and dz_V is an increment to a standard Wiener process. Note that these dynamics are adopted in a broadly similar fashion by major researchers in investment valuation, such as Majd and Pindyck (1987), Dixit and Pindyck (1994) and Schwartz and Moon (2000).

The dynamics of the second risk variable, project construction cost, is given by the following equation if we treat the total construction cost as another asset in the market.

$$\frac{dK_t}{K_t} = (\mu_K - \delta_K)dt + \sigma_K dz_K \tag{4}$$

where K_t is the market value of the expected total construction cost at t, and the parameters of the dynamics are defined similarly to those in Eq. 3. According to Eqs 3 and 4, the degree of uncertainty depends not only on how large the volatility is, but also on how far into the future one looks. Note that the correlation between V and K can be specified as

$$Cov\left(\frac{dV}{V}, \frac{dK}{K}\right) = \rho_{VK}\sigma_V\sigma_K dt \tag{5}$$

where ρ_{VK} is the correlation coefficient.

The 'current' values of V and K are defined by time 0 values, V_0 and K_0, respectively. Note that the dynamics in Eqs 3–5 are valid only during the construction phase. Therefore, the BOT-OV model will focus on the impacts of the early bankruptcy, the debt guarantee, and the negotiation option that may occur in the construction phase.

Estimating current values of the risk variables

The next task in step one is to estimate V_0 and K_0. Although this task is no different from those in other BOT evaluation approaches and can be delegated to experts in related areas, there are some particular concerns for BOT projects.

First, in a typical real options framework, V is defined as the project or firm value. There are many techniques for estimating the project value, as discussed in Copeland *et al.* (1996). However, in BOT project financing, since 'lenders look primarily to forecasted cash flow rather to project assets as collateral for the loan' (Beidleman *et al.*, 1990), the project value V is defined and measured by the NPV of the forecast net cash flows[4] during the operation, instead of the physical asset value. According to Copeland and Weston (1988), the discount rate used for computing V is the risk-adjusted weighted average cost of capital (WACC). Note that V is obtained by the risk-adjusted NPV; thus, the NPV approach is used to generate one of the primary inputs of the BOT-OV model.

Specifically, V_T can be computed by discounting all net cash flows from year T through T_E at the WACC, as shown in Figure 1(a), where T is the project completion time and T_E is the agreed termination date of the operation privilege. Moreover, according to the definition of V_t above, V_0 is obtained by first, pretending that the project will be completed at time 0 and will be operated from year 0 to year $T_E - T$, second, estimating the net cash flows assuming that the project begins operation at year 0, as shown in Figure 1(b), and third, discounting the estimated net cash flows above at the WACC. Note that μ_{Vr} can be obtained, given the values of V_0 and V_T, if we express the relationship between V_0 and V_T by

$$V_0 e^{\mu_{Vr} T} = V_T \tag{6}$$

Here μ_{Vr} can be defined as the 'appreciation rate' of the project value during the construction phase, and is an important parameter needed in step two of the model. Moreover, it can be shown that if the cash flow grows at a constant rate g, then the appreciation rate will equal the growth rate, that is, $\mu_{Vr} = g$.

Second, in estimating the construction or project cost, we assume that the project can always be completed on schedule by transforming the schedule

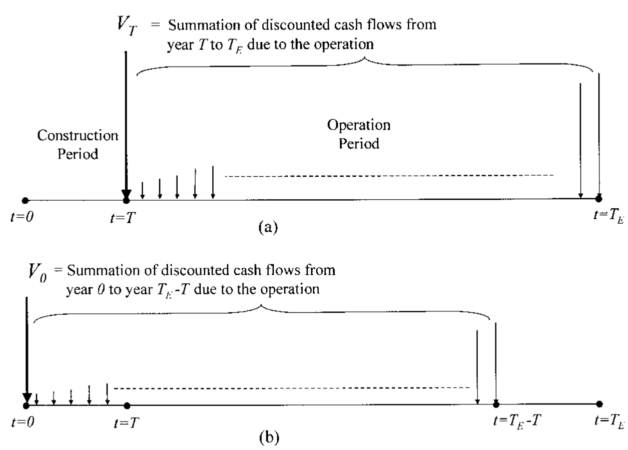


Figure 1 Cash flows and the estimation of V_t

differences into cost differences, so that we may focus on the uncertainty of the construction cost, rather than the uncertainty of the schedule or progress. Since Amram and Kulatilaka (1999) argued that the option based evaluation should be based on only a few relevant risk variables by bundling similar risks together, the construction cost fluctuations due to the schedule uncertainty and non-schedule uncertainty are bundled together as one risk variable, K. Note that K will also follow the concept described by Eq. 6.

