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REAL OPTIONS AND GAME THEORETIC VALUATION, FINANCING AND TENDERING FOR INVESTMENTS ON BUILD-OPERATE-TRANSFER PROJECTS

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THESIS

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ABSTRACT

Privatization has been recognized as an approach to solving the problem for governments in funding public works. The major technique applied in infrastructure privatization is non-recourse project financing, including the BOT (Build-Operate-Transfer) approach, a major scheme in infrastructure privatization practice. BOT is the marriage of public and private sector interests and is a win-win solution if successfully implemented. A BOT project has to demonstrate its financial and technical viability before it is undertaken. While it is relatively easy to demonstrate a project's technical viability, to evaluate a BOT project's financial viability is complex and challenging, mainly because of the uncertainties involved due to the scale, long concession period, and complexity of a BOT project. The problem of evaluating financial viability is further complicated by the fact that different parties in a BOT project have different perspectives. The developer will focus not only on the project profitability and cost, but also on the investment's side benefit, such as construction contract profit. For a government, since it is politically costly if a BOT project fails, the government's major concern falls on the project profitability and cost only. The two different perspectives may induce complicated interactions between the developer and government during the tendering or project procurement process.

Built upon modern option pricing theory and game theory, this thesis presents a quantitative BOT model that can dynamically evaluate the value of a BOT investment from the perspectives of the developer and government, and determine the BOT investment's developing decisions. By incorporating with signaling game analysis, the BOT model examines the BOT bidding strategy, valid signals of a developer's commitments and the effectiveness of the government BOT policies. Most importantly, the model considers the characteristics of a BOT project and analyzes the BOT problems more realistically. The model may provide the potential major BOT developers/contractors and other relevant participants a new perspective in the decision making process.

To My Dear Parents

and

My Beloved Brother, Shih-Ann, Who Now Rests in Heaven

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CHAPTER 1 INTRODUCTION

1.1 Background

Since the 1970s, governments have faced increasing difficulties in funding public works (Walker and Smith, 1995). Privatization has been recognized as an important approach to solving the funding problem (Augenblick and Custer, 1990; Walker and Smith, 1995). Government is no longer seen as the sole provider of public works. Instead, the role of government has evolved into one of partnership, coordinating the use of resources and providing infrastructure systems along with the private sector, which has brought vast funding and technology into the process. According to a World Bank report by Roger (1999), from 1990 to 1998, the private participation in infrastructure projects in the developing countries grew dramatically from about $16 billion in 1990 to $120 billion in 1997 and $95 billion in 1998.

The major technique applied in infrastructure privatization is "non-recourse project financing." Project financing can be defined as:
"*the raising of funds to finance an economically separable capital investment project in which the providers of the funds look primarily to the cash flow from the project as the source of funds to service their loans and provide the return of and a return on their equity invested in the project*" (Finnerty, 1996).

"Non-recourse project financing" deprives the lender's right to claim the developer's assets as debt compensation should a project fail. In other words, the developer is not required to provide any collateral. The only collateral to the debt holder is the assets obtained in a BOT project. In infrastructure privatization, the BOT (Build-Operate-Transfer) approach is one of the major schemes in practice. The BOT concession granted by the host government empowers the right to finance and build the project and to operate the project for a certain period of time. By the end of the concession period, the concessionaire transfers the project back to the host government at no cost or with limited compensation. Other variants may include BOO (Build-Operate-Own), which authorizes the concessionaire to operate the entity for an infinite period of time, or ROT (Rehabilitation-Operate-Transfer), which starts from rehabilitation instead of construction. From the viewpoint of a contractor, the BOT approach enhances the effectiveness and efficiency through pre-construction communication and close relations among the design, consultant, and construction groups. This feature of BOT approach favors the control of a project's completion risks, such as delay or cost overrun.

In the BOT approach, a concession agreement is established and a new "owning company" is formed to manage a single project and assume all responsibility for the project's success or failure. The new owning company can be called the "BOT firm," which is independent from the developer. In other words, the developer is not liable for the repayment of any bank loans borrowed by the BOT firm. Typically, in forming the owning company, the developer will become the initial equity holders and the lending banks will become the major debt holders. The capital needed for the project will be provided by the bank loan and equity. The creditors/lenders could include commercial banks, pension funds, and investment trusts, which are in the business of lending to such projects and willing to bear only low or moderate risks. The promoter/developer is considered as the major participant in the infrastructure privatization process. More often, they are the equity investors (shareholders) as well as the managers of the project. However, the promoters/ developers may not be the only shareholders; there could be other institutional investors (passive investors) interested in investing as shareholders and taking higher risk for higher return but not involved in the development or management. The equity hold by the investors who do not participate in the development or management of the project can be called as "passive equity." In many large-scale projects, the major part of the passive equity is publicly placed, that is, publicly sold in the stock market.

In the early stages of a BOT project, the major promoters/developers usually include the contractors, who may specialize in certain infrastructure construction techniques and/or be active in the construction industry nationally or internationally with abundant capital resources. These leading contractors inject the preliminary or most of the equity for the project and then seek other investors to invest in equity while negotiating with other lenders for debt financing. Typical equity ratio, according to the World Bank, is about 10% to 30% of the total project cost (Augenblick and Custer, 1990). The relationship among different parties participating in a BOT project is shown in Fig. 1.1.

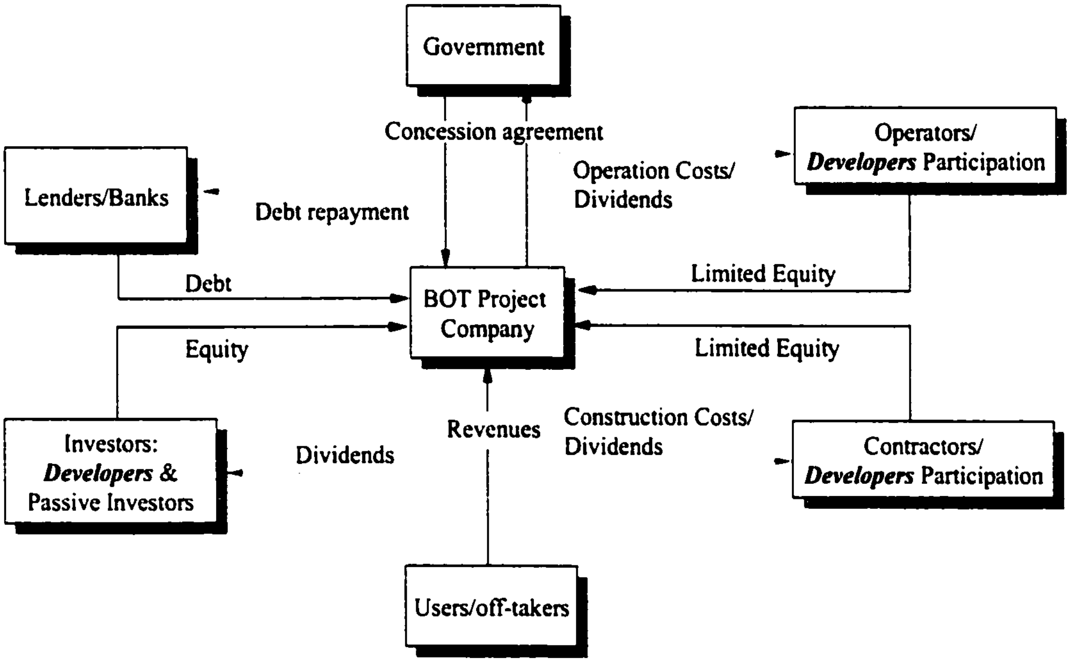

Fig. 1. 1 BOT's Relationship of Participants and Cash Flows

1.2 Motivation and Objectives

Recent research of BOT relies heavily on surveys. For example, Tiong et al. (1992, 1997) and Tiong (1996) identified critical success factors of winning BOT contracts from the survey results. Tiong (1995a, 1996) presented the importance of financial package and equity level. Using survey results, Tiong (1995c) further explained the roles of risks and debt guarantees in BOT tender or project procurement. Case study is another major methodology applied in BOT research. Shen et al. (1996) and Zhang et al. (1998) studied some applications of BOT approach in China and discussed the risk management of a BOT project. These efforts have provided a basic understanding of BOT projects on the basis of empirical studies. These empirical studies, although useful, lack a theoretical foundation to provide more insights into the intricacies/complexity of BOT projects.

1.2.1 BOT Investment Valuation

In a BOT investment, the overall profit of the investment may be the most important factor and has the deepest impact on the financial contents of a proposal. Because of the size, complexity, and uncertainty of BOT projects, it is relatively difficult to determine the value of a BOT investment. Different evaluation results of a project will have great impacts on both the promoters' and government's appraisals of the project's viability. For the developer, underestimating the investment value can make the financial package unattractive and lower the probability of winning the contract. The developer may then forgo a lucrative project and lose all prior investment (possibly millions of dollars) in preparing the bid. On the other hand, overestimating the investment value may seem to make the financial package attractive but will create many difficulties in either completing and/or operating the project, to an extent that the BOT firm may default or need the government's rescue. From the government's perspective, it is well recognized that the developer, as a bidder in the competitive bidding scheme, might strategically overestimate the project value and expect to benefit from re-negotiation after the project is awarded. In the case of a project default, the failure can be politically catastrophic to the government. Even if the default does not take place, to renegotiate with the developer or rescue a project when the developer encounters difficulties is costly from the monetary and political standpoint. Therefore, governments must be very cautious in evaluating BOT project proposals.

From the economic and corporate finance literature, traditional capital budgeting methods for evaluating an investment tend to underestimate the investment's value and overlook the investment's implementation flexibility (Mason and Merton, 1985; Trigeorgis, 1991; Trigeorgis and Mason, 1987; Myers and Majd, 1990; Dixit and Pindyck, 1994). There is a need to develop a better model to assess the investment in BOT projects, a model that is based on solid theory and encompasses relevant characteristics of BOT projects.

1.2.2 BOT Financing Strategies and Government Tendering Policies

In most cases, a BOT project's main equity investors are the promoting contractors. Contractors play one of the most important roles in a BOT project. Contractors are not allowed to transfer the equity ownership until the project has at least been completed and shown to be attractive (Walker and Smith, 1995). One reason is that the government and lenders treat the equity investment from contractors as a commitment or "signal" of good faith to complete the project. However, contractors bear extra risk beyond completion risk by equity investment and lose the flexibility of utilizing that capital for other new contracts, although the expected return from equity may be significantly higher than normal projects. It seems that those contractors who look to profit from a successful BOT project are willing to invest a level of equity "high" enough to convince the government and the lenders regarding the project viability. However, most contractors are thinly capitalized and highly leveraged and prefer not to have large amounts of their capital tied up for many years as equity investments (Walker and Smith, 1995). In economic and corporate finance literature, there is no theoretical framework to model the optimality of capital structure and government's equity level policy of a BOT project. It is important to derive a model that can help the developers select optimal capital structure and also help the government establish effective BOT policy concerning capital structures. We argue that without an adequate/accurate analysis on the optimality of capital structure and government policy, project participants may risk making improper decisions on equity levels that may jeopardize the project's development.

1.2.3 BOT Bidding Strategy and Government Tendering Policies

For most current BOT projects, project merits, not low bid, are used in selecting a party to undertake the BOT project. These merits may include cost, schedule, and proposal changing schemes, among others. Therefore, it is not uncommon for a contractor to earn merits by bidding low and making optimistic assumptions, such as high usage, low tolls, low construction cost, and short durations. By doing so, the bidder will have more chance to win the "admission ticket to the negotiation table" afterwards. After winning, the bidders may count on re-negotiation should a project do not go as well as originally assumed.

This kind of bidding scheme places government in direct conflict with the developers/promoters. Therefore, for government, a good theoretical framework can be used to develop policies that will enhance the quality of the procurement and reduce the hidden costs and risks. For the promoters, a theoretical bidding model can help them understand government's concerns, and therefore strengthen the proposal's competitive advantages and improve the probability of winning a project by applying appropriate strategies.

1.2.4 Need for a Theoretical Framework

As argued above, a theoretical foundation is needed to evaluate BOT projects objectively and scientifically. As Paul Samuelson (1958), who won the Nobel Prize in Economic Science, stated in his book:

"*The economic world is extremely complicated... Even if we had more and better data, it would still be necessary--as in every science--to simplify, to abstract from the infinite mass of detail... It is always necessary to idealize, to omit detail, to set up simple hypotheses and patterns by which the mass of facts are to be related, to set up the right questions before we go out looking at the world as it is... But if it is a good theory, what is omitted is greatly outweighed by the beam of illumination and understanding that is thrown over the diverse empirical data.*"

1.3 Significance and Contributions

The results of this thesis research contribute to

1. providing a theoretical framework for analyzing BOT projects and their related financial issues,
2. incorporating modern financial theory with game theory in a unified framework, which considers the characteristics of BOT projects,
3. helping the developers make better development decisions regarding BOT investment and bidding strategies, and
4. assisting the government to establish effective BOT tendering or procurement policies.

1.4 Organization of Thesis

Chapter 1 introduces the background and foundation of the BOT scheme, and identifies the problems, research objectives, significance, and contributions of this research. Chapter 2, Literature Review, reviews and summarizes existing research regarding BOT projects and introduces the building blocks for deriving the BOT model, namely, real options theory and game theory. Chapter 3 describes the research methodology. Chapter 4 investigates the nature of BOT investment problems. Basic assumptions regarding BOT projects that are critical in the derivation of the analytic model are formed. Chapter 5 applies the game theory to analyze the decision-making process in the BOT projects, and sets the foundations for the integration with the real options framework. In Chapter 6, a quantitative framework based on the real options theory and game theory is derived for the purpose of evaluating the BOT projects. Chapter 7 focuses on the information asymmetry problem and analyzes BOT government tendering or procurement policy on the basis of signaling game analysis. Chapter 8 is the case study on the Channel Tunnel project, one the world's biggest BOT projects. The case study analyzes the Channel Tunnel project, demonstrates how the BOT model derived in this research can be applied to this project, and verifies the BOT model derived. The verification relies on how well the model can analyze the Channel Tunnel project and whether our model can provide better solutions. Chapter 9 gives conclusions of this research and recommendations for future research.

