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Modelling project investment decisions under uncertainty using possibility theory

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Abstract

With the increasing popularity of privately financed and operated construction projects, a systematic evaluation of investment options is needed, especially if they are competing for the same capital resource. Traditional evaluation methods incorporating risk analysis techniques require the input of relative frequencies which are not easily available in construction. This paper proposes a method capable of modelling the effects of both monetary and non-monetary aspects of an investment option, using interval mathematics and possibility theory to handle the inherent uncertainty associated with such aspects. Two numerical examples are presented to demonstrate its application in the assessment and ranking of available investment options.

Keywords: Investment options; Project selection; Possibility distribution; Risk and uncertainty; Monetary factors; Non-monetary factors

1. Introduction

Success in the contracting business relies heavily upon selecting those investment options of most benefit to the organisation in both the short and long term. Whether these benefits are purely monetary or a combination of monetary and non-monetary gains, investment options must be compared objectively.

Contracting organisations and developers tend to concentrate on establishing the financial viability of a project through feasibility studies. A project is deemed economically feasible, if the expected revenue meets or exceeds an acceptable pre-determined level of return on the organisation's initial investment. As this procedure involves a degree of forecasting, decisions are frequently made based on past experience, either rationally or intuitively with some degree of uncertainty, and thus are made under risk [1]. If the total uncertainty is significant, as in the case of long term investments, not recognising it will often totally distort the predictions, in an unknown way, making any decisions based on these predictions highly suspect. Therefore, it is paramount for an organisation to be able to predict and compare all possible future monetary outcomes taking into account the inherent uncertainty associated with selected investment parameters including construction, operation and maintenance costs, interest rate, inflation, depreciation, tax rate, and operation life.

The estimation of values of investment parameters is undoubtedly crucial to the success of the feasibility study. As a result, the study must be detailed enough to allow major risk factors to be identified and critically assessed. It is not uncommon for organisations to commit considerable financial and human resources towards project appraisal. For example, the EuroTunnel project promoters spent approximately $1 million US dollars on the feasibility study before the tender was even won [2].

Traditionally, the net present value (NPV), internal-rate-of-return (IRR) and pay-back period investment appraisal techniques have formed the major component of feasibility studies. These three techniques are based upon the time-cost-of-money principle and use slightly varied procedures to forecast the expected monetary returns on an investment. The reliability of their output depends upon the accuracy of the deterministic cash flow values (benefits and costs), and their timing, as estimated by the organisation. A fundamental limitation of this assumption is that the various investment parameters

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cannot be practically assumed with a higher degree of certainty. The value of each parameter is affected by a myriad of risks and uncertainties which are often difficult to quantify. An element of uncertainty lies with each prediction which, alone or in combination, may have a significant impact on the outcome of the economic analysis. Uncertainty, emanating from the project itself or external factors, will always be present and needs to be accurately captured in the decision-making process.

In addition to the crucial uncertainty factor, the above techniques do not allow for the non-monetary (qualitative) factors to be considered in assessing the investment option. Non-monetary project aspects such as social, environmental, political and legal factors are deemed to be important; but these would usually be considered to lie outside the normal appraisal process [3]. Such aspects need careful analysis and understanding so that they can be managed [4]. In extreme cases, neglect of these aspects can cause the failure of a project despite very favourable financial components [5]. For example, dramatic change in government policy can substantially change project revenue to the extent that a once feasible project is rendered unprofitable. Therefore, it is recommended that the viability of a construction project should not be determined by monetary considerations alone.

A proper feasibility study should also provide the organisation with the opportunity to include factors related to the economic environment (boom or recession), project complexity, technical innovation, market share, competition, national significance and other strategic aspects. To provide for the effects of these qualitative aspects, the majority of organisations resort to estimating the necessary money contingencies without an appropriate quantification of the combined effects of monetary and non-monetary factors.

With the increasing popularity of privately financed, built and operated facilities in recent years, a systematic evaluation of investment options is needed, especially if these options are competing for the same capital resource. In a perfect world, an analyst contemplating an economic decision would have access to precise deterministic values. Unfortunately, this ideal state does not exist when investing in a project where there is uncertainty about nearly every estimate that is entered into an economic model [6].

This paper addresses the issue of combining both monetary and non-monetary aspects of an investment option. It proposes a method capable of modelling and ranking various investment options, specifically developed for construction projects. The proposed method utilises interval mathematics and possibility theory to handle the inherent uncertainty associated with investment parameters. Two numerical examples are presented to demonstrate its application in the assessment and ranking of available investment options.

2. Why possibility distribution?

The most commonly used risk analysis techniques in the construction industry today are sensitivity analysis and probability analysis. The former models the impact of change in variables on the project's IRR to identify the factors that are particularly risk sensitive whilst the latter is a more sophisticated technique which uses 'Monte Carlo' simulation to model the combined affect of numerous risk factors according to their relative frequencies.