Step two: align the dynamics of BOT risk variables with the capital market and project characteristics

The second step of the BOT-OV model is to estimate the parameters in the dynamics of risk variables, namely, μ_V, μ_K, δ_V, δ_K, σ_V, and σ_K. These parameters measure the project characteristics and represent investor opinion in the capital market. Thus this step will align the equity valuation process with the project characteristics and the capital market, and differentiate the BOT-OV model from existing BOT evaluation methods, such as Finnerty (1996), Walker and Smith (1995), and Ye and Tiong (2000). Particular concerns about the definitions and estimations of these parameters are discussed next.

Market equilibrium rate of return μ_V and μ_K

The first parameter is the market equilibrium rate of return μ. According to the capital asset pricing model (CAPM), the market equilibrium rate of return of a risky asset i in an efficient market is given by

$$\mu_i = r + \lambda\rho_{iM}\sigma_i \tag{7}$$

where

$$\lambda = \frac{r_M - r}{\sigma_M} \tag{8}$$

Here, r is the risk-free interest rate, λ is the risk premium per unit of risk, and $\rho_{iM}\sigma_i$ represents the total amount of risk. ρ_{iM} is the correlation of the capital market and risky asset i, and σ_i is the standard deviation of i. In Eq. 8, r_M is the expected return of the capital market, and σ_M is the standard deviation of the market return.

In BOT investments there are two rates of return related to the project, namely μ_V and μ_K, as shown in Eqs 3 and 4, respectively. According to Eq. 7, they may be estimated by

$$\mu_V = r + \lambda\rho_{VM}\sigma_V \tag{9}$$

$$\mu_K = r + \lambda\rho_{KM}\sigma_K \tag{10}$$

where ρ_{VM} and ρ_{KM} are defined similarly to ρ_{iM}.

It is worth noting that the parameters on the right hand side of Eq. 7 can be estimated reasonably from observable data in the market if the asset is publicly traded. If the asset is not publicly traded in financial market, the parameters in Eq. 7 can be estimated by other publicly traded assets with similar business or risk characteristics. As a result, the option based valuation can be consistent with investor opinion in the market. Note that in modern financial theory, various methods for estimating λ , ρ_{VM} , ρ_{KM} , ρ_V , ρ_K , r_M , and r have been developed, such as in Hull (1997); thus it is not necessary to repeat these methods in this section.

Rate of return shortfall δ_V and δ_K

In BOT projects, the rate of return shortfall of each risk variable can be defined, and is given by

$$\delta_V = \mu_V - \mu_{Vr} \tag{11}$$

$$\delta_K = \mu_K - \mu_{Kr} \tag{12}$$

where μ_V and μ_K are obtained by Eqs 9 and 10, and μ_{Vr} and μ_{Kr} are the expected appreciation rates of V and K, respectively. The concept of rate of return shortfall was first suggested by McDonald and Siegel (1984) for the valuation of options on real assets. The rate of return shortfall exists when the underlying asset is a non-financial asset or unfinished project. The expected appreciation rate can be inferred by different estimated asset values at different times during the construction, as shown in Eq. 6. Dixit and Pindyck (1994) argued that when the real asset is a project waiting to be constructed or completed, the rate of return shortfall may represent 'the opportunity cost of delaying construction of the project'. Since the concept is rather complicated, readers should refer to Trigeorgis (1993, 1996) and Dixit and Pindyck (1994) for detailed discussions.

Step three: construct a reverse binomial pyramid under two risk variables

Since two risk variables are selected in step one, one needs to construct a numerical option pricing model that considers two risk variables. The 'binomial tree' approach by Cox *et al.* (1979) is a numerical model for one risk variable. A reverse binomial pyramid can be constructed for evaluating BOT projects with two risk variables. In the following sections, we shall introduce the binomial tree method and then show how to extend it to the binomial pyramid.

N-Step binomial tree model

Cox *et al.* (1979) derived a discrete 'binomial tree' with n timesteps to represent the dynamics of the stock price. Given specific stock price dynamics, such as Eq. 2, one can transform the dynamics into a binomial tree as shown in Figure 2, where S is the stock price. After each timestep, the stock price can either go up by a certain percentage u, or down by a certain percentage d, with probability q and $1-q$, respectively. Here S_{uud} shown in Figure 2 indicates that S goes up in periods 1 and 2, and down in period 3.