CHAPTER 2 LITERATURE REVIEW

Relevant literature was reviewed and summarized in various sections in this chapter. Section 2.1 reviews the existing research on BOT projects. Section 2.2 explains the traditional capital budgeting theories. Section 2.3 introduces game theory and its applications. Section 2.4 presents the basic concepts of dynamic asset valuation, namely, the option pricing theory and real options theory. The numerical methods for solving the option pricing problems are introduced in section 2.5. Section 2.6 reviews several capital structure theories for financing a project/firm.

2.1 The Studies of BOT Project Financing and Tendering

Research on BOT capital structure/financing is limited. Most discussions are based on the experience and opinions of financial advisors or promoters, such as the empirical studies by Tiong and his colleagues, the book by Walker and Smith (1995), the report by Augenblick and Custer (1990), and UNIDO BOT Guidelines (1996) by the United Nations.

2.1.1 Survey and Case Study of Tiong and His Colleagues

Tiong et al. have conducted several surveys and case studies on BOT projects. Tiong's survey (1995a) regarding the competitive advantage of equity in BOT tender showed that high level of equity is necessary if competition is keen and financing for the project is uncertain. Typically, 20%-30% equity is considered to be high. This survey indicated that most of practitioners did not believe that as a higher level of equity is raised, the likelihood of winning the BOT concession is increased. This survey also showed that there may be a *minimum equity investment requirement* for winning a project, and the additional equity investment may *not* be a selection issue. Case study indicated that for the projects with relatively certain cash flows, the equity investment can be very low. Tiong's survey (1995c) concluded that the ability to provide an attractive financial package is critical when the project is technically certain, the competition is keen, and the project viability and financing are uncertain. Tiong (1995b) also concluded that the developers' ability to retain risks and offer guarantees does provide the competitive advantage in being awarded the project.

One major difficulty in BOT research is the differences between countries. Survey results may not be applicable from country to country. For example, the rate of inflation may be a crucial consideration in a country with an unstable economy, but less important in another country with a stable economy. Nevertheless, the survey results can provide useful insights about the BOT project and its capital structure. One insight is that the contractors do not want to invest equity much above the necessary or required amount. Obviously, opinions and surveys cannot explain the nature of problems or justify the optimality of a decision under different situations.

2.1.2 Dias and Ioannou's Theoretical Model

A recent theoretical study regarding a BOT project's capital structure was conducted by Dias and Ioannou (1995a, 1995b). Their model is a static trade-off model based on Modigliani and Miller's (1958) capital structure theory. However, this research followed the framework of Kim (1978) too closely and had similar results. Dias and Ioannou (1995b) extended the analysis to the valuation of government guarantees on the BOT project. One limitation of their framework is that it failed to consider the characteristics of a BOT project during the derivation of the optimal capital structure. Note that Kim (1978) and Dias and Ioannou (1995a) assumed that the objective of a firm is to maximize the firm's value. However, in BOT, the conventional assumption regarding the firm's objective may not be appropriate. It may be more appropriate to assume that the developer's objective is to maximize the joint profit from the construction contracts and equity investment. In fact, it is commonly recognized that the major interest of most developers toward a BOT project is the undertaking of construction contracts. In many cases, maximizing the profit from construction contracts is in conflict with maximizing equity investment/BOT firm value. Other important characteristics of the BOT projects include the completion risks because of the uncertainties of construction costs, and the participation of the host government in the project development. A model that fails to examine these characteristics will make the framework less practical and lose the clarity in understanding the true nature of BOT capital structure problems. A theoretical framework incorporating both the financial/economic theories and the characteristics of BOT projects will provide valuable insights to developers and the government in a BOT venture.

2.2 Conventional Capital Budgeting Theories

Capital budgeting is concerned with the selection of long-term investment projects. Capital budgeting decisions made under certainty are straightforward and easy, but few projects have certain outcomes. Therefore, the major concern in capital budgeting theories is how to treat the uncertainty and risk in the project/proposal evaluation. To a degree, Aggarwal (1993) argued that many problems will contribute to making the practice of capital budgeting "*closer to an art and farther from a science than is desirable.*" Part of the purpose of this research is to make capital budgeting in BOT closer to a science.

2.2.1 Net Present Value (NPV) Approach

In financial theory, it is typically assumed that a firm's objective is to maximize the market value of the firm's stock, and this objective can be proved to be consistent with maximizing each shareholder's utility regardless of the uniqueness of each shareholder's utility function. Net present value (NPV) approach is recognized as the only evaluation method that is consistent with the firm's assumed objective and is considered to be superior to other measures, such as internal rate of return (IRR), accounting rate of return (ARR), and payback period. For readers unfamiliar with these approaches, Copeland and Weston (1988) and Aggarwal (1993) have detailed introductions and discussions on traditional capital budgeting methods. Students of construction management should have a basic knowledge of NPV, IRR, and ARR. IRR and NPV are the only methods that consider the time value of cash flows and therefore are considered to be better approaches in capital budgeting. NPV is the net amount of the present value of cash inflows minus the present value of cash outflows. A firm should always undertake a project with positive and higher NPV among available alternatives. NPV can also be categorized as the discounted cash flow (DCF) method, since DCF also discounts cash flows.

NPV is perfect when the future cash flows are certain and there is no managerial flexibility in the investment. However, in the real world it is rare that the investment's cash flows are certain. When uncertainty is involved, the investment is considered to be "*risky*." The first step is to take the *expectation* of the uncertain cash flows. Once risk comes into play in investment, the problem is complicated by the investor's risk attitude. The second step is to adjust for risk attitude by using the "*risk-adjusted discount rate*" (RADR) to discount the expected cash flows. The search for the *good* RADR has been an important but difficult issue in financial research. In practice, people sometimes use either the firm's average cost of capital or opportunity cost of capital as the discount rate if the project has the same risk class as the firm's. Many firms, rather than determining a unique RADR, classify different *risk categories* of projects and assign each category different discount rate to reflect the risk involved (Trigeorgis, 1996). For more accuracy, one could use different discount rates in different periods in order to reflect the change of nominal rates of interest. However, Aggarwal (1993) argued that the much lower degree of accuracy of the estimated cash flows in capital budgeting overwhelms the need of accuracy in estimating the change of interest rates in different periods.

There is one critical problem with NPV that is related to some implicit assumptions in typical capital budgeting when NPV is applied in the construction industry. Typical capital budgeting assumes that the cash outflow is certain and occurs at the very beginning of a project (e.g., the purchase of special equipment). Even when there are cash outflows in different time periods other than time 0, they are implicitly assumed to have the same risk characteristics as the cash inflows. Typical capital budgeting focuses on the riskiness of the future cash inflows when RADR is used to address the uncertainty. The result is that the higher the risk, the higher the RADR. The high RADR will subsequently reduce the PV of the cash inflows reflecting the uncertainty/risk involved. However, in a construction project, the future cash inflows are considered to be certain, since they are paid according to the bid and construction contracts. On the other hand, the uncertainty mainly comes from the cost or cash outflows. The first problem of applying NPV is deciding what discount rate should be used for the certain cash inflows. The second problem is that if the cost cash outflows are discounted by a high RADR that reflects the riskiness of cost, then it would produce a very unreasonable conclusion that higher risk would lower the PV of the cost and subsequently raise the NPV of the project. These problems will be addressed in this research.

Even when NPV is adjusted for the application on construction projects, the NPV method loses its attractiveness when the project has managerial flexibility. Trigeorgis (1996) argued that the basic inadequacy of the NPV is that NPV ignores or cannot properly capture "*management's flexibility to adapt or revise later decisions when, as uncertainty is resolved, future events turn out differently from what management expected at the outset.*" Mason and Merton (1988) pointed out that the cumulative error of ignoring *all* the operating/managerial options embedded in a project can cause a significant underestimate of its value. Especially for the evaluation of *long-horizon* projects in which future profitability can only be imprecisely estimated, it is critical to consider the associated managerial or strategic options. As a result, the option pricing or real options approach is suggested by many researchers because of its ability to incorporate these strategic options into the project valuation. It is worth noting that Trigeorgis (1996) called this new value obtained by option pricing approach "*expanded/strategic NPV*," which implies that Contingent claims analysis (CCA) does not abandon NPV. Instead, it incorporates new features into NPV and uses NPV as the initial and primary input as well. The NPV from traditional approach is then called "*static/passive NPV.*"

2.2.2 Internal Rate of Return (IRR) Approach

IRR is another popular capital budgeting approach. One major reason is because of its intuitiveness regarding return rate, although this is in fact misleading. IRR is defined as "the rate of return on project investment reflected in its set of future cash inflows" (Aggarwal, 1993). One can easily compute the IRR by searching the discount rate that makes the present value of all cash inflows equal to the present value of all cash outflows. Software such as Excel can easily do the computation. If IRR is higher than a preset hurdle rate, then the project is considered acceptable. This approach may seem to eliminate the problem of deciding the discount rate required in NPV; however, it creates the new problem of deciding the hurdle rate, which is used as a standard to decide if the IRR is high enough for acceptance. The most serious criticism of IRR is that the calculation of IRR implicitly assumes that all project cash inflows are *reinvested* in other projects at a rate of return that equals IRR. Due to this assumption, when a project's IRR is higher than a firm's normal rate of return, it will be difficult to justify how the reinvestment could have the same *high* rate of return. For a long-term project, which is typical in capital budgeting, the problem will be even more serious. NPV on the other hand implicitly assumes that the reinvestment's rate of return is its opportunity cost, NPV's discount rate. The assumption of reinvestment return underlying the NPV method can be justified by business practice. Other criticisms of IRR in comparison with NPV include the violation of value-additivity principle and the situations of multiple rates of return (Copeland and Weston, 1988).

2.2.3 Decision-Tree Analysis (DTA) and Simulations

It is argued that DTA may be successfully used to incorporate the operating options. DTA is a very well-known and developed method that can handle the interaction between random events and management decisions. DTA helps management analyze the decision problems by identifying all alternative actions with respect to the possible random events in a hierarchy/tree structure. In a very complicated setting, it is argued that DTA can perfectly replicate the result of the CCA. Trigeorgis (1996) argues that DTA still has some practical limitations in the real world:

- DTA would become an unmanageable "*decision-bush analysis*" when it is applied to the real-world problem in which the number of different paths through the tree expands geometrically.
- More seriously, the presence of the operating options in the future decision nodes "*changes the payoff structure and the risk characteristics of an actively managed asset in a way that invalidates the use of a constant discount rate.*" DTA will then fall into the confusions and difficulties of selecting the discount rates.

Applying simulation in capital budgeting to solve more complex problems was first suggested by Hertz (1964). This method tries to identify the key variables that determine the cash flows of a project and then simulates these variables to obtain the distribution of the resulting cash flows or NPVs. This distribution is sometimes named "*risk profile.*" The most serious criticism was from Myers (1976):

"*If NPV is calculated using an appropriate risk adjusted discount, any further adjustment for risk is double-counting. If a risk-free rate of interest is used instead, then one obtains a distribution of what the project's value would be tomorrow if all uncertainty about the project's cash flows were resolved between today and tomorrow. But since uncertainty is not resolved in this way, the meaning of the distribution is unclear.*"

2.3 Game Theory and Its Applications

Game theory can be defined as "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers" (Myerson, 1991). It can also be called "conflict analysis" or "interactive decision theory," which can more accurately express the essence of the theory. However, game theory is still the most popular and accepted name. In economics, game theory has been applied to several important topics such as the analysis of oligopoly market, incomplete contracting, and financial contracting and debt, and has been recognized to be a very powerful approach. In construction, conflicts among builders and owners are very common and tend to increase. A BOT project involves series of highly complicated contracts compared with the already complicated conventional construction contracts. The incompleteness of a contract is no surprise. For example, it is difficult to implement the default provisions, since the government or lenders may tend to rescue the project due to the political and financial cost of the failure of a BOT project. Negotiations involved in the BOT project's procurement and operating process further complicate the formation and implementation of the contract. Therefore, game theory is very appealing as an analytical framework to investigate the interaction and dynamics between the BOT participants. In this research, game theory will be used to analyze the promoter's equity level strategy and government's equity level policy in BOT approach and to investigate the promoter's bidding strategy and government's procurement policy. The following introduction of the fundamentals of game theory basically follows Gibbons (1992) and Binmore (1992). Fudenberg and Tirole (1991) and Myerson (1991) have excellent in-depth discussions on game theory.

2.3.1 Static Game of Complete Information

In this section, some essential concepts and definitions in game theory shall be illustrated by examples of a two-player game. General cases of *n*-players definitions are omitted for convenience. The first example is the prisoner's dilemma, as shown in Fig. 2.1. Two suspects are arrested and held in separate cells. If both of them confess, they will be sentenced to jail for 6 years. If neither of them confesses, they will be sentenced for only 1 year. However, if one confesses and the other does not, the honest one will be rewarded by being released (in jail for 0 years) and the other will be punished with 9 years in jail. Note that in Fig. 2.1, the first number in each cell represents player 1's payoff and the second number is for player 2. We use the left side of the table to represent player 1 and use the top of the table to represent player 2.

		Player 2	
		Confess	Not confess
Player 1	Confess	$(\underline{-6}, \underline{-6})$	$(\underline{0}, -9)$
	Not confess	$(-9, \underline{0})$	$(-1, -1)$

Fig. 2. 1 Static Game: Prisoner's Dilemma

Figure 2.1 is a "normal form representation" of a game that specifies the players in the game, the strategies available to the players, and the payoff of each player for his strategy. The normal form representation is usually used in representing a "static game" in which they act simultaneously, or more generally, each player does not know the other player's decision before he makes his own decision.[i] If the payoff matrix as shown in Fig. 2.1 is known to all players, then the payoff matrix is a "common knowledge" to all players in a game and this game is called a game of "complete information." Conversely, if each player's possible payoff is privately known by himself only, it is a game with *incomplete information or asymmetric information.* Players in a game are assumed to be rational. This is one of the most important assumptions in any economic analysis. In other words, it is assumed that the players are always maximizing their payoff. Also, for clarity and

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[i] A typical example in construction is the competitive bidding played by bidders in which the competing bidders submit their bids "simultaneously."

convenience we shall assume that the players are *risk neutral*, that is, the utility function is: $u(x) = x$.