One of the troublesome issues associated with probability analysis is the utilisation of a probability measure to evaluate uncertainty. Therefore, much effort is needed in defining and developing each contributing risk factor's probability distribution using historical data in estimating relative frequencies. Since each construction project is affected by different risk factors, accurate knowledge of relative frequencies cannot simply be assumed from another project, as would be possible in other industries such as manufacturing where events have a repetitive nature.

Most analysts take it for granted that uncertainty is a model associated with randomness [7]. While probability theory can be a powerful tool in the appropriate circumstances, many times the type of uncertainty encountered in construction projects does not fit the axiomatic basis of probability theory. Simply because, uncertainty in these projects is usually caused by the inherent fuzziness of the parameter estimate rather than randomness [6]. Uncertainty involved in real risk situations is often epistemic (relating to the knowledge of things) rather than alearotic (depending on chance) [8]. The use of probability for the purpose of investment appraisal is well documented [9].

Another limitation of using probability analysis is that the influence of non-monetary (qualitative) aspects on projects is often difficult to quantify. The lack of know-how in measuring strategic and intangible (qualitative) costs and benefits led current models to ignore their contribution to the overall economic analysis.

One way to alleviate the above shortcomings is to use the possibility theory where the user needs only to determine a possible range, and perhaps even a most likely value for each investment parameter, without the input of each factor's relative frequency. The possibility theory is an appropriate vehicle as it is based on the concept that all values within a certain range are possible, with the exact value being unknown. A range of values, or an interval, is assigned subjectively, but the individual values in the interval are not assigned a relative belief value. An expert may feel that a given parameter is within a certain range and may even have an intuitive 'feel' for the 'best' value within that range. However, seldom will the analyst have an empirical foundation for the estimate based on frequency of occurrence [6]. Mak

[10] argues that normative theories in probability are not as applicable in the construction industry as some may perceive, and considers possibility theory to be superior to probability theory in analysing problems where subjective judgements dominate the risk analysis process. This viewpoint is shared and supported also by others [11,12].

The possibility theory has been used successfully in a wide range of engineering and scientific fields including: project scheduling and network analysis [13,14] contract selection and decision making [15,16] and safety perfomance [17].

In view of the above, the authors concluded that the possibility theory appears to be more appropriate, under the above circumstances, for modelling project investment decisions under risk than probability theory.

3. Methodology and model input

It is evident that the appraisal of any construction project as an investment must incorporate the effect of both monetary and non-monetary aspects on the project's overall attractiveness and that these aspects are most appropriately represented. The purpose of this section is to develop a methodology, and hence a computer program using possibility theory, for combining both of these aspects for individual investment options. As economic decision-making often requires the relative ranking of alternatives under consideration [1], it was decided to allow the developed program to prioritise available options. As a result, the decision-maker can select the 'best' option based on comparing the corresponding combined 'aggregate' possibility distributions of their expected monetary and non-monetary performance. To develop the program, a number of programming languages were considered, however, the authors opted for a combination of the Visual Basic for Applications (VBA) and the widely used Microsoft Excel software. This combination proved to be adequate and user friendly. The program allows the user to perform the following tasks, which are presented diagrammatically in Fig. 1:

- *Identify the number of monetary factors applicable to the project under investigation*. The user can identify up to 150:
  Monetary factors (prior to operation) such as design, material, labour and construction costs;
  Monetary factors (annual payments during operation) such as revenue, loan repayment, and operation and maintenance costs; and
  Monetary factors (lump sum payments during operation) such as replacement costs.

- *Define the selected monetary factors as possibility distributions*. This step allows users to define the appropriate possibility distribution for each factor in dollar values. For the purpose of modelling, it is assumed that each factor will be entered into the model in any one of the following four forms see (Fig. 2):
  A single deterministic value (with 100% certainty; e.g. design cost is a lump sum of \$100,000);
  A closed interval (defined by an equally likely range; e.g. design cost is somewhere between \$80,000 and \$130,000);
  A triangular distribution (defined by a most likely value; e.g. design cost is about \$100,000, with a lower and upper least likely values of \$80,000 and \$130,000, respectively); and
  A trapezoidal distribution (defined by a most likely range; e.g. design cost is most likely in the range of \$100,000–\$120,000 with a lower and upper least likely values of \$80,000 and \$130,000, respectively).

- *Develop the resultant monetary distribution*. Applying the conventional time-cost-of-money principle, the NPV for all monetary factors can be calculated. To facilitate the arithmetic manipulation (addition and multiplication) of the possibility distributions, the vertex method [18] has been utilized (see Appendix A). Also, the following four assumptions were made:
  Monetary factors (prior to operation) take place in Year (0). That is, the length of the construction period is minimal compared to the period of investment (operation);
  Monetary factors (during operation) are of constant annual value (+ve cash in-flow and −ve cash out-flow) throughout the period of investment;
  Monetary factors (lump sum payments during operation) are discounted back to Year (0); and
  Cash flow discount and tax rates can be represented by any of the above four forms of possibility distribution.