Second, Cox *et al.* constructed an 'option valuation tree', and performed the calculation as shown in Figure 3, where F is the value of a European call option. Figure 3 shows that the option price is solved backwards recursively from the maturity date. Upon maturity, the option price is $Max[0,S-X]$, where X is the exercise price, for example, $F_{uuu} = Max[0,S_{uuu} - X]$. For period 2, the option value at each node is obtained by computing the discounted expected period 3 option value. For example, F_{uu} can be computed from

$$F_{uu} = \frac{1}{R}[qF_{uuu} + (1-q)F_{uud}] \quad (13)$$

where R is the discount factor, which equals $e^{r\Delta t}$ when the interest is compounded continuously, Δt is the length of a timestep, and $q \equiv \frac{R-d}{u-d}$. Readers may refer to Cox *et al.* (1979) for the derivation of Eq. 13. It is worth noting that the probability q in Eq. 13 is a pseudo probability. Solving backward, F can be obtained from

$$F = \frac{1}{R}[qF_u + (1-q)F_d] \quad (14)$$

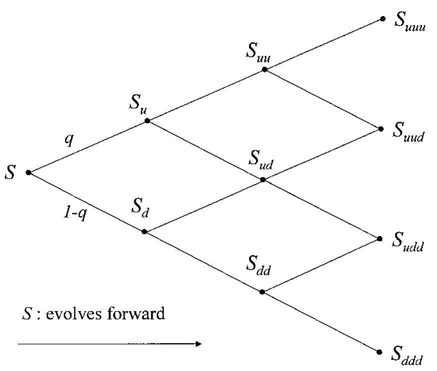


Figure 2 Binomial tree of the dynamics of stock prices

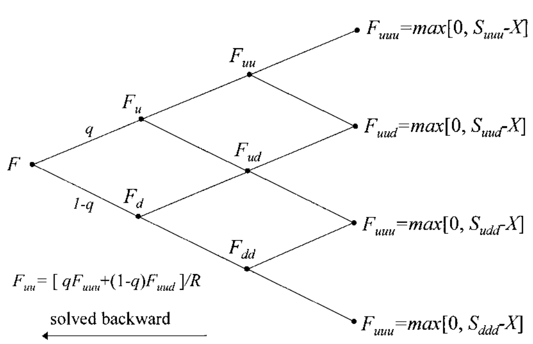


Figure 3 Binomial tree for solving stock option value

Note that one advantage of the model is that it can readily solve options with early exercise features, such as American options.

For an n-step binomial tree, the solution can be solved backwards recursively. The remaining question in using the binomial model is how to decide the jump amplitudes u and d, and the jump probability q.

Jump probability q and jump amplitudes u and d

Under the binomial tree framework, u and d are needed in order to compute the jump probability $q \equiv (R-d)/(u-d)$. Suitable values for u, d, and q could be found by matching both the mean and variance of the logarithm of a price change (Luenberger, 1998). By imposing $u = 1/d$ for convenience, the jump amplitudes may be found from

$$u = \exp\ (\sigma\sqrt{\Delta t}) \quad (15)$$

$$d = \exp\ (-\sigma\sqrt{\Delta t}) \quad (16)$$

where terms of higher order than Δt are ignored. Equations 15 and 16 show that the jump amplitude is determined by the risk variable's volatility, σ. Note that alternatively one can obtain u and d by imposing a fixed pseudo probability, $q = 0.5$ (Hull, 1997). This alternative will be adopted later in our binomial pyramid model. In this case, the jump amplitudes are given by

$$u = \exp\ \left[(r - \tfrac{1}{2}\sigma^2)\Delta t + \sigma\ \sqrt{\Delta t}\ \right] \quad (17)$$

$$d = \exp\ \left[(r - \tfrac{1}{2}\sigma^2)\Delta t - \sigma\ \sqrt{\Delta t}\ \right] \quad (18)$$

Binomial model for dividend-paying stocks

The binomial model can be used to price options on dividend-paying stocks. In principle, the stock price will fall by the amount of dividend paid on the date of payment, or ex-dividend date. Thus, dividend payment can be considered as a stock price reduction, and the stock price fall will reduce the value of an option. Assuming that the stock pays a dividend yield δ_S continuously, the stock dynamics can be modelled as

$$\frac{dS}{S} = (r - \delta_S)dt + \sigma dz \quad (19)$$

As a result, the jump amplitudes u and d in Eqs 17 and 18 will become:

$$u = \exp\ \left[(r - \delta_s - \tfrac{1}{2}\sigma^2)\Delta t + \sigma\ \sqrt{\Delta t}\ \right] \quad (20)$$

$$d = \exp\ \left[(r - \delta_s - \tfrac{1}{2}\sigma^2)\Delta t - \sigma\ \sqrt{\Delta t}\ \right] \quad (21)$$

This dividend-paying feature is crucial in the BOT-OV model, since the rate of return shortfall, δ, of the BOT project is analogous to the stock dividend yield,