To answer how each prisoner will play/behave in this game, *Nash equilibrium*, one of the most important concepts in game theory, will be introduced. If game theory makes a unique prediction about each player's choice, then it has to be that each player is willing to play the strategy as predicted. Logically, this prediction should be the player's *best response* to the other player's predicted strategy. No single player wants to deviate from the predicted strategy, that is, the strategy is *strategically stable* or *self-enforcing* (Gibbons, 1992). This prediction is called a "*Nash equilibrium.*" In the prisoner's dilemma although the (Not confess, Not confess) may seem better for both players, it is *unstable* since every player wants to deviate from this solution to get extra benefit or avoid the other's betrayal. Any suspect who deviates from (Confess, Confess) will be hurt and any suspect who deviates from (Not confess, Not confess) will be rewarded. Therefore, the only predicted strategy that no player wants to deviate from is (Confess, Confess) and this is the Nash equilibrium in the prisoner's dilemma.

An easy way to check the Nash equilibrium in a static game is as follow and also shown in Fig. 2.1:

- Check prisoner 1's best response first: Check first column. If prisoner 2 plays "not confess," prisoner 1's best response is to play "confess." Therefore, the payoff 0 in (0, -9) is underlined. Then check the second column. If prisoner 2 plays "confess," prisoner 1's best response is to play "confess" too. The first -6 in (-6, -6) is then underlined.
- Then check prisoner 2's best response: Check first row. If prisoner 1 plays "not confess," prisoner 2's best response is to play "confess." The payoff 0 in (-9, 0) is underlined. Similarly, check the second row, and it is obvious that the second -6 in (-6, -6) should be underlined.
- Find the cell in which both elements are underlined, such as (-6, -6) in prisoner's dilemma. This is the Nash equilibrium of the prisoner's dilemma.

In some games there will be multiple Nash equilibria, that is, the uniqueness of Nash equilibrium is not guaranteed. Fortunately the existence of multiple equilibria will not be a problem. Much of the game theory is an effort to identify a *compelling* equilibrium in different classes of games to make the prediction appealing (Gibbons, 1992). The static game's powerful applications in duopoly have been recognized in economic literature.

2.3.2 Dynamic Games of Complete Information

Most of the analysis in this research will be using the dynamic game with complete or incomplete information. However, the previous introduction of static game is essential because those concepts will be used repeatedly in other classes of games and in this research. In contrast to static games, players in a dynamic game move *sequentially* instead of simultaneously. Since the moves are sequential, it will be easier and more intuitive to represent a dynamic game by a tree-like structure, called an "*extensive form*" representation. In a dynamic game, suppose that the player who moves in a later stage can fully observe previous player's moves and know his own location in the game tree. This assumption is called "*perfect information*" assumption and this assumption can be relaxed and become "*imperfect information*".

The *Price War* example can clearly demonstrate the spirit of a dynamic game, the *credibility*. The firm, "New Inc.," wants to enter a monopoly market to compete with the solely existing firm, "Old Inc." Old Inc. does not want New Inc. to enter the market, as it will decrease the firm's monopoly profits. Old Inc. threatens New Inc. that they would start a price war by reducing the price by 30% if New Inc. does enter the market. The extensive form game tree is shown in Fig. 2.2. The game tree shows: 1. New Inc. chooses to enter the market or not, and then Old Inc. chooses to start a price war or not, and 2. the payoffs of each combination. A possible conclusion is that Old Inc. can use the strategy: to play "start a price war" if New Inc. plays "enter," so that Old Inc. can threaten New Inc. not to enter the market. As a result, it may seem that "stay out" and "start a price war if enter" is a Nash equilibrium. Nevertheless, as shown in Fig. 2.2 the threat to start a price war is *not credible* because Old Inc. will not start a price war if New Inc. does enter; instead, Old Inc. will maximize the payoff by playing "no price war" after New Inc. enters the market unless the Old Inc. is behaving *irrationally*. New Inc. knows the threat's incredibility and therefore will maximize the payoff by playing "enter." Thus, the Nash equilibrium of the Price War game is (Enter, No price war), an equilibrium that does not rely on the player to carry out an incredible threat.

The game in Fig. 2.2 is called a "*dynamic game of complete information.*" We shall apply this type of game in the analysis of BOT projects. The game's extensive form is a tree diagram as shown in Fig. 2.3, which shows the players of the game, player 1 and player 2; possible actions the players could choose such as "Up" or "Down" for player 1 and "Left" or "Right" for player 2; possible random events that could occur; the timing of actions such as player 1 first then followed by player 2 and then followed by player 1 again; and the payoffs (U_1,U_2) for each players in different combinations of actions. The game shown in Fig. 2.3 is a dynamic game with perfect (and complete) information, which is the simplest case in a dynamic game. In Fig. 2.3, the payoffs for each player such as (2,0), (1,1) and the sequence of moving and the actions available for each player are common knowledge.

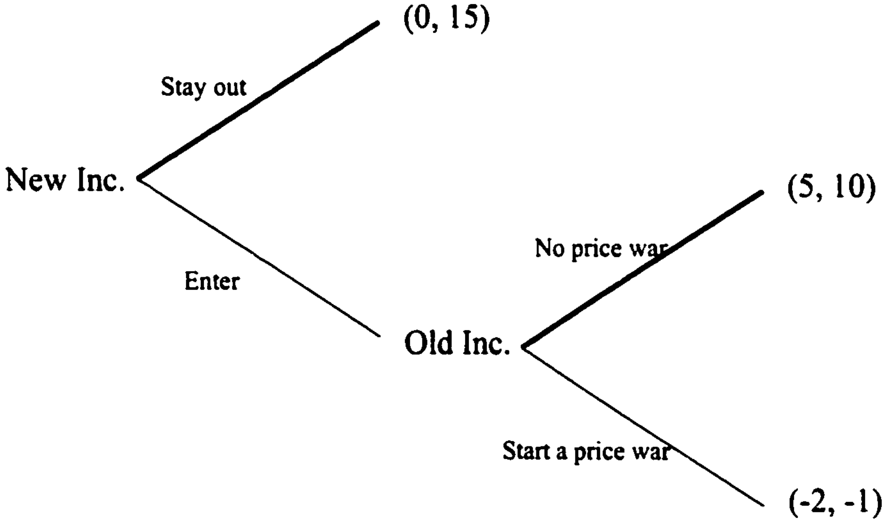

Fig. 2. 2 Dynamic Game: Price War

Thus player 1 knows that player 2 knows the common knowledge and player 1 also knows that player 2 knows that player 1 knows...and so on.

The way to solve a dynamic game is to solve the game backward recursively and obtain the backwards-induction outcome (Gibbons, 1992). Mathematically, the solution can be found through the following problem set up for a two-stage game:

1. Player 1 chooses action $a_1 \in A_1$ and then player 2 chooses action $a_2 \in A_2$, where A_1 and A_2 are the sets of possible actions taken by player 1 and player 2, respectively.
2. Payoffs for player 1 and 2 are $U_1(a_1, a_2)$ and $U_2(a_1, a_2)$ respectively.
3. Solving player 2's problem first: given any a_1, solve for $\underset{a_2 \in A_2}{Max} U_2(a_1, a_2)$, then get $a_2^* = R_2(a_1)$, where $R_2(a_1)$ is a function that specifies player 2's best response for each a_1.
4. Then solving player 1's problem: solve for $\underset{a_1 \in A_1}{Max} U_1(a_1, a_2^*) = U_1(a_1, R_2(a_1))$
5. An equilibrium solution $(a_1^*, R_2(a_1^*))$ can be found.

For games with more than two stages, only these five steps are needed to obtain the solution. The solution can be easily found through the aid of game tree following the same rationality, *payoff maximization behavior*, used in the mathematical model. For example, in Fig. 2.3, first the last subgame is solved and "UP" will be player 1's choice because payoff '3 by UP' > '0 by DOWN.' Therefore, the subgame just solved can be replaced by the payoff vector (3, 0) as shown in Fig. 2.4. Then the subgame for player 2 is solved through 2's choice of "LEFT." The final equilibrium solution is shown in Fig. 2.5.

Applications of dynamic games of complete information include the analysis of negotiation in unionized firms by Espinosa and Rhee (1989), sequential bargaining regarding the decision to strike or not by Fernandez and Glazer (1991), and sovereign debt by Bulow and Rogoff (1989).

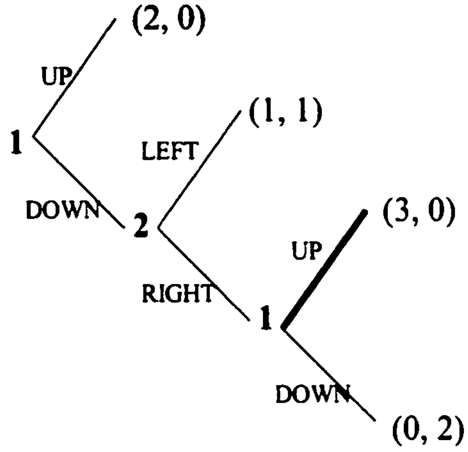

Fig. 2. 3 Solving a Dynamic Game: Step 1

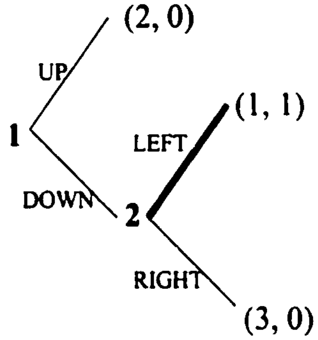

Fig. 2. 4 Solving a Dynamic Game: Step 2

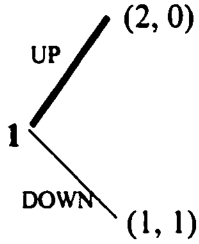

Fig. 2. 5 Solving a Dynamic Game: Step 3

2.3.3 Static Games of Incomplete Information

Games of incomplete information or asymmetric information are also called "Bayesian games" because it involves using Bayes' rules in solving for the equilibrium. Therefore, static games of incomplete information are also called static Bayesian games. The core issue of incomplete information is the existence of "*private information*" that is known to specific players instead of to all players. This private information is regarding the payoff functions of the players. The players with private information are called informed players. Similarly the players who are uncertain of the other players' payoff functions are uninformed players. The equilibrium of a Bayesian game is called "*Bayesian Nash equilibrium.*" The concept of the equilibrium is the same as previous games: to find the equilibrium so that no player has incentives to change his strategy. Since it is relatively complicated to solve for a Bayesian Nash equilibrium, the details will be deferred until they are needed later in the analysis.

Typical and very appealing applications of static Bayesian games are the design of auctions, because auctions involve the bidders with their distinct valuations, which are private information. McAfee and McMillan (1987) analyzed the famous phenomenon: winner's curse. Baron and Myerson (1982) discussed how to regulate a monopolist with unknown costs. Sappington (1983) discussed agency theory.

2.3.4 Dynamic Games of Incomplete Information

After the introductions of dynamic game and incomplete information above, no extra explanation is required on what a dynamic game of incomplete information is. However, the equilibrium of this class of games is much more complicated than previous classes of games, yet closely related. Furthermore, most games in the BOT approach fall into this category. Therefore, it is critical to fully understand their characteristics and the games' equilibrium concept: *perfect Bayesian equilibrium*. Nevertheless, the details of dynamic games of incomplete information will be deferred until Chapter 7.

Just as in the subgame-perfect Nash equilibrium in a dynamic game of complete information, the perfect Bayesian equilibrium has to rule out incredible threats and time-inconsistent promises. In other words, the equilibrium has to be *sequentially rational*, that is, the sequential moves and the subsequent players' rationality of maximizing their payoff must be considered. In contrast to subgame-perfect Nash equilibria, perfect Bayesian equilibria cannot be obtained through backwards induction because it needs to be checked back and forth circularly. The primary steps to solve for perfect Bayesian equilibria are to:

- find possible candidates of perfect Bayesian equilibria, and
- check each candidate or each class of candidates for the satisfaction of perfect Bayesian equilibria's sufficient conditions.

These conditions will be specified and discussed later during the analysis because of their complicated and lengthy arguments. For detailed explanations of why these conditions should be satisfied, one may refer to Gibbons (1992) and Fudenberg and Tirole (1991).

Applications of this class of games are fruitful in economic and corporate finance literature. The main applications include signaling games, cheap-talk games, sequential bargaining games, and reputation problems. Signaling games are especially broadly applied to analyze problems such as job market (Spence, 1973, 1974), capital structure, and optimal contract (Leland and Pyle, 1977; Myers and Majluf, 1984; Dybvig and Zender, 1991). As the BOT approach involves many bidding and operating strategies and government's policies, it is very appealing to apply the game theory as an analytical framework of the research on BOT approach.

2.4 Options Pricing Theory and Real Options

2.4.1 Stochastic Processes: Brownian Motions and Diffusion Processes

Black and Scholes' option pricing theory is based on the assumption that stock prices follow diffusion processes. They model the option price as a continuous-time stochastic processes by applying Itô 's Lemma. The literature at a mathematically rigorous level about continuous-time stochastic processes and Itô 's Lemma can be easily found. The following introduction is presented at an intuition level and is based on the lecture notes of Pennacchi (1997). Pure Brownian motion process or a Wiener process is the basic stochastic building block for the general continuous-time stochastic processes, also known as diffusion processes. Arithmetic Brownian motion and geometric Brownian motion are the diffusion processes. Itô 's Lemma, also referred to as fundamental theorem of stochastic calculus, is then used to determine the differential of a function of a diffusion process, which is often used in dynamic models in finance.