Fig. 3 shows a hypothetical output of this stage; i.e. a resultant possibility distribution representing a hypothetical project's NPV. The NPV distribution is defined by a most likely range of \$86,000–\$162,000 and a lower and upper limit of \$60,000 and \$190,000, respectively.

- *Identify the number of non-monetary factors applicable to the project under investigation*. These non-monetary factors may include political, environmental, social, technological and strategic aspects of the project as well as that of the organisation.
- *Define the selected non-monetary factors as possibility distributions*. Due to the qualitative nature of these factors, users need to reflect the *level of*

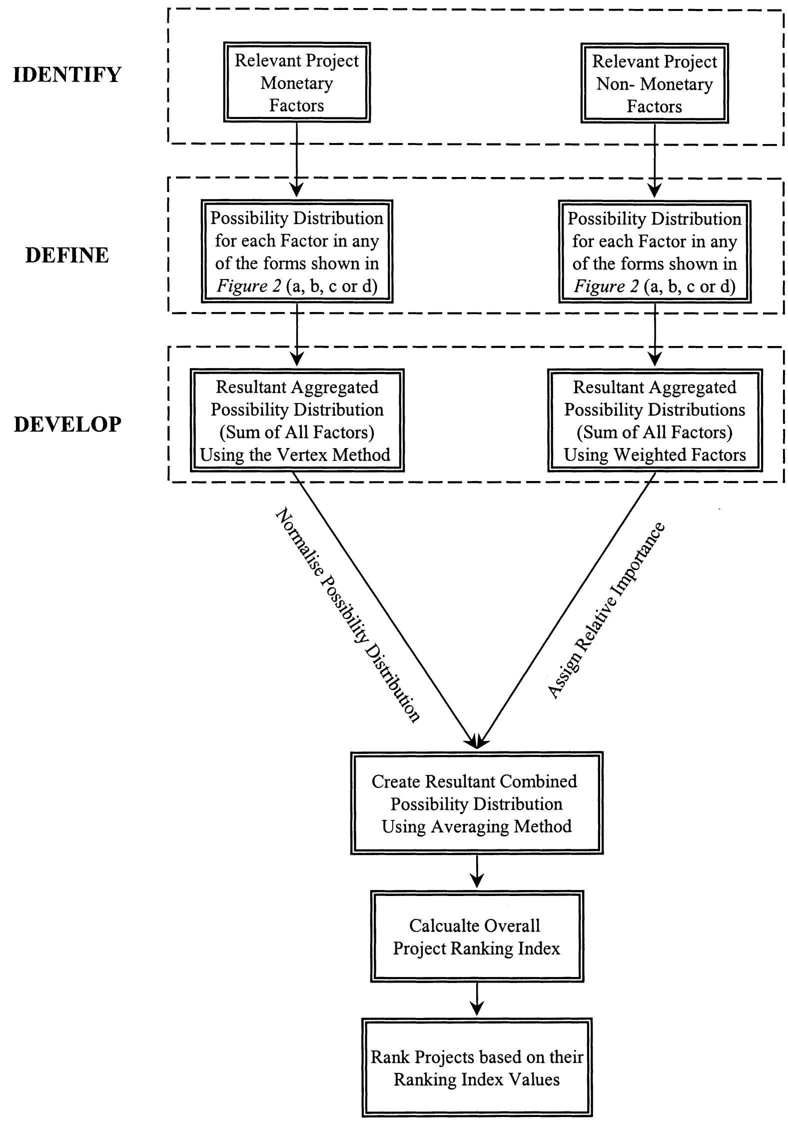

Fig. 1. The process of modelling project investment decisions.

satisfaction for each factor on an interval scale from 0 to 1, with 1.0 indicating a maximum positive project or organisation outcome for this particular factor. For example, if the project is 100% compatible with the organisation's strategy, a score of 1.0 may be used. Scores will be entered into the model in a similar manner as with monetary factors, the user can use any of the following distributions:

A single deterministic value (with 100% certainty; e.g. project compatibility with the organisation's strategy is 0.8);

A closed interval (defined by an equally likely range; e.g. project compatibility with the organisation's strategy is somewhere between 0.7 and 1.0);

A triangular distribution (defined by a most likely value; e.g. project compatibility with the organisation's strategy is about 0.8 with a lower and upper least likely values of 0.6 and 1.0, respectively); and

A trapezoidal distribution (defined by a most likely range; e.g. the most likely range for project compatibility with the organisation's strategy is 0.8–0.9 with a lower and upper least likely values of 0.6 and 1.0, respectively).