δ_S. As a result, for the BOT risk variables V and K, the jump amplitudes are given by

$$u_V = \exp\left[(r - \delta_V - \tfrac{1}{2}\sigma_V^2)\Delta t + \sigma_V\sqrt{\Delta t}\right] \quad (22)$$

$$d_V = \exp\left[(r - \delta_V - \tfrac{1}{2}\sigma_V^2)\Delta t - \sigma_V\sqrt{\Delta t}\right] \quad (23)$$

$$u_K = \exp\left[(r - \delta_K - \tfrac{1}{2}\sigma_K^2)\Delta t + \sigma_K\sqrt{\Delta t}\right] \quad (24)$$

$$d_K = \exp\left[(r - \delta_K - \tfrac{1}{2}\sigma_K^2)\Delta t - \sigma_K\sqrt{\Delta t}\right] \quad (25)$$

where u_V and d_V are the jump amplitudes of the first risk variable V, and u_K and d_K are the jump amplitudes of the second risk variable K.

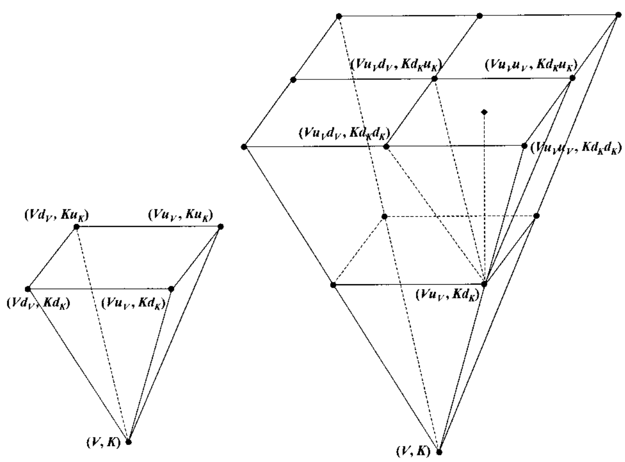
Reverse binomial pyramid for BOT investments

Here we derive a binomial pyramid for BOT investments that has a two-jump process for each risk variable by following Hull and White's (1994) procedure. The first task is to assume the jump probabilities as shown in Table 1 by first imposing an equal probability of jumping up or down and then adjusting the jump probability by the correlation between the risk variables, ρ_{VK}. The probability adjusting procedure can be found in Ho (2001) and Hull and White (1994).

The second task is to construct a reverse binomial pyramid with the sharp end on the bottom. The model starts with a point or a node from the bottom. The point represents the current values of the risk variables (V, K). After each time increment, four branches will emanate from each node as shown in Figure 4(a), and

Table 1 Binomial pyramid's jump probability

	V – move	
K – move	Down	Up
Up	$0.25(1-\rho)$	$0.25(1+\rho)$
Down	$0.25(1+\rho)$	$0.25(1-\rho)$



(a) One Step Jump for two Risk Variables (b) Two Step Binomial Pyramid

Figure 4 Reverse binomial pyramid: one step and two steps

a corresponding jump probability will be associated with each branch. Figure 4(a) shows the possible price movements or jumps in the pyramid after one time increment. Figure 4(b) shows the dynamics after two jumps. Each node in the previous level will generate another four nodes in the current level. Figure 4(b) shows the four nodes generated by the middle level node (Vu_V, Kd_K). To obtain the real option price, one needs to compute the expected option payoff discounted at risk-free rate backwards recursively from the top of the reverse pyramid to its bottom. The backward computation process is similar to that shown in Eqs 13 and 14 in a regular binomial model.

Step four: determine the payoff functions of the BOT equity

Payoff functions give the option payoffs upon the exercise. In a BOT project, the exercise of an option is analogous to the bankruptcy or completion of the project. The equity payoff can be expressed as a function analogous to the option payoff functions. Thus, it is critical to formulate appropriate payoff functions. Two types of payoff function are needed. The first type is the 'terminal' payoff function, which is used when the project is completed. The second type is the 'time *t*' payoff function, which is used when the project is under construction and the project can be bankrupted before its completion.

Bankruptcy conditions

Under the bankruptcy risk, the early termination of a project is similar to the early exercise of an option. The difference is that in an American option, the early exercise timing is determined simultaneously with the value of the options. However, in BOT projects, the early exercise condition is determined in advance, and it is imposed by the lending agency or credit agreement for the protection of the debt holders. As a result, in BOT projects, the default or bankruptcy conditions become the early exercise conditions. Therefore, it is critical to determine the bankruptcy conditions.