Pure Brownian motion process: Consider the following stochastic process $z(t)$. The change in $z(t)$ over Δt is given by

$$z(t+\Delta t)-z(t)\equiv\sqrt{\Delta t}\tilde{\varepsilon} \tag{2.1}$$

where $E[\tilde{\varepsilon}]=0$, and $Var[\tilde{\varepsilon}]=1$. Assume T is make up of N intervals of length Δt. Then

$$z(T)-z(0)=\sum_{i=1}^{N}\Delta z_i\equiv\sum_{i=1}^{N}\sqrt{\Delta t}\tilde{\varepsilon}_i \tag{2.2}$$

The mean of $z(T)-z(0)$ is 0 and the variance is T, because

$$E[z(T)-z(0)]=\sqrt{\Delta t}\sum_{i=1}^{N}E[\tilde{\varepsilon}_i]=0 \tag{2.3}$$

$$Var[z(T)-z(0)]=(\sqrt{\Delta t})^2\sum_{i=1}^{N}Var[\tilde{\varepsilon}_i]=\Delta t\cdot N\cdot 1=T \tag{2.4}$$

Following from the Central Limit Theorem, $z(T)-z(0)$ can be said to have a normal distribution with mean 0 and variance T when $N \to \infty$. Without loss of generality, one can assume that each $\tilde{\varepsilon}_i$ has a standard normal distribution and define

$$dz(t) \equiv \lim_{\Delta t \to 0} \Delta z = \lim_{\Delta t \to 0} \sqrt{\Delta t}\tilde{\varepsilon} \qquad (2.5)$$

dz is referred to as a pure Brownian motion or a Wiener process that is normally distributed with a mean of 0 and a variance of dt. $z(T)-z(0)$ can be calculated from stochastic integral and the result is a distribution

$$z(T)-z(0)=\int_0^T dz(t) \sim N(0,T) \qquad (2.6)$$

Diffusion processes: Diffusion processes can be generalized from pure Brownian motion process. The first one is arithmetic Brownian motion with a non-zero drift as well any desired variance or volatility. First consider a new process $x(t)$,

$$dx(t)=\sigma \cdot dz(t) \qquad (2.7)$$

Then $x(t)$ is distributed as a normal distribution with mean 0 and variance $\sigma^2 T$, and the distribution is obtained by

$$\int_o^T dx = x(T)-x(0)=\int_0^T \sigma \cdot dz(t) = \sigma\int_0^T dz(t) \sim N(0,\sigma^2 T) \qquad (2.8)$$

Arithmetic Brownian motion process $x(t)$ is defined as

$$dx = \mu dt + \sigma dz \qquad (2.9)$$

where μ is a deterministic change per unit of time to the $x(t)$ process.

It follows that the arithmetic Brownian motion process is distributed as

$$\int_0^T dx = x(T)-x(0)=\int_0^T \mu dt - \int_0^T \sigma dz(t) = \mu T + \sigma\int_0^T dz(t) \sim N(\mu T,\sigma^2 T) \qquad (2.10)$$

However, in general, μ and σ could be a function of time or even $x(t)$. In this case, the arithmetic Brownian motion's stochastic differential equation (SDE) is

$$dx(t)=\mu[x(t),t]dt+\sigma[x(t),t]dz \qquad (2.11)$$

$dx(t)$ is described as being instantaneously normally distributed with mean $\mu[x(t),t]dt$ and variance $\sigma[x(t),t]dt$. The second diffusion process is geometric Brownian motion process. It is defined as

$$dx = \mu x dt + \sigma x dz \quad \text{or} \quad \frac{dx}{x} = \mu dt + \sigma dz \tag{2.12}$$

where μ and σ are constants. It can be proved that geometric Brownian motion process will follow a *lognormal distribution*, which is appropriate for modeling the price of a limited-liability security such as a common stock.

Itô's Lemma: Itô's Lemma gives the rule for finding the differential of a function of one or more variables, some of which follow a diffusion process. The function could be stated such as $F(x(t),t)$ where $x(t)$ is a variable of a diffusion process. Give the variable $x(t)$ following $dx = \mu[x,t]dt + \sigma[x,t]dz$, then the differential of $F(x(t),t)$ will be

$$dF(x(t),t) = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(dx)^2 \tag{2.13}$$

where $(dx)^2 = \sigma^2 dt$. Substituting in for dx and $(dx)^2$, equation (2.13) can be rewritten as

$$\begin{aligned} dF(x(t),t) &= \frac{\partial F}{\partial x}(\mu[x,t]dt + \sigma[x,t]dz) + \frac{\partial F}{\partial t}dt + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}\sigma[x,t]^2 dt \\ &= (\frac{\partial F}{\partial x}\mu[x,t] + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}\sigma[x,t]^2)dt + \frac{\partial F}{\partial x}\sigma[x,t]dz \end{aligned} \tag{2.14}$$

Generally, let $F(x_1, x_2, \ldots, x_m, t)$ be at least a twice differentiable function. By Itô's Lemma, the differential is given by

$$dF(x_1, x_2, \ldots, x_m, t) = \sum_{i=1}^{m}\frac{\partial F}{\partial x_i}dx_i + \frac{\partial F}{\partial t}dt + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\partial^2 F}{\partial x_i \partial x_j}dx_i dx_j \tag{2.15}$$

where $dx_i dx_j = \sigma_i \sigma_j dt$. Equation (2.15) can be rewritten as

$$\begin{aligned} dF(x_1, x_2, \ldots, x_m, t) = [&\sum_{i=1}^{m}\frac{\partial F}{\partial x_i}\mu_i[x,t] + \frac{1}{2}\frac{\partial^2 F}{\partial x_i^2}\sigma_i[x,t]^2 + \frac{\partial F}{\partial t} + \sum_{i=1}^{m}\sum_{j>1}^{m}\frac{\partial^2 F}{\partial x_i \partial x_j}]dt \\ &+ \sum_{i=1}^{m}\frac{\partial F}{\partial x_i}\sigma_i[x,t]dz_i \end{aligned} \tag{2.16}$$

In a BOT project, considering the growing trend of market demand for the facility and the demand's uncertainty during the concession period, the demand can be modeled as a "geometric" Brownian motion process in equation (2.12). The non-zero drift, μ, of geometric Brownian motion represents the trend of the growing demand, and the variance or volatility, σ, represents the uncertainty or the risks. $F(x,t)$ can be a project valuation function or profit function and can be further analyzed using *Itô 's Lemma*. The introduction of continuous-time stochastic process provides the tool for a more dynamic and precise analysis.

2.4.2 Black, Scholes, and Merton's Option Pricing Theory

The option pricing theory by Black and Scholes (1973) and Merton (1973) is the building block of the modern dynamic asset pricing theories. Option pricing theory recognizes the interactions among option holder's profit maximizing behaviors, asset's uncertainty, and market disciplines. Recently, the option pricing theory is often applied in the evaluation of non-financial assets or physical investments. Researchers sometimes termed it "real options." This dynamic pricing process overcomes difficulties in discounting approach, and successfully computes the value of an investment more realistically.

A "European call option" is a type of contract giving the right to buy a specific asset, such as a common stock, at a specific price on a specific date in the future. An example could be a 3-month IBM stock call option. If today is September 20, this call option may specify that the option issuer gives the holder the right to buy an IBM stock from option issuer at the exercise price $120 on the maturity date, December 20. Therefore, on December 20, the option holder must decide whether to buy an IBM stock at $120 or not. If the stock price of IBM is greater than $120 on December 20, say, $150, the option holder will certainly "exercise" his option, that is, to buy an IBM stock and make $30 profit. If the stock price is $110, the option holder will just abandon his option and make $0 profit. The question is how to set a price for this stock option.

Another type of option is called "American call option." The American style option allows the holder to exercise the option *before* its maturity date. Therefore, an American call option on IBM allows the holder to buy an IBM stock at $120 on any day before December 20. It can be proved that under certain conditions even when the stock price rises above $120, it may not be optimal to exercise the option immediately. For example, if on September 20, the stock price is $121, it is not optimal to exercise the option immediately and earn only $1 profit. The holder should wait and keep the option alive. It turns out that the problem of *when* to optimally exercise an American option has to be solved simultaneously with the price of the option.

Suppose the price of a share of stock follows geometric Brownian motion process:

$$\frac{dS}{S} = \mu dt + \sigma dz \tag{2.17}$$

where S is the stock price, μ is the instantaneous rate of return and σ^2 is the instantaneous variance of rate of return. Suppose further that the value of a European call option on this stock is $F(S, t)$, a function of $S(t)$ and t. Itô's Lemma says,

$$dF(S,t) = [\frac{\partial F}{\partial S}\mu S + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2]dt + \frac{\partial F}{\partial S}\sigma S dz \tag{2.18}$$

Black and Scholes (1973) further assume "ideal conditions" in the market that the short-term interest rate is known and constant, that the stock pays no dividends or other distributions, that there are no transaction costs, and that the market is frictionless. They argue that by forming a hedging portfolio and adjusting the portfolio continuously, the return on the portfolio becomes certain. The return on this portfolio is instantaneously riskless and the rate of return must be risk-free assuming no arbitrage. Consider a portfolio with the hedge position containing one option short and $\frac{\partial F}{\partial S}$ shares of stock long. Let $W(t)$ be the value of this portfolio at time t, then

$$W(t) = -F(S,t) + (\frac{\partial F}{\partial S})S \tag{2.19}$$

The instantaneous change of $W(t)$ is

$$dW(t) = -dF(S,t) + (\frac{\partial F}{\partial S})dS$$

$$= -[\frac{\partial F}{\partial S}\mu S + \frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2]dt - \frac{\partial F}{\partial S}\sigma S dz + [(\frac{\partial F}{\partial S})\mu S dt + (\frac{\partial F}{\partial S})\sigma S dz]$$

$$= -[\frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2]dt \tag{2.20}$$

As shown in equation (2.20), $dW(t)$ is certain with the uncertain term being dropped out. As argued by Black and Scholes, the riskless return under continuous hedging is equal to risk-free return,

$$dW(t) = rW(t)dt = r[-F + (\frac{\partial F}{\partial S})S]dt \tag{2.21}$$

where r is the risk-free rate and is assumed to be constant. The basic idea in equation (2.21) to solve the price consistent with capital market is the economics' "no arbitrage opportunity" argument, that is, if the asset is mispriced, the arbitrage transactions will adjust the prices until the market equilibrium is reached and there exists no arbitrage opportunity. From equations (2.20) and (2.21),

$$-[\frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2]dt = r[-F + (\frac{\partial F}{\partial S})S]dt \tag{2.22}$$

Thus, Black and Scholes partial differential equation (PDE) is obtained by rewriting equation (2.22)

$$\frac{\partial F}{\partial t} + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}\sigma^2 S^2 + rS(\frac{\partial F}{\partial S}) - rF = 0 \quad \text{or} \quad F_t + \frac{1}{2}\sigma^2 S^2 F_{ss} + rSF_s - rF = 0 \tag{2.23}$$

The price of an option is obtained by solving equation (2.23) subject to one terminal condition set by the value of options at maturity date T, and two boundary conditions. For example, a European option's terminal condition is

$$F(S(T),T) = S(T) - X, \qquad S(T) \geq X$$

$$F(S(T),T) = 0, \qquad S(T) < X$$

where X is the exercise price. This condition can also be rewritten as

$$F(S(T),T) = \max[0, S(T) - X] \tag{2.24}$$

The boundary conditions are

$$F(0,t) = 0 \tag{2.25-1}$$

$$F(S,t)=S\,,\qquad S\to\infty \tag{2.25-2}$$

Black and Scholes solved this PDE and obtained an exact and unique solution, which is also the option valuation formula

$$F(S(t),t)=S(t)N(d_1)-Xe^{-r(T-t)}N(d_2) \tag{2.26}$$

where

$$d_1=\frac{\ln(S(t)/X)+(r+\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} \tag{2.27}$$

$$d_2=d_1-\sigma\sqrt{T-t} \tag{2.28}$$

and $N(d)$ is the cumulative normal density function. It is also worth noting that $N(d_1)$ can be proved to be the hedging ratio of shares of stock to options, $\frac{\partial F}{\partial S}$, and $N(d_2)$ can be interpreted as the probability that the option will finish in-the-money (Copeland and Weston, 1988).

Since Black and Scholes (1973) and Merton's (1973) breakthrough in the valuation of stock options, theories regarding *asset valuation* concept and process have advanced into a new era. The most powerful feature in their pricing approach is that the price can be solved independently of individual investor's risk attitude.

2.4.3 Real Options Theory

Option pricing theory recognizes the interactions among option holder's optimizing behaviors, asset uncertainty, and market disciplines. Although, a more general term for various option pricing theories is "contingent claims analysis." The term "real options theory" will be used mostly in this paper. This dynamic pricing process overcomes difficulties in "discounting approach," and successfully computes the value of an investment more realistically, making modern option pricing theory a better choice for analyzing BOT projects.

Real options are collections of options on real assets such as strategic or financial flexibility or options. The diffusion process assumptions of the operating cash flow and the value of firm/project are considered to reasonably represent their stochastic behaviors (Brennan and Schwartz, 1984; Fischer et al., 1989; Mason, 1978; Trigeorgis, 1991; Leland, 1994; Dixit and Pindyck, 1994). In BOT research, real options framework provides the capability of modeling BOT scheme's characteristics including cost uncertainty, operating risks and flexibility, and other constraints. Real options can also be incorporated with game theory in studying the interaction or negotiation between creditors and borrowers. However, the mathematical treatment in this framework is quite complicated and analytical solutions are limited to some special cases only. Thus, numerical methods are needed in most of the cases.