- *Develop the resultant non-monetary distribution*. As it is highly unlikely for selected non-monetary factors to have the same importance, weights of importance need to be utilized. The assessment of a weight for each factor is an important decision for the analyst to make in view of the investment opportunity at hand. The analyst must decide, which factors are most important, weights are then assigned according to the relative importance of

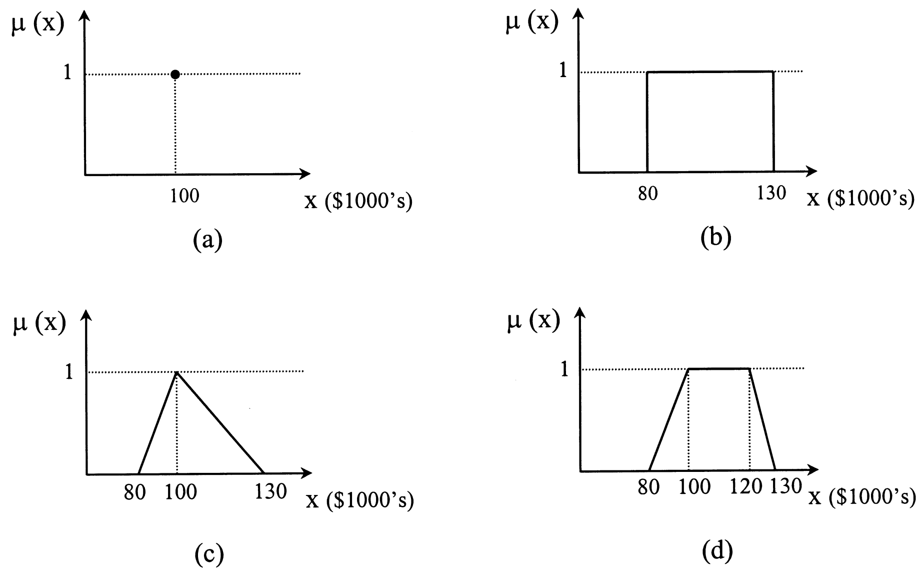

Fig. 2. Analyst's perception of design cost: (a) single value; (b) closed interval; (c) triangular distribution; (d) trapezoidal distribution.

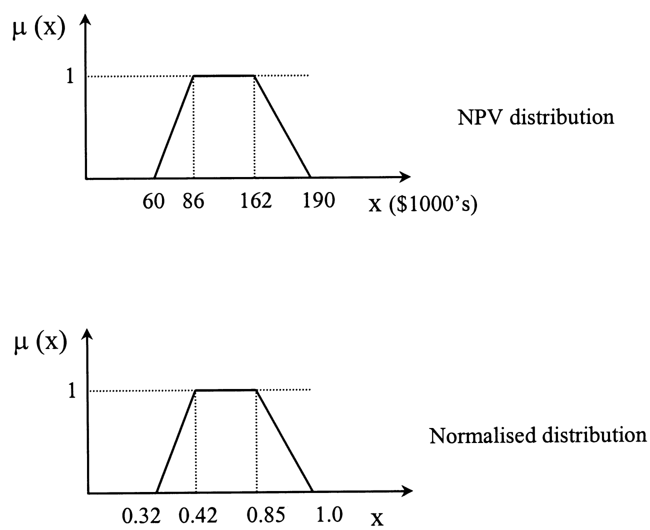

Fig. 3. Resultant NPV possibility and its normalised distributions.

factors. A recommended method for eliciting criteria weights is the analytic hierarchy process method, which is a hierarchical scaling method proposed by Saaty [19]. In the proposed method herein, the weight of importance varies between 0 and 1.0, with 0 indicating no importance and 1.0 indicating a very high importance. Each possibility distribution is multiplied by respective importance weights and the resultant project non-monetary distribution values are simply equal to the sum of the weighted factor values, divided by the sum of weighting values. This is otherwise known in its various forms as the fuzzy weighted averaging method [20–22] (see Appendix B). This technique seeks to find the 'average' of two or more possibility distributions by modifying their shape through fuzzy arithmetic.

- *Create the combined 'aggregate' project distribution.* In this step, both the monetary and non-monetary distributions are combined using the fuzzy weighted averaging method. Prior to combining both distributions, however, the user needs to assign a relative importance level among them. For example, if both distributions were of equal importance, then each distribution gets multiplied by 0.50. This step is crucial in highlighting the

contribution of the non-monetary factors to the overall attractiveness of the investment option. As both distributions (range values) differ in magnitude and unit, the monetary distribution values must first be modified. 'Normalisation' is a common modification process [12] involves dividing each value of the monetary distribution by the largest value so the range is between 0 and 1, (see Fig. 3). The normalised distribution is then combined to the non-monetary distribution to form one resultant 'aggregate' distribution representing the overall possibility distribution for the project, which may or may not resemble its predecessors [23]. This distribution, in turn, can be compared to other developed projects' distributions facilitating their ranking as explained in the following step.
- *Calculate the overall project ranking*. This step allows the decision-maker to select the most appropriate investment option. For each option, a ranking index is developed reflecting its overall possibility distribution. The ranking index method [24] (see Appendix C) was selected above other ranking procedures [20,25] to perform the final task of ranking the various project investment options.