Note that the purpose of bankruptcy is to protect the debt holders. A very common bankruptcy condition in the debt indenture is when the borrower cannot meet the repayment schedule. However, in construction phase, since there is no revenue generated from the project, the repayment schedule is delayed until the project completion. As a result, the BOT project will never be bankrupted during the construction if the lender does not specify other bankruptcy conditions. Thus, to prevent this problem and protect the lender, the lender will impose other conditions to trigger the bankruptcy should adverse events occur, especially in a large project with a long construction period. For example, the lender may specify the upper limit of cost overrun or the milestone upon each loan drawdown during the construction phase. When the drawdown requirement is not met, the lender can cease to provide the further loan required. Under such circumstances, unless the developer can justify the cost overrun or schedule delay, or arrange other funding sources, such as new equity injection or government rescue, the BOT firm will be bankrupted. This is why in BOT projects the loan is always arranged as a 'facility', so that the loan can be terminated under certain conditions specified ex ante.

In the BOT-OV model, we assume that lenders will try to prevent the project value from being below the value of the total estimated debt. It is assumed that the loan is a facility and the lender will estimate and monitor the project value V_t and the expected total cost K_t frequently, to ensure that the project value is larger than the estimated total debt at any time t. This practice is incorporated in the credit agreement of the Channel Tunnel project (Ho, 2001). Therefore, in the BOT-OV model the bankruptcy condition can be modelled as

$$V_t - D_t(K_t)e^{-r_d(T-t)} < 0 \qquad (26)$$

Here, $D_t(K_t)$ is defined as the total outstanding debt at time T prices estimated at time t, and thus $D_t(K_t)e^{-r_d(T-t)}$ is the total estimated debt at time t prices obtained by discounting $D_t(K_t)$ at the loan interest rate r_d for the period $T-t$. Note that if we assume the interest is compounded and accrued continuously and the interest payment is delayed until project completion, the total debt at time t prices can be approximated reasonably by the zero-coupon bond formula shown in Eq. 26.

Equation 26 suggests that if the project value estimated at t is less than the estimated required total debt at time t prices, the lending bank will force the bankruptcy of the BOT firm to prevent further loss. Note that $D_t(K_t)$ is expressed as a function of K_t and formulated by considering the project cost schedule, the loan indenture, and the time value. There are many ways to specify the debt function, $D_t(K_t)$, to approximate the total outstanding debt at any time t. Since it is lengthy and trivial to specify $D_t(K_t)$, we shall forego the derivation details in this paper. Particular concerns regarding how to model $D_t(K_t)$ for a BOT project can be found in Ho (2001).

Payoff functions of the BOT project

The payoff function upon completion is the terminal payoff function and is given by equation (27). The 'if condition' in Eq. 27 is obtained by substituting $t = T$ into Eq. 26. Note that this payoff function will be applied to the top level of the reverse binomial pyramid and computed in the first round of backward computation.

$$\begin{cases} V_T - D_T(K_T), & \text{if} \quad V_T - D_T(K_T) \geq 0 \\ 0, & \text{if} \quad V_T - D_T(K_T) < 0 \end{cases} \qquad (27)$$

The time t payoff function for BOT equity holders under default risk is given by

$$\begin{cases} \text{Payoff obtained by backward computations} & \text{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} \geq 0 \\ 0, & \text{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} < 0 \end{cases} \qquad (28)$$

If the bankruptcy condition is not triggered before T in Eq. 28, the payoff will be obtained from previous rounds of backward computation. Note that the 0 payoff in Eq. 28 is because of the limited liability of equity.

Step five: plug the terminal and time t payoff functions into the reverse binomial pyramid to compute the BOT equity value

First, computation begins at the time T layer by substituting V_T, K_T, and other parameters obtained from steps one to three into the terminal payoff function, Eq. 27, and obtaining a payoff for each node in the time T layer. Second, one needs to compute the equity payoff of each node at the next layer, time $T-\Delta t$, by Eq. 28. Then the computation performed for the time $T-\Delta t$ layer is repeated backwards recursively until time 0, and then the equity value is obtained.

Illustrative example: financial viability and debt guarantee of BOT projects

This section presents an illustrative example of how to apply the evaluation model to analyse a BOT investment. We shall demonstrate the advantages of applying the BOT-OV model and the weaknesses of the NPV method. Note that this section should be considered as an illustration of the model implementation, instead of an empirical case study.

Project description

SmartCorp Inc. is a leading construction firm in Canada. The management is evaluating a BOT airport terminal investment. Suppose that SmartCorp will fund an independent BOT firm, SmartAir, to develop and operate the airport if SmartCorp wins the BOT bid. In the BOT airport terminal project, the BOT firm, SmartAir, will obtain a 40-year ground lease from the government to develop and operate the terminal. The development of the airport includes the terminal building with retail shops, restaurants, two airport hotels, and parking lots. SmartCorp's preliminary analysis shows the following.