2.4.4 "No-Arbitrage" Principle and Risk-Neutral Valuation Solutions

In a more general setting by Cox and Ross (1976), the price of an option will follow

$$\frac{1}{2}\sigma^2(S,t)F_{SS} + (rS - b(S,t))F_S - rF + F_t = 0 \qquad (2.29)$$

where $b(S, t)$ is the continuous pay out stream of the underlying asset, S. In this setting, the underlying asset value is governed by

$$dS = \mu\,(S, t)\,dt + \sigma\,(S, t)\,dz \qquad (2.30)$$

When $\mu(S,t) = \mu S$, $\sigma(S,t) = \sigma S$ and $b(S, t)=0$, equation (2.29) becomes equation (2.23). From equations (2.23) and (2.29), it is worth noting that the underlying asset's drift, μ, does not enter equations (2.23) and (2.29), instead, only the risk-free interest rate, r, enters. Therefore, if the drift term in equation (2.30) is replaced by r, the solution will not be affected at all. As a result, one could pretend that the world is risk neutral, that is, S evolves with drift r instead of μ, and calculate the solution of the contingent claim, and the solution will be consistent with the real world. This valuation technique is called "*Risk Neutral Valuation.*" Cox and Ross argued that from the fact that option price can be obtained uniquely by creating a risk-free hedge portfolio, one could conclude that the solution of the contingent claim does not depend directly on the investor's preferences/risk attitudes. Therefore, a convenient choice of risk attitudes is *risk neutrality*. In a world with risk-neutral investors, equilibrium requires that the expected returns on both the underlying assets and the contingent claims are equal to risk-free returns. This risk-neutral valuation is also known as "*equivalent measures*" in probability theory (Harrison, 1985). For example, in the derivation of Black and Scholes' option pricing formula, we may pretend that we were in a risk-neutral world and replace the stock's real dynamics $\frac{dS}{S} = \mu dt + \sigma dz$ in equation (2.17) with the risk-neutral dynamics

$$\frac{dS}{S} = rdt + \sigma dz \tag{2.31}$$

Going through the same derivation process in 2.4.2, we may obtain Black and Scholes' PDE in equation (2.23) and option price formula in equation (2.26).

The risk-neutral valuation concept has a simple interpretation: by pretending the world is risk neutral and taking the expectation of the contingent claim's payoff and discounting at risk-free rate, one could compute the price/value of a contingent claim. Risk-neutral valuation is used in almost all kinds of numerical computation methods.

2.5 Numerical Methods for Solving Real Options Problems

A major approach to solving complex option pricing problems is to apply numerical techniques. Basically, there are two categories of the numerical methods. The first category is probabilistic and economic related. This category solves for the solution based on risk-neutral valuation and no-arbitrage theory. Binomial (Cox et al., 1979) and trinomial/lattice (Boyle, 1988) methods and simulation methods (Boyle, 1977) are in this category. They usually do not involve advanced mathematical treatment and have probabilistic and economic interpretations. The second category is PDE related. This approach tries to solve the PDEs similar to equation (2.29) directly by using typical numerical methods, such as finite difference methods (Brennan and Schwartz, 1977) or techniques of solving free boundary problems (Parkinson, 1977; Mason, 1978) when early exercise is allowed.

Among the numerical methods for solving option-pricing problems, the first one proposed is the "*binomial model*" by Cox, Ross, and Rubinstein (1979). They proposed a binomial option pricing formula, which requires only elementary mathematics, yet precisely and efficiently solves the option problem. Cox et al. proved that the binomial method solution is indeed the Black, Scholes, and Merton solution in the limiting case, that is, when the discrete time interval approaches 0. It is shown that the binomial method can also solve American option problems, which do not have analytical solutions and require complicated computation in other numerical methods such as finite difference method. Most applications of real options possess the early exercise characteristic; for example, a project can always be abandoned before the project completion date.

2.6 Capital Structure Theories

2.6.1 The Development and Categories of Capital Structure Theories

Capital structure can be defined as how the amount and sources of the required capital of a firm or project can be arranged. Debt and equity are the essential elements discussed in the literature. Other extensions also include some hybrid securities such as preferred stock. Researches on this topic try to conclude whether there exists an optimal capital structure for a firm and what the optimal capital structure is. Modigliani and Miller (1958) published their pathbreaking work on capital structure. Twenty-six years after their work, Myers (1984) still offered the criticism,

"*... we know very little about capital structure. We do not know how the firms choose the debt , equity or hybrid securities they issue... we have inadequate understanding of corporate financing behavior, and how that behavior affects security returns.*"

No one theory can fully explain the actual financial behaviors regarding capital structure decisions of a firm. However, some researches did give us valuable insights during the process of solving this puzzle. Research that sets the milestones during the development of capital structure theories will be introduced in this section, and insights from the research related to the optimal capital structure of BOT projects will be emphasized as well. It is worth noting that during the past, researchers focused on debt or firm valuation and how to maximize the returns of the shareholders. However, this research will reexamine the problem from the viewpoint of the BOT project developer and how the overall return from the BOT investment can be maximized.

Myers (1984) categorized the development into three major approaches, namely, a static framework of tradeoff of the costs and benefits of borrowing (Modigliani and Miller, 1958, 1963; Baxter, 1967; Kim, 1978), a pecking-order framework and the managerial theories including agency theory and managerial capitalism. Copeland and Weston (1988) summarized several important works in capital structure explaining the reasons of the existence of an optimal capital structure. These works include the frameworks considering bankruptcy costs, *signaling hypotheses* (Myers and Majluf, 1984), option pricing implications—the bondholder wealth expropriation hypothesis (Black and Scholes, 1973), and the effect of agency costs (Jensen and Meckling, 1976). Recent studies reexamined the capital structure problem from the perspective of security or contract design. These studies include the incomplete contracting model (Hart, 1995), game theory applied in debt re-negotiations (Mella-Barral and Perraudin, 1997) and applied in capital structure (Ziegler, 1999), and the control rights allocations (Aghion and Bolton, 1992).

2.6.2 Recent Approaches

Recent studies considered the existence of tax shields, financial distress costs, agency costs, and the incompleteness of contracts, and then further focus on modeling debt indenture provisions, the relationship among the parties or other special constraints by applying contingent claim analysis framework or game theory. Aghion and Bolton (1992) showed that contracts with contingent transfer of control rights may minimize inefficiencies and provide rationale for debt contracts. Bolton and Scharfstein (1993) studied the trade-offs involved in making debt more or less easy to renegotiate. Recent works based on game theory, or incorporated with other theories, such as option pricing theory were done by Ziegler (1999), Fischer et al. (1989), Leland (1994), Leland and Toft (1996), Anderson and Sundaresan (1996), and Mella-Barral and Perraudin (1997). An excellent paper by Leland (1994) considered the capital structure policy in a unified analytical framework that incorporates the tax shields, financial distress costs, debt indenture provisions, firm specific risks, and agency problems.

CHAPTER 3 RESEARCH METHODOLOGY

This study was conducted as an iterative approach shown in Fig. 3.1. The first step was to identify the problem and its significance. A thorough literature review followed the problem formation to build upon existing research efforts. After the literature review, modern option pricing theory/real options analysis and game theory were identified as the building blocks or fundamental theoretic frameworks for the BOT model. A BOT model of project pricing, capital structure, and tendering were derived through the theoretical analysis. Finally, the model was verified and validated by using a real-world BOT project, the Channel Tunnel.

3.1 Literature Review and Theoretical Framework

The review of literature included the infrastructure privatization, project financing and financial decision theories, game theory and the mathematics of diffusion process and numerical methods for solving stochastic SDEs. The infrastructure privatization literature review focused on BOT scheme and its critical success factors, the risk and return of a BOT project, the financial relationship among all participants, and its capital structure. Project financing and financial theory literature review focused on the project financing scheme, the determinants and calculation of the costs of capital, firm theories, static trade-off capital structure theories, agency theories, incomplete contracts, and modern financial contingent claims analysis on project valuation and decision making.

The game theory analysis focused on dynamic games with complete and incomplete information and signaling games. The review of mathematics of diffusion process centered on the Brownian motions and their application on finance, and the techniques of solving the SDEs. Also, the literature on the real-world BOT project, the Channel Tunnel, was reviewed. Various kinds of information on the Channel Tunnel project, such as the prospectus, recent restructuring proposal, and case studies performed by different researchers, were collected to verify and validate the model.

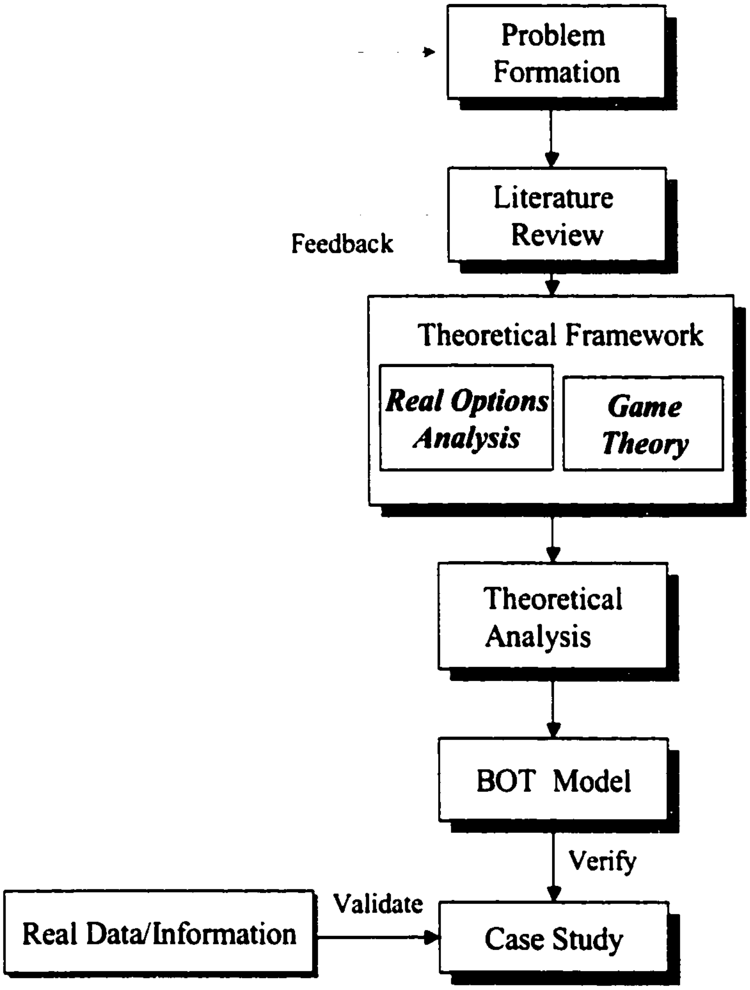

Fig. 3. 1 Research Procedure

3.2 Theoretical Analysis

Theoretical studies include making reasonable assumptions, assigning relationship or constraints to relevant parameters, building up BOT pricing and evaluation model and numerically solving it, and analyzing game theoretic equilibrium solutions including bidding behaviors and effective signals. To keep the clarity of the analysis, a simplified study example will be used to demonstrate the analytical framework. The theoretic frameworks used in this research are the real option analysis and game theory. Real option analysis will be applied in the BOT investment's valuation. The BOT valuation model includes the analyzing of the developer's BOT profit structures, the maximization of the profits from the structure, and the balancing of costs and benefits in a multi-period and dynamic manner. The BOT valuation model is not expressed in an analytical form due to the complexity of a BOT project and the limits of current mathematical techniques. Therefore, the derivation involved the process of solving the mathematical equations by applying *numerical methods*. Game theory was applied to analyze the participants' interactions in BOT, and incorporated with CCA to evaluate a BOT project. The existence of effective signals of the developer's private information concerning the BOT project was investigated by applying both game theory and real options analysis.

3.3 BOT Valuation, Financing and Tendering Model

A BOT model was derived and generalized through the theoretical analysis. The BOT model includes 1. the BOT investment rationale and decision framework, 2. the BOT profit structures valuation framework, and 3. the BOT tendering/procurement framework. These sub-frameworks are part of the unified BOT model. The model identified critical parameters in BOT projects and built the relationships and dynamics of these parameters.

3.4 Verification and Validation Using a Case Study

The verification was performed through the study of one real case. The verification looked into how well the model could interpret the numbers/results from the case study and how the model could prescribe more effective decisions. The case study is the Channel Tunnel project in the U.K. and France, which suffered from serious cost overrun during construction and insufficient revenue during operation. The government BOT policies on this case are discussed as well. Case study is used to compare and verify the real-world data with the results from the theoretical investigations. Explanations will be given regarding the consistency or inconsistency.

CHAPTER 4 BOT INVESTMENT PROBLEM AND BOT MODEL

The first step in building a BOT model is to investigate the BOT investment problem. This step is also the first step of the theoretical analysis, as shown in the research methodology. The results of this chapter provide the fundamental framework of the BOT model. The tasks of this chapter included the following. Section 4.1 discusses the BOT investment characteristics and basic assumptions. Section 4.2 investigates the risk characteristics of BOT projects. Section 4.3 introduces the investment valuation problem and the concept of the "profit structures." The problems of measuring financial project viability are discussed in section 4.4. Section 4.5 discusses the dynamics or stochastic behaviors of the cost and future operating cash flow of BOT projects. Last, section 4.6 outlines the BOT model derived, which contains three analytical frameworks.

4.1 Assumptions Regarding the BOT Investment Problem

Among various schemes of infrastructure privatization, the BOT approach will be the representative scheme in this research. The major differences between BOT and other variants are the duration of the concession period and the ownership of the project. Many variants may be analyzed in a similar manner according to the models derived in this research.

In a normal infrastructure tendering process, the government provides the specifications to the contractors and sometimes divides the project into many relatively small subcontracts. However, for a privatized infrastructure project, the proposals from different promoting teams are distinct, mainly at financial packages and technical solutions. The winning bid is not necessarily the lowest bid. Specialized contractors with sufficient capital to bid the project are limited. Therefore, the leading contractors are those who possess the unique expertise and resources toward the BOT project development. As a result, for government, it is difficult to replace the leading contractors should a project fail.

From the economic perspective, the government has to incur significant costs to replace the leading contractors.

By virtue of the BOT practice, this research assumes that the leading contractors are often also the developers or promoters. This assumption plays an important role in the BOT model derivation.