The program output includes the resultant monetary and non-monetary distributions as well as the overall distribution for each investment option. Up to ten options could be ranked according to their overall possibility distributions with an output of a tabular summary of monetary, non-monetary and combined ranking index values of the projects.

4. Numerical application 1

Moselhi and Deb [1] presented a method for selecting a project under risk. Their method uses multi-objective decision criteria through the probability based multi-attribute utility theory and takes into account the uncertainties associated with each individual objective. In this section, the numerical example detailed in Ref. [1] is used to demonstrate the applicability of the possibility-based method proposed above by comparing its prediction with that of Ref. [1]. In this example, the selection decision was assumed to be based upon selected five factors. The single monetary factor (X_1) and four non-monetary factors ($X_2 - X_5$) are listed below in decreasing preference. Where in the context of this paper, non-monetary is taken to describe those factors unaffected by the time value of money.

X_1 net present value (NPV)
X_2 number of new jobs created
X_3 number of employees from minority groups
X_4 number of additional staff to owner's management team
X_5 prestige of the agency

The base data given in Table 1 is that of predicted cash flows for Projects *A*, *B* and *C* with each project having an economic life of 25 years. For utility matrix generation, Moselhi and Deb [1] used the formulae, given below, to utilise the cash flow data and then determine both the expected *E*(NPV) and associated standard deviation σ(NPV) for each project.

Table 1
Base data for Projects *A* and *B* (after Ref. [1])

Objective	Project	Period (years)	Minimum (optimistic)	Maximum (pessimistic)	Likely (most likely)
X_1 net cash flow \$(10)^6	*A*	0	−0.875	−1.15	−1.0
		1–25	0.46	0.35	0.40
	B	0	−1.7142	−2.285	−2.0
		1–25	0.9	0.60	0.70
	C	0	−2.25	−3.90	−3.00
		1–25	1.0431	0.727	0.9458
X_2 number of new jobs created	*A*		145	120	130
	B		175	145	160
	C		300	180	200
X_3 number of minority employees	*A*		18	10	15
	B		11	7	10
	C		22	15	18
X_4 number of new staff on team	*A*		2	6	4
	B		4	9	6
	C		7	10	8
X_5 prestige of agency	*A*		5.5	4.5	5.0
	B		7.5	6.0	7.0
	A		3.5	2.5	3.0

$$E(\mathrm{NPV}_i) = \sum_{t=0}^{n} \frac{E(C_{ti})}{(1+I_f)^t}$$

$$\sigma(\mathrm{NPV}_i) = \sum_{t=0}^{n} \frac{\sigma(C_{ti})}{(1+I_f)^t}$$

where $E(C_{ti})$ is the expected net cash flow for project i in period t, I_f is a risk-free interest rate, and n is the economic life.

The verification of the methodology presented in the previous section required cash flows to be represented by triangular distributions similar to that shown in Fig. 2(c). The reason for choosing triangular distributions is to simply utilise all given data in Ref. [1]. The relative importance of the sum total of the non-monetary factors was taken to be 0.60 to reflect the weighting of net cash flow assumed by Moselhi and Deb [1]. Additionally, the interest rate is assumed to be a risk-free, single rate of 9.0% and the least likely range of results was defined as being within three standard deviations of the mean. Table 2 contains a comparison of the NPV (monetary) results obtained by the proposed methodology to those gained by the utility method [1]. From this table, it is evident that the expected values of Projects *A*, *B* and *C* are in a good agreement (±4.0%).

4.1. Selecting the best option

As for the *E*(NPV), the mean and standard deviation of every variable is determined from the base data for optimistic, pessimistic and most likely values presented in the example. The standard deviation around the mean represents the level of risk or uncertainty associated with that variable. Base data for the variables X_2 to X_5 are presented in Table 1. The resulting characteristic values for each variable are then used as program input to determine the project expected utilities [1]. Once a full analysis of the entire data set has been completed, it was found that the combined project ranking index values were 0.73, 0.63 and 0.5 for Projects *C*, *B* and *A*, respectively. This is in complete agreement with the project ranking order given by the utility method.

Table 2
Summary of monetary results using possibility vs. the utility method

Project	Model	Expected value ($)	Deviation (%)
A	Possibility	2,929,000	−0.1
	Utility	2,933,000	
B	Possibility	4,876,000	−3.0
	Utility	5,040,000 [a]	
C	Possibility	6,292,000	+4.0
	Utility	6,058,000	

[a] The formulae for calculating *E*(NPV) given in Ref. [1] were used to verify the tabulated result for Project *B*. The above correctly calculated value actually differs from that presented in Ref. [1].