- SmartCorp has only one alternative to finance the BOT project, which is to invest \$100 million as equity in forming SmartAir. The other source of funding for the project cost is a bank loan. The interest rate is 10% per annum, compounded continuously.
- The estimated project construction cost K_0 is \$400 million in year 0's prices, not including the net financing cost. The estimated nominal cost growth rate is 4% per annum. The estimated cost volatility σ_K is 0.2 per annum.
- The estimated optimal construction time is 4 years.
- The estimated accounting rate of return during the operation is 17%. The estimated net cash flow of the first operating year, year 5, and that of the final operating year, year 40, are \$54.82 million and \$315.46 million, respectively, with an estimated 5% constant annual growth rate g, in nominal terms. It is estimated that the 5% growth rate will prevail if the project is completed before the scheduled completion time T. In other words, if the project begins operation at $t = 0$, the net cash flow of year 1 will be $\$54.82e^{-g\times 4}$ million. The estimated volatility σ_V is 0.4 per annum. The project value is uncorrelated with the project cost, that is, $\rho_{VK} = 0$.
- The WACC of SmartAir is 12.65% per annum.
- If the government provides a debt guarantee, the loan interest rate will equal the risk-free interest rate, 4%.

There are two major issues. Is the BOT airport financially viable from the perspective of a SmartAir shareholder? What are the impacts of the government debt guarantee and the developer's negotiation option? In the next section, we shall use the BOT-OV model to analyse this BOT investment. Additional assumptions or estimates needed for the analysis will be given during the analysis.

Implementation of the BOT-OV model

The implementation follows the five steps introduced above. Particular concerns about the implementation of the illustrative project are discussed below. Interested readers may refer to Ho (2001) for details regarding the implementation of the BOT-OV model on an actual project.

Step one. Select the risk variables and determine their dynamics and current values. As argued previously, the BOT project value and the construction cost are selected as the risk variables. The dynamics are given in Eqs 3–5. V_0 may be computed from the estimated cash flows, assuming that the project begins the operation at year 0. Considering that the cash flow growth rate g = 5% and the WACC = 12.65%, we obtain $V_0 = \int_1^{36} 54.82e^{-g\times 4}e^{-(WACC-g)(t)}dt$ = \$532.2 million. Note that if the project begins operation at scheduled completion time, $T = 4$, the total discounted cash flow at year 4's prices will be given by $V_{t=4} = \int_5^{40} 54.82e^{-(WACC-g)(t-4)}dt$ = \$610 million. Then, according to Eq. 6, we obtain $\mu_{Vr} = 5\%$, which will be needed in step two. As for K, we have K_0 = \$400 million, and $\mu_{Kr} = 4\%$, as stated in the project description.

Step two. Align the dynamics of the risk variables with the capital market and project characteristics. Suppose that the analysis by SmartCorp showed that the expected rate of return and volatility of the capital market are 0.13 and 0.3 per annum, respectively, and that the correlations ρ_{VM} and ρ_{KM} are equal to 1.2 and 0, respectively. By Eqs 9 and 10, we obtain $\mu_V = 18.4\%$ and $\mu_K = 4\%$. Second, the rates of return shortfall, δ_V and δ_K, are determined by Eqs 11 and 12, and we obtain $\delta_V = 13.8\%$ and $\delta_K = 0\%$.

Step three. Construct a reverse binomial pyramid for the two-risk-variable problem. Considering the accuracy and computing time, here we use 300 timesteps for the pyramid. The most important parameters in the binomial pyramid are the jump probabilities and jump amplitudes. The jump probabilities are computed by substituting ρ_{VK} into the matrix in Table 1. The jump amplitudes for V and K are obtained by Eqs 22–25.

Step four. Determine the payoff functions. After the binomial pyramid has been constructed, one can generate the values for V_t and K_t for each time increment level. These values will be substituted into the payoff functions. Two types of payoff function are needed. At time T, the terminal payoff function is given by Eq. 27, and the time t payoff function is given by Eq. 28.

Step five. Plug the terminal and time t payoff functions into the reverse binomial pyramid to compute the equity value of the BOT firm. Substituting all the parameters obtained from the previous four steps and computing the equity payoff for each time layer backwards recursively, we obtain the equity value of the BOT firm, SmartAir: \$73 million.