In the model derivation, some necessary environment or conditions for proceeding with a BOT project are assumed to exist. These conditions are summarized by Augenblick and Custer (1990):

- Legal environment: The host government's legal system is fairly mature and ready to support the contractual issues of the infrastructure privatization.
- Economic environment: The host country has a developed banking system and an organized capital market that provides equity investors and lenders enough financial tools to make investments or hedge risks.
- Host country credit rating: At least an intermediate credit rating is required for the BOT investment.
- Political environment: The political stability and continuity of the host country is satisfactory.

In other words, those risks that are caused by the legal, economic and political environment, and host country credit rating, are ruled out so that this research can focus on more subtle issues. However, the imperfections of the above conditions are not ruled out. For example, it is possible that the government's regulation regarding the BOT approach is not appropriate or mature enough because of the lack of the understanding and experiences of the BOT approach. One objective of this research is to build an understanding of the BOT approach.

4.2 BOT Investment's Risk Characteristics

Risks are inherently present in any construction project. Walker and Smith (1995) grouped risks into three headings: financial risks, political risks, and technical risks. They also grouped risks by the different stages of a project. Augenblick and Custer (1990) summarized risks as completion risk, performance and operating risk, cash flow risk, inflation and foreign exchange risk, insurable risks, uninsurable risks (Force Majeure), and political risk. Dias and Ioannou (1995c) categorized the risks as: country risks (political & regulatory risks), force majeure risks, physical risks, financial risks, revenue risks, promotion risks, procurement risks, development risks, construction risks, and operating risks. There is no unified definition for each risk category. Sometimes the same category name can refer to different risks depending on the researchers. For example, "financial risk" grouped by Walker and Smith (1995) refers to all risks related to costs and revenues, such as market risks and interest rate risks, etc., whereas Dias and Ioannou (1995c) include no income or market risk, but do include risks of default on interest payment or loan repayment. In this research, the risks inherent in a BOT project are categorized as:

- Construction/completion risks: The completion risks mainly occur at the construction phase, and include completion delays, cost overruns, and technical difficulties, although many insurable risks are also in this category, such as plant and equipment casualties, physical loss or damage, and workmen's compensation. These insurable risks do not concern this research because they can be fairly priced and hedged.
- Operating/economic risks: These risks are primarily related to the economic situations or changes during the operation and construction phase, and include price, demand or supply changes, management, business cycle, and liability. Uninsurable force majeure risks may be included in construction/completion and economic/operating risks categories.
- Financial risks: Financial risks could result from interest rate, foreign exchange rates or inflation rates, and could be hedged by utilizing certain financial tools in a developed financial market or by contractual arrangement. Financial risks also include a firm's

default risk that is related to the firm's financial structure arrangement, such as debt service, repayment, and equity investment.

- Political and environmental risks: These risks are the hardest to predict and control among all categories. They could include political support risks, forced buy-out risks, cancellation of concessions, and changes in environmental regulations.

During the analysis, the focus will be on quantifiable risks such as completion risks and operating risks. Those risks that can be hedged by using financial tools or contractual arrangement or that can be insured by commercial insurance would be considered irrelevant during the analysis.

4.3 Valuations of BOT Investments and Profit Structures

Traditional corporate financial theories assumed that the firm's objective is to maximize the value of the firm. However, in BOT it is natural and reasonable to assume that the developers' objective is not to maximize a BOT firm's value; instead, the objective is to maximize the value of the developers' firms, that is, the developers are maximizing the overall profits from the BOT investment. The main reason is that the value of the BOT firm is not the only payoff to the developers. The BOT investment includes the project's future operation, and other potential business activities, such as the construction and operating related services. Figure 4.1 shows BOT investment's profit components through the flowchart of the BOT's participants and cash flows. We will call the three profit components together the BOT's "profit structures" in this research. Therefore, from the developer's perspective, the net profit from a BOT investment is the value of the profit structures minus the initial equity investment. In other words, the valuation of a BOT investment is equivalent to the valuation of the profit structures minus initial equity investment. This observation plays a critical role in the developer's investment valuation, financing decisions, and bidding strategies.

4.3.1 First Component: Equity Value

The first component in the profit structures is the value of the BOT firm's equity. The developers will become the shareholders by holding some fraction of the total equity of the BOT firm and receive dividends. In this research, the fraction shall be defined as the "developer's equity owning ratio," ψ_E. The other $1-\psi_E$ fraction of equity will be defined as the "passive equity" invested by the "passive investors." Passive investors mainly focus on the dividend return from equity investment instead of the development of the projects. Note that if the debt/loan is fairly priced, the equity value will amount to the project's net present value plus the amount of equity investment.

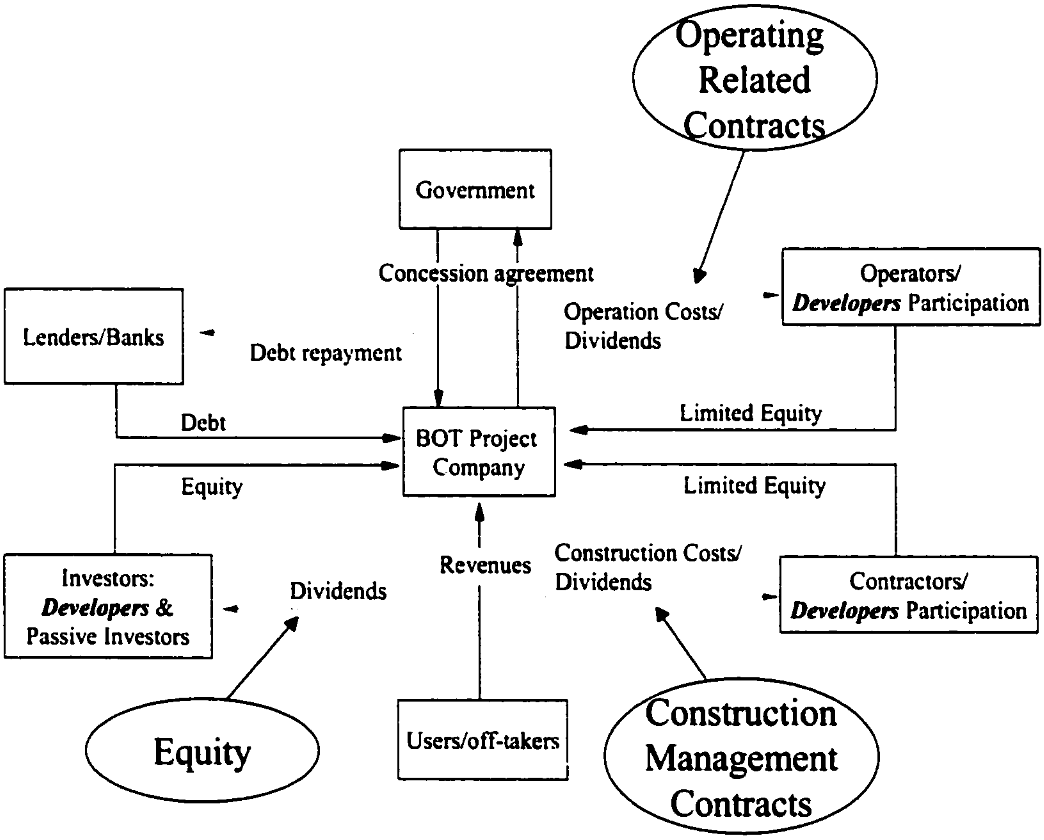

Fig. 4. 1 BOT Developer's Profit Structures

4.3.2 Second Component: Construction Contract Value

The second and third components of the profit structures are from the construction contract and the operating related contract, respectively. These two components are often a major reason why the developer is interested in a BOT project. Most construction firms are thinly capitalized, that is, they rely heavily on short-term debt financing for their capital needs. As a result, they are reluctant to invest their limited and expensive capital in BOT equity without significant profits from the contracts, especially when the construction period is long and the return from the equity investment is slow. Here we call the market value of the profit from the construction contract the "construction contract value."

It is worth noting that in Fig. 4.1, the construction contract value can be subdivided into the "budgeted" construction profit and the "over-budget" construction profit. The budgeted profit here is defined as the least profit assumed by the contractors. We assume that the budgeted profit is included in the budgeted construction cost as a certain percentage, called markup ratio. In a BOT project, it is assumed that the budgeted profit, or markup profit, can be justified even when there is a cost overrun. However, the markup ratio may not be applied to the portion that is beyond the budgeted cost without justifications. If the cost overrun is justifiable, the developer/contractor may be entitled to certain markup profits on the added costs. The profits on the added costs are referred to as over-budget construction profit in this research. If the cost overrun cannot be justified, the developer/contractor cannot make any markup profits over the added costs. In this case, the over-budget construction profit is zero. Because of the flexibility of the real options valuation framework, the contract value can be assessed with respect to different types of contracts, such as the lump sum or target cost contract.

4.3.3 Third Component: Operating Related Contract Value

The third component of the profit structures is the profit from operating related contract, and will be called "operating related contract value" herein. Note that the operating related contract value is different from the operating profit. The operating profit refers to the project's profitability, that is, the project's operating revenue less operating cost. However, here the operating related contract value is due to the *fee-based* profit. The operating related contract typically involves certain fixed charges of fees according to certain formula/criteria, whether the net operating profit is positive or negative. For example, the insurance policies on the properties during the operation are the operating related contracts. The operating related fee has higher claim priority than the debt/bank loan. The operating related contract may be undertaken by the developer or other equity holders. The profit from the operating related contract is other than the project itself and often treated as another side benefit by taking equity risks. In this research, for convenience, it is assumed that the developer will undertake the major operating related contracts. An important characteristic of the operating related contract value is that if the project is forced into bankruptcy or the developer is replaced before the project's completion, the developer will lose this profit from the contracts. Therefore, the profit from the operating related contract is *contingent* on the success of developer's continuing management throughout both construction and operating phases.

4.4 Measurement of Financial Viability: Different Perspectives

It is non-trivial to define and assess a BOT project's financial viability. Moreover, different participants have different perspectives concerning an investment's viability. This research discusses the financial viability from two different perspectives, the perspective of the government and the BOT firm's shareholders, and the perspective of the BOT developer.

We argue that the BOT firm's shareholders and the government share the same perspective in measuring a project's financial viability. First, from the financial theoretical point of view, a firm's financial viability can be judged by the value of the firm's equity since the firm's equity value is the value of the firm's tangible and intangible assets minus the firm's debt. If the value of the firm's assets is sufficiently greater than the debt, the firm will have satisfactory positive net worth, and be considered as "financially viable", and vice versa. Because the BOT project is the only asset of a BOT firm, the BOT firm's equity value can effectively measure the BOT project's financial viability. Second, from the political viewpoint, if a BOT project fails and the BOT firm is bankrupted, the government will incur significant political cost. Thus, the government will try to avoid any development failures in the BOT projects. Since a firm's bankruptcy condition is determined by the value of the firm's assets and the firm's debt, by definition, the BOT firm's equity value is closely related to a BOT project's bankruptcy probability. Moreover, in this research, we will assume that one of the government's major roles is to protect public interests. This assumption will be explained in details in Chapter 7. Thus, the government needs to assure that the project is viable enough so that the equity sold to the passive shareholders is not over-priced. In other words, the investor should not have equity loss for the BOT equity investment. Therefore, from either the financial theoretical or the political viewpoint, the value of the equity or stocks of a BOT firm defines the BOT project's financial viability from the BOT shareholder and government's perspective. We argue that if the equity value is greater than the equity investment amount there will be positive profit from equity investment and the project can be considered "financially viable," and vice versa. Note that this argument is true only when the debt/bank loan is fairly priced, that is, the loan interest rate fairly reflects the project characteristics. Otherwise, the equity investment gain/profit may come from the expense of the debtholder instead of the viability of the project itself. In this research, we assume that the debt is fairly priced. Thus from the shareholder and the government's perspectives, given specific amount of equity investment, the value of the equity measures the project's financial viability.

However, from the developer's perspective, since the overall profit is the profit structures minus initial equity invest, the financial viability of an "investment" is measured by profit structures, instead of equity value. To differentiate the two perspectives, we will use "BOT investment's financial viability" to represent the developer's perspective, and use "BOT project's financial viability" to indicate the shareholder and government's perspective. These two different perspectives will play a critical role in the procurement/tendering process of a BOT project.

4.5 Dynamics/Stochastic Behaviors of the BOT Projects

The construction/completion risks discussed in previous sections are the major uncertainties in the construction phase of a BOT project. Operating/economic risks are the major risks in the operating phase. However, it is not clear which type of risk is more significant. This may depend on the type of BOT project and the concession agreements. For some technically simple projects, the completion risks may be small. For some projects that are exposed to market competition, the operating/economic risks may be large. In fact even if a project is predicted to have low risk, surprises can occur. For example, in the Channel Tunnel project, the construction costs were expected to be less risky due to its technical simplicity, but the actual costs were doubled (Finnerty, 1996). We argue that BOT projects are different from typical industrial investment projects because the amount of capital investment needed, or the construction cost, is uncertain or risky, as is the project's future profitability. The importance of cost uncertainty is no less than the investment's operating/profitability risk. However, traditional capital budgeting theory does not take into account the uncertainty of the project's construction cost. In a BOT investment, it is critical to consider the uncertainties of both construction cost and future operating profit. The modeling of the dynamics of the BOT project's value and construction cost will be discussed later in Chapter 6.

4.6 The BOT Model: Three Analytical Frameworks

Modern option pricing theory, game theory, and the characteristics of BOT projects will be incorporated in a unified BOT model to evaluate a BOT investment. This quantitative BOT model includes three analytical frameworks: (1) the BOT Investment Rationale and Decision Framework, (2) the BOT Profit Structures Valuation Framework, and (3) the BOT Tendering/Procurement Framework. These implementations of the three frameworks will be discussed in the following chapters.

Figure 4.2 shows the BOT Investment rationale and decision framework. This framework is the first element of the BOT model. Rational developers are assumed to maximize their overall profit by maximizing the profit structures minus initial equity investment amount. Under this objective, the developer will search for optimal developing decisions that can maximize the profit structures minus initial equity investment. Therefore, it is crucial to compute the profit structures with respect to various developing strategies. The profit structures could be computed using real options analysis with respect to different financing and bidding decisions. For example, the profit structures may be solved with respect to different initial equity investment or proposed construction costs.