5. Numerical application 2

The objective of the developed program was to model and rank a number of investment options through the application of possibility theory. Its effectiveness at meeting this objective was tested using two projects of similar nature, which will be referred to as Project *A* and Project *B*. The two projects *A* and *B* had different concession (operation) periods of 30 and 24 years, respectively. Both projects were affected by different risk factors. Project *A* was surrounded by lower financial risk. Accordingly, the final monetary possibility distribution for Project *A* shows more certainty than Project *B*. It was to be entirely funded by a consortium of national banks and there was also little chance that a competitive facility would be built nearby due to the environmental sensitivity of the region. However, factors such as disturbance to the environment, especially the local tourism industry, and the risk in adopting an innovative construction method had to be considered. Cash flow discount rate was assumed to be in the range of 6.0–8.0% (closed interval, Fig. 2(b)). Tables 3 and 4 show a summary of monetary and non-monetary factors considered for Project *A*. It should be noted that the net annual revenue accounts for both gross revenue and costs.

Project *B* would be located in another country with relatively unstable political environment. Additionally, the promoting organisation will be required to form a contractual arrangement with the host country. The

Table 3
Project *A* — monetary input and output

Monetary factor	Year	Defining possibility distribution values			
		a	*b*	*c*	*d*
Discount rate (%)	1–30	6.0	6.0	8.0	8.0
Estimated construction cost (M$)	0	−260	−250	−250	−240
Net annual revenue (M$)	1–30	40	42	42	44
Resulting NPV (M$)		60.6	86.4	161.2	190.6
Normalised NPV value		0.32	0.45	0.85	1.00

Table 4
Project *A* — non-monetary input and output

Non-monetary factor	Weighing	Defining possibility distribution values			
		a	*b*	*c*	*d*
Political	0.40	0.90	0.95	0.95	1.00
Environmental	0.75	0.50	6.60	0.70	0.75
Social	0.85	0.80	0.80	0.95	0.95
Technological	0.80	0.25	0.50	0.50	0.70
Financial	0.50	0.40	0.40	0.60	0.60
Non-monetary distribution value		0.55	0.64	0.73	0.80

Table 5
Project *B* — monetary input and output

Monetary factor	Year	Defining possibility distribution values			
		a	*b*	*c*	*d*
Discount rate (%)	1–24	8.0	9.0	9.0	10.0
Estimated construction cost (M$)	0	−300	−280	−280	−250
Net annual revenue (M$)	1–24	51	51	53	53
Resulting NPV (M$)		9.13	54.9	67.5	133.8
Normalised NPV value		0.07	0.41	0.50	1.00

Table 6
Project *B* — Non-monetary input and output

Non-monetary factor	Weighing	Defining possibility distribution values			
		a	*b*	*c*	*d*
Political	1.00	0.20	0.50	0.50	0.80
Environmental	0.40	0.80	0.80	0.90	0.90
Organisational	0.85	0.40	0.50	0.50	0.60
Competition	0.80	0.50	0.50	0.70	0.70
Market Share	0.90	0.80	0.90	0.95	1.00
Financial	0.90	0.70	0.80	0.70	0.80
Non-monetary distribution value		0.53	0.63	0.71	0.79

difference in political uncertainty is reflected in Tables 4 and 6 whereby Project *B* assigns a higher weighting and uncertainty to the political risk factor. Other risks to the project include; working with local contractors, the effects of high inflation and also the high possibility of competitive facilities being constructed. However, the project could potentially reap great benefits for the promoting organisation, in the form of increasing future market share in this particular country. Cash flow discount rate was assumed to be in the range of 8.0–10.0% with 9% is the most likely value (triangular distribution, Fig. 2(c)). Tables 5 and 6 show a summary of monetary and non-monetary factors considered for Project *B*. For both projects, a tax rate of 36.0% was assumed and a relative importance of non-monetary factors was set at 0.35, implying a 0.65 contribution of monetary factors.

The computed overall combined project distributions for Project *A* and *B* were [0.41, 0.52, 0.81, 0.93] and [0.22, 0.41, 0.48, 0.93], respectively. Fig. 4 contains a representative diagram of the two project's overall possibility distributions. The ranking index for Project *A* was higher than its counterpart for *B* (see calculations in Appendix C), therefore, Project *A* represents the better investment option.

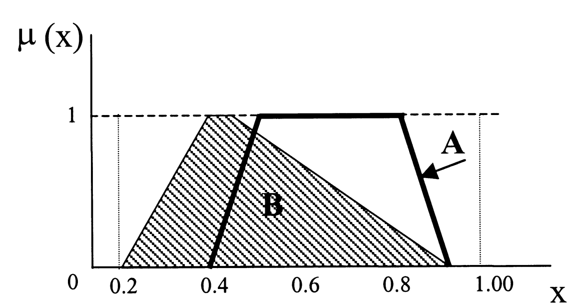

Fig. 4. The overall 'combined' possibility distributions for Projects *A* and *B*.