Financial viability of the BOT airport project

Since the equity value, \$73 million, is less than the equity investment amount by \$27 million, SmartAir shareholder will have a loss in the equity investment. Therefore, as argued in the section on evaluation criteria, from the perspective of either the shareholder or the government the project is financially unviable. If the developer's major profit in the BOT investment comes from the equity investment, the developer should forego the BOT Airport investment if there is no government subsidy. Note that according to the NPV method, the NPV of the project is $V_{t=4}e^{-WACC\times 4} - K_0 = -\$32.2$ million. This result also suggests that the project should not be undertaken. However, three major problems are associated with this result. First, the impact of cost uncertainty is not considered. Second, the NPV does not reflect the fact that the developer's investment returns are realized and measured through equity returns. Third, the NPV method cannot analyse further the impacts of the government debt guarantee and the developer negotiation option. In the following section, we shall address how can we use the BOT-OV model to analyse the impacts of the government guarantee and the negotiation option on the financial viability.

Government debt guarantee: valuation and impacts

In some cases, the government will provide various types of guarantee towards the project financing or operation, to strengthen the financial viability of an investment. Government debt guarantee is desired when the lenders feel that the project is financially unviable and cannot be financed by debt. Under this circumstance, the government needs to know the value of the guarantee subsidy, and the developer needs to include the value of the guarantee in the investment decision making.

Since the financial viability of a BOT investment is measured by equity in this paper, the value of the guarantee should also be assessed by the equity value. The difference in equity values between the project with and without debt guarantee will be the value of the guarantee. For simplicity, it is assumed that the guarantee will assure the lending banks that the debt will be fully repaid by the government when the project fails. Thus, under the guarantee, we assume that the lenders will consider the debt risk free, and that the loan interest rate will be the risk free rate, 4%. Also, since the debt is risk free, we assume that the banks will continue to support the project even when adverse events occur. Suppose that it is costly for the government to replace the developer should adverse events occur. It is reasonable to assume that the only time that the project is bankrupted under debt guarantee is at completion time, T. Based on these characteristics and assumptions, the value of the debt guarantee can be evaluated by the BOT-OV model. The difference in implementing the model is that for the layers from time 0 to time $T - \Delta t$, the payoff is obtained by backward computation, instead of Eq. 28. After the modification, the equity value under the debt guarantee is obtained and equal to \$124.5 million. As a result, the value of the government debt guarantee G can be computed by

$$G = E_G - E = \$124.5 - \$73 \text{ million} = \$51.5 \text{ million} \quad (29)$$

where E_G and E are the equity values with and without the debt guarantee, respectively. Since under the debt guarantee the equity value is greater than the equity investment, \$100 million, the BOT airport becomes financially viable from a shareholder perspective. In fact, this new equity value also indicates that the government oversubsidizes the BOT investment.

One advantage of the BOT-OV model is that the model can accommodate different guarantee situations or clauses. For example, under debt guarantee, if the government has a strict policy to enforce the bankruptcy and replace the developer when the BOT firm meets the default conditions, we obtain a lower E_G, \$99.9 million, and a lower G, \$26.9 million. Here, E_G = \$99.9 million is obtained by using normal payoff functions, Eqs 27 and 28, in steps four and five. The debt guarantee value, \$26.9 million, results mainly from the reduction of loan interest and the lower bankruptcy possibility due to the lower accrued interest. Note that in the following sections we shall go back to the previous guarantee situation and assume that, under debt guarantee, the government will not enforce the bankruptcy because of the cost of replacing project developers.

Traditional techniques like the NPV method cannot assess options like debt guarantee. The magnitude of the guarantee value in this example shows that failing to evaluate the guarantee value will have a significant negative impact on the decision making of the developer and government.

Negotiation for rescue: valuation and impacts

In some cases, the developer may negotiate for a government rescue in adverse circumstances, and the government may or may not agree to a rescue. Ho (2001) solved the conditions for the government's rescue decisions. Here we simply examine the impact of one possible outcome of the negotiation. According to Ho (2001), under certain conditions the developer may 'expect' that requests for rescue in all adverse circumstances will be justified and granted, and thus it is reasonable to assume that the BOT firm will not be bankrupted before time T. Then we can evaluate the new equity value by the approach used for evaluating the government guarantee, except that the loan interest rate will be the original 10%,[5] instead of 4%. We obtained the new equity value, \$95.5 million, under the aforementioned negotiation condition. Thus, the value of the negotiation option is \$95.5–73 million = \$22.5 million. Note that the government needs to be aware of the value of the negotiation option, since the value is at the expense of government rescue and yet without being explicitly considered by the government.