The relationships of the developing decisions and profit structures are obtained by quantitative computation and sensitivity analysis. The relationships are used in determining the optimal developing decisions such as bid price, financing package, and other financial related plans, which will maximize the investment's overall profits. For example, one of the most important factors in different financial alternatives is the amount of initial equity investment given that the total construction cost is funded by equity and debt.

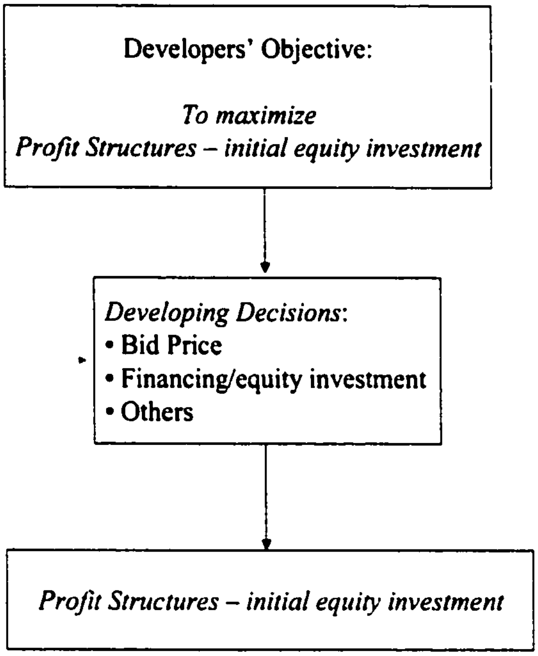

Fig. 4. 2 BOT Investment Rationale and Decision Framework

Figure 4.3 shows the BOT profit structures valuation framework, the second element of the BOT model. It integrates the real option theory and game theory, and computes the value of a BOT investment with respect to different decision variables. The analysis starts by applying game theory on BOT investments. Equilibria and implications are derived from the analysis for obtaining the developer's payoff functions. Participants' payoff functions will become the boundary conditions for analyzing BOT investments in the real options framework. The boundary conditions and the parameters are the inputs in the real options framework. The BOT profit structures valuation framework solves for profit structures, namely, equity value, construction contract value, and operating related contract value. Note that the framework shown in Fig. 4.3 assumes that the BOT investment is a dynamic game with complete information, that is, there is no information that is private to any party. As a result, the parameters used in real options framework, such as the estimated project cost and value and their dynamics, are assumed as common knowledge in the framework. This assumption will be relaxed later in the "signaling game" analysis in Chapter 7 when the possibility of conveying false information regarding the project's financial viability is considered.

The third analytical framework of the BOT model is the BOT tendering/procurement framework shown in Fig. 4.4. The procurement framework focuses on the interactions between the government and the developers under asymmetric information. The framework is based on the game theoretic signaling analysis. Figure 4.4 indicates that the government's BOT policies and the developer's developing and tendering decisions will interact with each other and may have some equilibrium solutions under certain conditions. The game theoretic signaling analysis is adopted to obtain the equilibria. The developer's tendering/bidding strategies and government's procurement policies can be derived from the BOT tendering framework. The implementation of the framework is discussed in Chapter 7, and the case study on the bidding strategies and procurement policies is presented in Chapter 8.

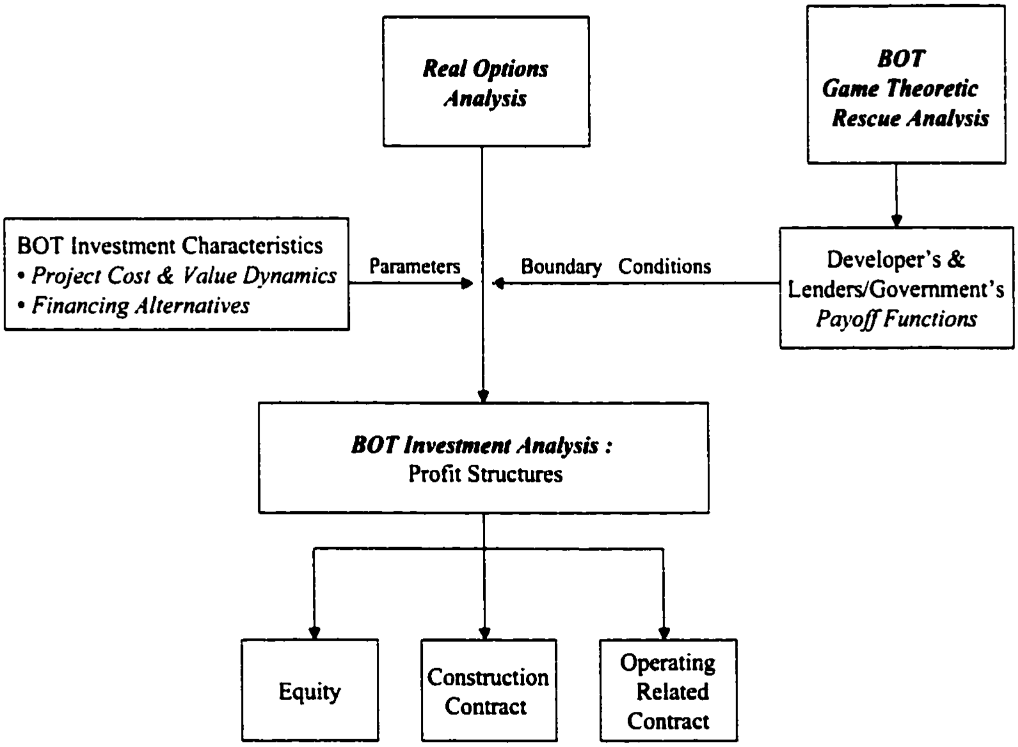

Fig. 4. 3 BOT Profit Structures Valuation Framework

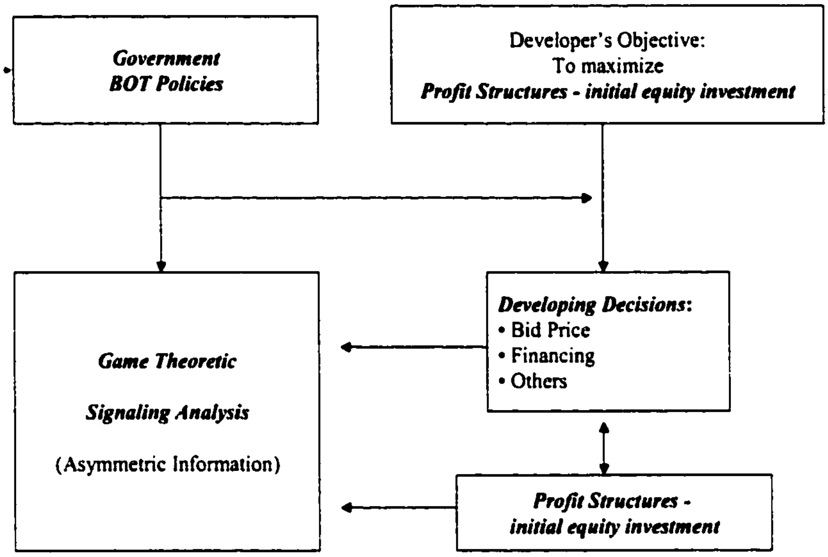

Fig. 4. 4 BOT Tendering/Procurement Framework

CHAPTER 5 GAME THEORETIC RESCUE ANALYSIS ON BOT INVESTMENTS

The game theoretic rescue analysis is the first element in the BOT profit structures valuation framework shown in Fig. 4.3. The purpose of the analysis is to derive the developer's payoff functions used in the real options model. As mentioned in Chapter 4, a BOT investment is in a risky environment. Except for political risk, most risks are related to the uncertainty of cost and project value. In some extremely adverse situations, such as drastic cost overrun or economic changes, the investment may become financially unviable, and cannot be continued or completed. In these adverse developments, the lending bank may refuse to provide extra capital needs/debt for covering cost overrun, or the developer may learn that the changes in economic situations have made the BOT investment depreciate too much to be worth continuing. In view of the spirit of the BOT scheme, the developer should fully assume the completion and operating risk and reflect the risk on the valuation of the investment. The only exception is when the government grants debt guarantee or other subsidies that will ensure the completion or continuation of the project should the project fail. Fully assuming the completion and operating risk by the developer will reduce the value of a BOT investment and thus lower the investment's financial viability or raise the investment's threshold (e.g., a higher project value).

Nevertheless, it is naïve to presume that the developer will fully assume such risk in the investment valuation process. In many cases it is not rational for the developer to fully assume the completion and operating risk due to the developer's option to negotiate for subsidies should unforeseen problems arise. This fact has important impacts on the investment valuation and government BOT policies. These impacts are analyzed and discussed in Chapters 6 and 7. In this chapter, game theoretic framework is applied to analyze the responding actions of the developer and government when extremely adverse situations occur.

5.1 BOT Game Theoretic Model: Dynamic Game with Complete Information

The game theoretic framework for analyzing BOT investments shown in Fig. 5.1 is a dynamic game expressed in an extensive form, where D is the developer and G is the government.

Suppose the BOT investment concession agreement does not specify any government debt guarantee or subsidies in the face of critical adverse events. Neither does the law prohibit the government from rescuing a failing BOT project by providing debt guarantee or extending the concession period. However, the government is not encouraged to rescue a project without compelling and justifiable reasons. For example, cost overrun caused by inefficient management or incorrect cost estimate is difficult to justify for government rescue, whereas cost overrun caused by unexpected unusual equipment/material price escalation may be justified more easily. If the government grants a subsidy to a failing project on the basis of unjustifiable reasons, a loss of public trust or suspicion of corruption is a possibility.

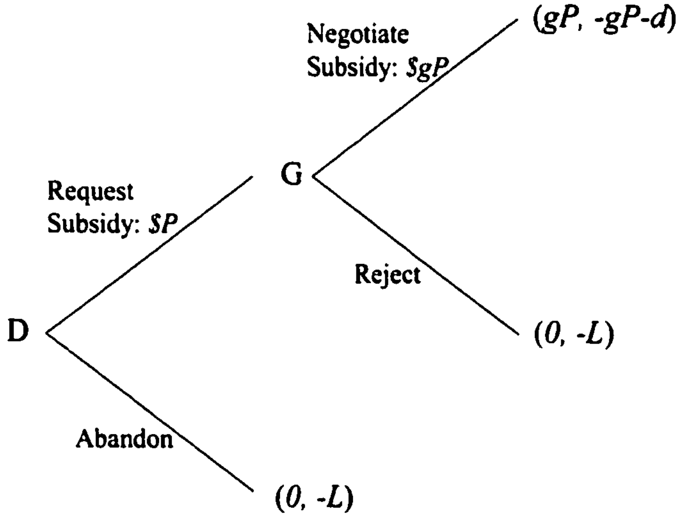

Fig. 5. 1 BOT Negotiation Game under Adverse Development

The dynamic game starts from adverse situations such that the developer has to abandon the project if the government does not rescue the project. As shown in Fig. 5.1, the developer can decide to abandon the project. More importantly, the developer can also request the government to rescue and subsidize for the amount of \$*P* even when the contract clause specifies no government debt guarantee and requires the developer to take full completion/economic risk. If the developer abandons the failing project, the payoff will be $-\varepsilon$, where $\varepsilon \to 0$. The main reason is that if the situations come down to the needs of bankruptcy or abandonment, the value of the *equity* should approach zero before the abandonment; therefore, the developer, being an equity holder, will lose little if the failing project is abandoned. Here it is assumed that $\varepsilon = 0$. On the other hand, in the game model if a BOT project is abandoned, the payoff of the government is $-L$, where L is the opportunity cost incurred due to the abandonment. Generally, from either a financial or political perspective, it is very costly for the government to abandon a BOT project. Financially, when a BOT project is abandoned, millions or billions of investment funds will be wasted. Even if the project is being purchased or taken over by another new developer, the transaction cost and schedule interruption cost will be significant. Politically, the abandonment of a BOT project that has spent millions or billions of dollars signifies nothing but the incompetence of the government.

Alternatively, the developer can request subsidy in the amount of \$*P* from the government as shown in Fig. 5.1. The subsidy can be in various forms. For example, in serious cost overrun, the bank typically will not provide extra capital needs without government debt guarantee. The debt guarantee is a liability to the government and an asset to the BOT firm. Therefore, debt guarantee is a subsidy from the government. Other forms of subsidy may include the extension of concession period, the future tax exemption for a certain number of years, and the extra loan from government instead of banks.

After the developer's request for subsidy, the game proceeds to its subgame, shown in Fig. 5.1: negotiate subsidy or reject. If the government, G, rejects the developer's request, the developer will abandon the project and the payoff for both parties will be $(0,-L)$ again.

If the government decides to negotiate a subsidy ratio, *g*, the payoff of the developer and government will be (gP, $-gP-d$), respectively, where *d* is the cost other than the subsidy, gP. Note that *d* can be seen as the political cost. For example, to rescue a BOT project from bankruptcy or abandonment usually will bring criticism toward government. If the government lacks compelling reasons for the subsidy, the criticism will cause significant *political* cost measured by the magnitude of project cost. In some cases, *d* can be very small if the subsidy is justifiable. A reasonable conjecture is that the government will be comparing the cost of "negotiate subsidy," $gP+d$, and the cost of "reject," L. If $gP+d \leq L$, the government will "negotiate subsidy" with the developer, and vice versa. This BOT game model is analyzed in next section.

5.2 Negotiate or Abandon: Nash Equilibria

According to section 2.3, the BOT dynamic game tree derived above will be solved backward recursively and its equilibrium solution, the Nash equilibrium, will be obtained. Since the variables of the game's payoff matrix are undetermined, the payoff comparison and maximization cannot be done to solve for a unique solution. However, we can analyze the conditions of possible Nash equilibria of the game tree in Fig. 5.1. Our first conjecture is that there may be three candidates for Nash equilibrium, namely (1) developer "request subsidy," and government "negotiate subsidy," (2) developer "request subsidy" and government "reject," and (3) developer "abandon."