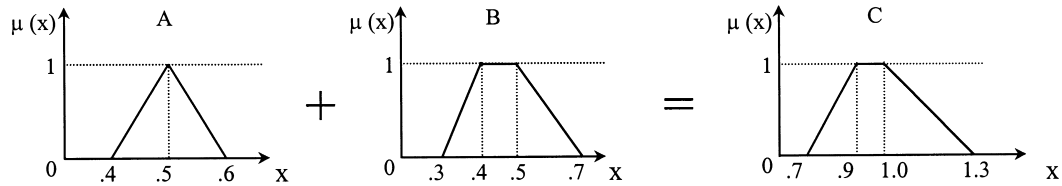

Fig. 5. The use of the vertex method.

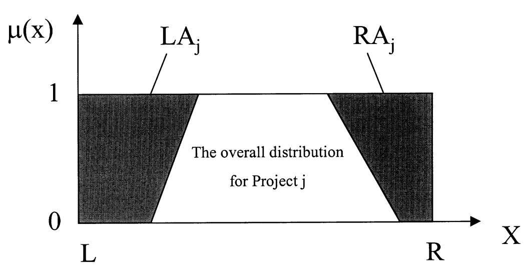

Fig. 6. The area used in the ranking index is represented by shaded area.

6. Conclusion

The use of interval mathematics and possibility distribution is a departure from conventional probability-based techniques which rely rather heavily on randomness and frequency to quantify the effects of risks on projects. This paper presented a methodology capable of modelling the effects of both monetary and non-monetary factors on a construction project, through the use of possibility distributions. The proposed methodology has provided an accurate and convenient method for comparing different project alternatives. Possibility theory appears to offer an even less calculative intensive method whilst still providing accurate and transparent results. The developed program is designed to conduct an analysis of each investment option and to provide an overall ranking of available investment options. Two numerical examples were presented; the first as a comparison to the probability-based utility method [1], and the second to demonstrate the successful application of the program to rank two construction projects.

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Appendix A. The vertex method

This method makes use of the α-cut representations of fuzzy sets. It is an approximate computational technique but is highly efficient compared with the exact method of non-linear programming, with an accuracy that is much better than the conventional discretisation approach [18]. Suppose y is a function of n variables; i.e. $y = f(x_1, x_2, \ldots, x_n)$ and each x_i, $i = 1, \ldots, n$ is an interval variable represented by $X_i = [a_i, b_i]$. Assuming that y is continuous in the n-dimensional rectangular region with 2^n vertices, then the value of interval function Y can be obtained by:

$$Y = f(X_1, X_2, \ldots, X_n)$$

$$Y = \left[\min_j(fc_j), \max_j(fc_j))\right], \quad j = 1, \ldots, 2^n$$

where c_j is the ordinate of the jth vertex.

The algorithm consists of the following steps:

1. Select an α value where $0 < \alpha < 1$.
2. Find the interval(s) in X and Y which correspond to this α, these intervals are known as the α-cuts.
3. Using the binary algebraic operations on intervals, compute the interval(s) of $f(x)$ corresponding to those of X and Y.
4. Repeat the above steps for different values of α to complete an α-cut representation of the solution. Processing more α-cuts, however, increases the computational requirements.

Example. A and B are a triangular (0.4, 0.5, 0.6) and trapezoidal (0.3, 0.4, 0.5, 0.7) possibility distributions, respectively. If $C = A + B$, an approximate calculation of C is as follows:

1. Take an α-cut at 0.0, $A_{0.0} = [0.4, 0.6]$ and $B_{0.0} = [0.3, 0.7]$, thus $C_{0.0} = [0.7, 1.3]$
2. Take an α-cut at 1.0, $A_{1.0} = [0.5]$ and $B_{1.0} = [0.4, 0.5]$, thus $C_{1.0} = [0.9, 1.0]$
3. The resulting distribution [0.7, 0.9, 1.0, 1.3] is shown in Fig. 5.

Appendix B. The averaging method

Fuzzy averaging is the aggregation of opinions (given as fuzzy numbers), regarding the uncertainty associated with the various criteria, in order to obtain an overall picture or conclusion about the situation. The fuzzy average [22], V_i, is given by:

$$V_i = \frac{\sum_{j=1}^{J} w_j \times p_{ij}}{\sum_{j=1}^{J} w_j}, \quad i = 1, \ldots, I$$

where V_i is the fuzzy aggregate assessment of the project, w_j is the weight given to the criterion j and p_{ij} is the characteristic value associated with each criterion.