Effects of initial project value and cost on the project viability and debt guarantee

We can also analyse the impacts of the project characteristics on the project viability and debt guarantee. One major concern of shareholders and the government is the possibility and impact of the overestimation of project value V_0 and the underestimation of the project cost K_0. It is critical to understand the degree of over- or under-estimation of V_0 and K_0 for which the project is still financially viable. Figures 5 and 6 show the equity values with respect to the adjusted V_0 and K_0, respectively. Note that the project is financially viable only when the lines lie above \$100 million. For example, Figure 5 shows that, without debt guarantee, V_0 needs to be adjusted by at least 1.12 so that the project can become viable. Moreover, the equity value difference between the two lines in the figures represents the value of the debt guarantee. Figure 6 shows that the debt guarantee is more valuable when the estimated project cost is higher.

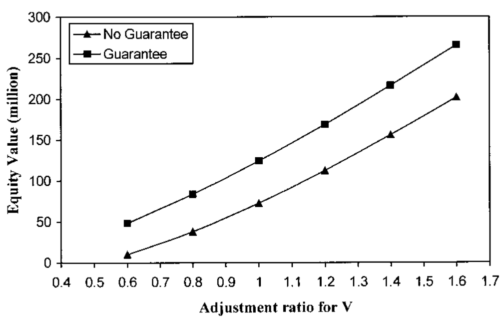


Figure 5 Impacts of initial *V* on the equity value

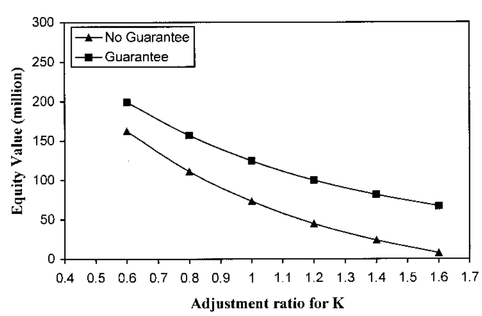


Figure 6 Impacts of initial *K* on the equity value

By fixing the initial cost ratio to 1.0 as shown in Figure 5, we may find a critical point, where the equity value equals \$100 million. If we vary the initial cost by ratios from 0.6 to 1.6 and find a critical point for each initial cost estimate, we may obtain a financial viability profile, as shown in Figure 7. Those projects with initial estimates lying above the curve in Figure. 7 represent a financially viable project, and vice versa. For example, Figure 7 shows that project A is financially viable with and without debt guarantee, whereas project B is financially viable only under debt guarantee. This type of 'viability profile' can help the project participants evaluate the financial viability when there are different opinions or estimates regarding the initial estimates. Moreover, the viability profile can assist the participants in examining how sensitive the viability is with respect to initial estimates. These scenario analyses are very valuable in the project evaluation process.

Conclusions

It is complex and challenging to evaluate the financial viability of BOT projects. Traditional capital budgeting methods cannot account appropriately for the asymmetric payoff under the bankruptcy risk, and neither can they correctly assess the impact of the risks and project characteristics on the financial viability of a privatized infrastructure project. This paper presents an option pricing theoretical model for evaluating the financial viability of BOT projects more realistically and accurately. This model considers the uncertainties of both the construction cost and the project net cash flows, and aligns the evaluation process with capital market disciplines and project characteristics. Furthermore, the model can quantitatively assess the value of a government debt guarantee and its impact on the project financial viability. The major contribution of this model is to provide a theoretical and quantitative framework for evaluating the financial viability of privatized infrastructure projects from the perspectives of both the project developer and the government.

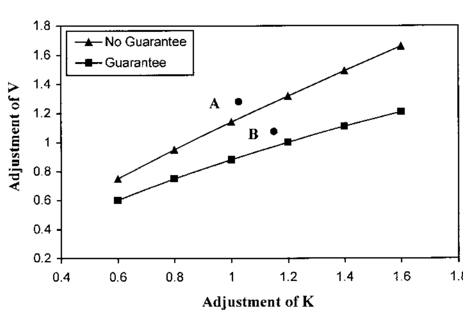


Figure 7 Project financial viability profile

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Endnotes

1 See Copeland and Weston (1988) or Luenberger (1998) for an introduction to and derivation of CAPM.

2 Note that for a leveraged firm, the value of the firm or project is different from that of equity.

3 The Wiener process increment *dz* is normally distributed with a mean of 0 and a variance of d*t*. See Dixit and Pindyck (1994, p. 63).

4 The 'net cash flow for capital budgeting purposes' is defined by the net amount of the company's after-tax operating earnings, plus non-cash charges such as depreciation cost, less investments in operating working capital, property, and other assets. See Copeland and Weston (1988, p.39) and Copeland *et al.* (1996, p.139).

5 This is because the lenders have no ground to trust the developer's negotiation capability.