Example. Supposing Factor A was defined by a trapezoidal distribution [0.6, 0.7, 0.8, 0.9] and it had a relative importance of 0.4 compared to Factor X [0.4, 0.5, 0.55, 0.7], also a trapezoidal distribution. The resultant combined distribution is:

$$=0.4(A) + 0.6(X) = [0.24,\ 0.28, 0.32, 0.36] + [0.24,\ 0.3, 0.33, 0.42] = [(0.24 + 0.24),\ (0.28 + 0.3), (0.32 + 0.33),\ (0.36 + 0.42)] = [0.48,\ 0.58, 0.65, 0.78]$$

Appendix C. The ranking index model

An index for ranking fuzzy numbers that is suitable for economic analysis has been proposed [20]. This index is based on the difference of area of a rectangle and the area under the possibility distribution of each alternative. The following equation for this ranking index is given by Choobineh and Behrens [6].

$$K_j = 0.5\left[1 - \frac{RA_j - LA_j}{R - L}\right]$$

The R and L correspond to the maximum and minimum of the domain of the utility function of the decision-maker, respectively. R, L, RA_j and LA_j are shown in Fig. 6. The value of the ranking is bounded by zero and one.

Example. The two projects A and B covered in Section 5, need to be ranked. The overall distribution is A [0.40, 0.52, 0.81, 0.93] and B [0.22, 0.41, 0.48, 0.93], as shown in Fig. 4. Using the Ranking Index Model:take a range of $L = 0.2$ and $R = 1.0$

Project A: $RA_A = 0.13,\ \ LA_A = 0.27,\ \ K_A = 0.59$
Project B: $RA_B = 0.13,\ \ LA_B = 0.27,\ \ K_B = 0.39$

Therefore, $K_A > K_B$, and project A dominates project B.

References

[1] Moselhi O, Deb B. Project selection considering risk. Construction Management and Economics 1993;11:45–52.
[2] Levy SM. Build, operate, transfer: paving the way for tomorrow's infrastructure. Canada: Wiley, 1996.
[3] Lopez MDS, Flavell R. Project appraisal — a framework to assess non-financial aspects of projects during the project life cycle. Int J of Project Manage 1998;16(4):223–33.
[4] Tweedale HM. Maximising the usefulness of risk assessment. In: Melchers, RE, Stewart, MG editors. Probabilistic risk and hazard assessment. Balkema: Rotterdam, 1993, p. 1–11.
[5] Toakley AR. Risk analysis of project development portfolios: a review of research needs. Australian Institute of Building Papers 1997;8:135–43.
[6] Choobineh F, Behrens A. Use of intervals and possibility distribution in economic analysis. J Operations Res Society 1992;43(9):907–18.
[7] Behrens A, Choobineh F. Can economic uncertainty always be described by randomness. Proceedings of the IEE Conference, Toronto:116–20.
[8] Williams TM. Risk management infrastructure. Int J of Project Manage 1993;11(1):5–10.
[9] Gregory G. Decision analysis. UK: Pitman, 1988.
[10] Mak SW. Risk analysis in construction: a paradigm shift from a hard to soft approach. Construction Manage and Economics 1995;13:385–92.
[11] Andersson L. The theory of possibility and fuzzy sets: new ideas for risk analysis and decision making. Document D8: Swedish Council for Building Research, Stockholm, Sweden, 1988.
[12] Schmucker KJ. Fuzzy sets, natural language computations, and risk analysis. Rockville, USA: Computer Science Press, 1984.
[13] Chisaki T, Tatish M, Tatsumi H. Project scheduling under fuzziness, FPERT. Proceedings of the 3rd Int. Conf. on Modern Techniques on Construction, Engineering and Project Management Singapore, 24–26 March.
[14] Lorterapong P, Moselhi O. Project-network analysis using fuzzy set theory. J of Construction Engng and Manage 1996;122(4):308–18.
[15] Wong KC, So ATP. A fuzzy expert system for contract decision making. Construction Manage and Economics 1995;13:95–103.
[16] Wang W, Hawash KIM, Perry JG. Contract type selection (CTS): a KBS for training young engineers. Int J of Project Manage 1996;14(2):95–102.
[17] Tam CM, Fung I. Assessing safety performance by fuzzy reasoning. Asia Pacific Building and Construction Manage J 1996;2(1):6–13.
[18] Dong W, Chiang W, Wong F. Propagation of uncertainties in deterministic models. Computers and Structures 1987;26(3):415–23.
[19] Saaty T. The analytic hierarchy process. New York: McGraw Hill, 1980.
[20] Smith PN. Multicriterion project evaluation involving uncertainty and imprecision. Transactions of Multi-Disciplinary Engineering 1995;GE19(2):43–53.
[21] Dong WM, Wong FS. Fuzzy weighted averages and implementation of the extension principle. Fuzzy Sets and Systems 1987;21:183–99.
[22] Bojadziev G, Bojadziev M. Fuzzy logic for business, finance, and management. London: World Scientific, 1996.
[23] Kaufmann A, Gupta MM. Introduction to fuzzy arithmetic: theory and applications. New York: Van Nostrand Reinhold, 1985.
[24] Choobineh F, Li H. An index for ordering fuzzy numbers. Fuzzy Sets and Systems 1993;54(3):287–94.
[25] Tseng XY, Klein CM. New algorithm for the ranking procedure in fuzzy decision making. IEEE Transactions on Systems, Man and Cybernetics 1989;19(5).

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