1. Developer "request subsidy" and government "negotiate subsidy."

   Here, since the government chooses to "negotiate subsidy," this equilibrium will be called "rescue" equilibrium. Solving backward from the government's node first, if the payoff from negotiation is greater than that from rejection, that is, $-gP-d \geq -L$, the government will "negotiate subsidy" with the developer. Therefore, the condition for negotiation or rescue is $-gP-d \geq -L$, and can be rewritten as

$$gP \leq L-d \tag{5.1}$$

This condition is straightforward: the negotiation offer should not be greater than the cost saving from rescuing the project. The payoff for the developer and government will be (gP, $-gP-d$), respectively, as shown in Fig. 5.2. Figure 5.2 is the subgame obtained by solving Fig. 5.1 backward.

The next step is to solve Fig. 5.2 backward again and obtain the final solution. According to Fig. 5.2, the developer will request subsidy if $gP \geq 0$. Since g and P will not be negative numbers, the condition for the developer to negotiate will always be satisfied. In other words, it is always to the developer's benefit to negotiate subsidy if equation (5.1) is satisfied. Therefore, for both government and developer, there must exist $g \in [0, 1]$ if and only if $L-d \geq 0$. In other words, the "rescue" solution will be the Nash equilibrium if and only if equation (5.2) is satisfied.

$$L-d \geq 0 \tag{5.2}$$

Figure 5.3 shows the equilibrium path expressed in bold lines that goes through the game tree followed in equilibrium. Note that equation (5.1) also shows the possible range of the negotiation offer. That is, when the developer requests subsidy for P, the final settlement for the subsidy will be a portion of P, gP, that satisfies equation (5.1). The range of negotiation offer can be expressed as

$$gP \in [0,\ L-d] \tag{5.3}$$

Alternatively, the negotiation offer can be expressed by its ratio, g:

$$g \in \{x : 0 \leq xP \,\&\, xP \leq (L-d)\} \tag{5.4}$$

From equation (5.3) or (5.4), we know that there must exist $g \in [0, 1]$ if and only if $L-d \geq 0$. In other words, as long as $L-d \geq 0$, the "rescue" equilibrium will be the Nash equilibrium of the game. No one can be better off by deviating from this equilibrium.

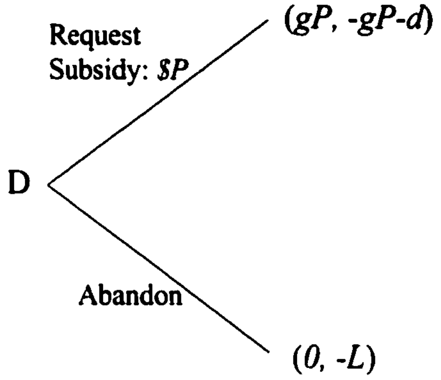

Fig. 5. 2 Developer's Decision Node and Payoffs

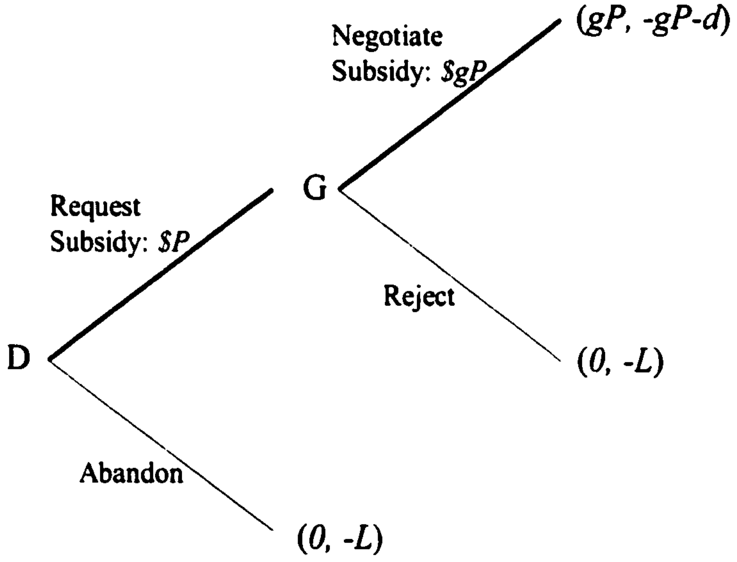

Fig. 5. 3 BOT Negotiation Game's Equilibrium Path

2. Developer "request subsidy" and government "reject."

   If equation (5.1) is not satisfied, "reject" would be a preferable decision to the government. As a result, the payoff matrix for both parties is $(0, -L)$. Now turn to the developer's node: it seems that the payoff of either "request subsidy" or "abandon" is $\$0$, and the developer is indifferent to the two actions. From the game tree, it is not obvious which action the developer will choose if equation (5.1) is not satisfied. Nevertheless, if the developer recognizes the existence of the cost incurred in the process of requesting subsidy, although it may be small, the developer should "abandon" the project instead of requesting subsidy. From this perspective, the cost of requesting subsidy should be recognized whenever there is a tie between "request subsidy" and "abandon," although the cost of requesting subsidy is suppressed in the game tree for clarity. To summarize, if the developer knows the government will "reject" the subsidy request, the developer should choose "abandon" instead of "request subsidy" in the first place, and this is exactly our third possible equilibrium, developer "abandon." The second equilibrium solution cannot exist.

3. Developer "abandon."

   Here, since the developer knows that the government will choose to "reject" the subsidy request, the developer will abandon the project in the first place. We shall term this equilibrium the "no rescue" equilibrium. As argued above, the developer will abandon the project if and only if it is optimal for the government to "reject" the subsidy request. The condition of this Nash equilibrium is $-gP-d < -L$, which can be rewritten as:

   $$gP > L-d \tag{5.5}$$

   In fact, this condition is not very helpful, and a stronger condition is needed. For example, in the first equilibrium solution, we conclude that if $L-d \geq 0$, the Nash equilibrium will be the first solution. However, according to equation (5.5), one may argue that it is possible that $L-d \geq 0$ and yet equation (5.5) is satisfied as well, so that the Nash equilibrium becomes the third one. This would conflict with our previous analysis. Thus, a deeper analysis is needed for the conditions of "no rescue" equilibrium. Examining the tree in Fig. 5.1, we find that the only two possible choices for the developer are to "request subsidy" and to "abandon," with payoff matrix $(gP, -gP-d)$ and $(0, -L)$, respectively. For the "abandon" to be an equilibrium solution, it must be that it is impossible to achieve the "rescue" solution. In other words, g must belong to an empty set. According to equation (5.4), we obtain

$$g \in \phi \quad \text{if and only if} \quad L\text{-}d < 0 \tag{5.6}$$

Therefore, the conditions for the "abandon" to be a Nash equilibrium are equation (5.5), $gP > L\text{-}d$, and $L\text{-}d < 0$. These two conditions can be combined into one condition:

$$L\text{-}d < 0 \tag{5.7}$$

Equation (5.7) can also be obtained by negating the condition of the "rescue" Nash equilibrium, equation (5.2).

To conclude this section, we find two conditions, equations (5.2) and (5.7), for BOT game's two possible Nash equilibria, "rescue" and "no rescue," respectively. Both equilibria depend solely on the knowledge of government's abandon cost and rescue's political cost. We shall assume that the BOT game is a game with *complete information.* As a result, L and d are common knowledge and both parties can easily solve for the game's Nash equilibrium. However, from the practical perspective, it is not easy for both parties to quantify L and d, because it is difficult to measure the political cost in L and d in terms of monetary units.

5.3 Refinement of Nash Equilibria

Fortunately, the game depicted above can be analyzed without knowing the exact amount of L and d. To perform this analysis, we need to examine the characteristics of the BOT project, especially its bankruptcy conditions and the bankruptcy's associated political costs.

5.3.1 BOT Project's Bankruptcy/Abandonment and the Associated Costs

To analyze the adverse conditions that place the BOT on the edge of bankruptcy or abandonment, we will use some concepts and notations from Chapter 4. A very common bankruptcy condition in debt indenture is the inability of the borrower to meet the repayment schedule. In the BOT project's construction phase, since there is no revenue generated by the project, the repayment schedule is mainly the payment of loan interest. Since the interest is relatively small compared with the actual capital from the loan providers and equity holders, the BOT project will never be bankrupted under standard debt indenture. As a result, the lending bank will impose other conditions to trigger the bankruptcy and protect the loan should severely adverse events happen. For example, the lenders could specify the upper limit of cost overrun during the construction. In general, rational lenders will want to prevent the *net value of the project* from being below the *debt outstanding*. Therefore, one feasible assumption for the bankruptcy condition is

$$V_t - D_t(K_t)e^{-r_d(T-t)} < 0 \qquad (5.8)$$

where V_t is the project value, computed by the expected operating cash flows resulting from the project if the project is completed at time *t*. $D_t(K_t)e^{-r_d(T-t)}$ is the estimated total outstanding debt discounted by the loan interest rate, r_d. In other words, $D_t(K_t)e^{-r_d(T-t)}$ is the estimated total debt at time *t* prices. Equation (5.8) suggests that if the project value estimated at *t* is less than the estimated required total debt at *time t prices*, the lending bank will bankrupt the project to prevent further loss. Note that similar to V_t, D_t is uncertain and depends on estimated construction cost, K_t. More specifically, D_t is a function of K_t and is denoted as $D_t(K_t)$. We shall analyze the relationship between D_t and K_t in Chapter 6. If we assume that the lending bank can effectively monitor the progress and update V_t and K_t during construction, we may infer that at the time of bankruptcy, the value of the unfinished project is very close to the outstanding debt; that is, it is at the brink of bankruptcy conditions. In other words, the bank may want to retract and protect the lender or debt holder by bankruptcy when $V_t - D_t$ is close to zero. It is not wise for the bank to continue providing additional capital, because it is very likely that the BOT firm will not be able to repay any further lending in the future. Unless the government guarantees the repayment of the extra loan, or subsidizes the project by extending the concession period or by other means to secure the additional debt, the lending bank will deny further capital needs and bankrupt the BOT firm.

If a project is abandoned or bankrupted, it can be sold to other private developers or the government. To somewhat simplify our analysis, we assume that the project will be sold to the government upon bankruptcy, since the government may want to regain the control of the project after previous failing development. Another reason is that typically the BOT contract cannot be directly transferred to a new developer without a new negotiation with the government, that is, a new unique contract has to be signed between the new developer and the government. Therefore, one may consider the abandoning process as the government's "re-purchasing" and "re-tendering" a project. That is, one may consider the bankruptcy as a costly replacement of the developer. Thus under normal situations, the failing project purchased by the government will still be financed mainly by debt, and the subsidies for securing the lending bank's new loan is still essential in order to complete the project. As a result, when a project is abandoned, the cost of the government, L, can be modeled as

$$L = G + C \tag{5.9}$$

where G is the least required subsidy that can persuade the lending bank to support a failing project, and C is the political cost from the criticism toward the government on the actual failure or bankruptcy of a BOT project.

On the other hand, if the government negotiates the subsidy with the existing developer and rescues the project, the cost incurred would be the negotiated subsidy, gP, and the political cost, d. Here we model the rescue's political cost as

$$d = C' + Y \tag{5.10}$$

where C' is the political cost from the criticism toward the government on the *failing but rescued* BOT project, Y is the political cost due to the unjustifiable subsidy. Y can be further modeled as

$$Y = \begin{cases} 0 & \text{if } gP \leq J \\ \infty & \text{if } gP > J \end{cases} \tag{5.11}$$

where J is the amount of the subsidy that can be justified. We shall call J the "justifiable claim," which can be described by imagining that if the request went to court, what amount of "claim" by the developer the court would have granted. If the subsidy is less than the justifiable claim amount, the government will not be blamed for over-subsidy, and the amount of Y will be assumed as 0. If the subsidy is greater than J, the government may be criticized and accused of corruption, and the extra political cost, Y, besides the project's failure, is considered as very large.

5.3.2 Refined Nash Equilibria

Equation (5.3) shows that the range of negotiation offer is $[0, L\text{-}d]$. This range could be so wide that the equilibrium cannot offer insightful conclusions. However, it can be refined by considering more characteristics specific to BOT projects. The first refinement is due to the lending bank's constraints. As argued above, the lending bank will not provide extra capital needed unless the government's subsidy can secure the loan within reasonable risk range. Any subsidy less than the least required subsidy, G, will not be accepted by the bank even if the developer is willing to accept it. This constraint refines the negotiation offer range to

$$gP \in [G, L-d] \tag{5.12}$$

The second refinement is due to the characteristics of the bankruptcy conditions and government's bankruptcy cost. Combining equations (5.9) and (5.10), we obtain

$$L-d = G + C - (C' + Y) = G + (C - C') - Y \tag{5.13}$$

Together with equation (5.12), we have

$$gP \in [G, G + (C - C') - Y] \tag{5.14}$$

Note that if we assume that the political cost of a failing but rescued project, C', is slightly less than the political cost of a failing and bankrupted project, C, we will have $C - C' = \varepsilon$, where ε is a positive number. We can rewrite the upper bound of gP as $G + (C - C') - Y$ to $G + \varepsilon - Y$. Equation (5.14) will become $gP \in [G, G + \varepsilon - Y]$, and gP will have a solution only when $G + \varepsilon - Y \geq G$, that is, $\varepsilon - Y \geq 0$. According to (5.11), $\varepsilon - Y$ will be greater than 0 only when $Y = 0$ or $gP \leq J$. For simplicity, we assume that ε is *small* and can be ignored. Then, gP will have solutions, $gP \in [G, G + \varepsilon] \rightarrow G$, when $gP \leq J$ or equivalently, $G \leq J$. In other words,

$$gP = G \quad \text{if and only if} \quad J \geq G \tag{5.15}$$

$$gP = \phi \quad \text{if and only if} \quad J < G \tag{5.16}$$

Equations (5.15) and (5.16) are the refined Nash equilibria of the BOT rescue game. First, when $J < G$, the government cannot justify the least subsidy, G. Because the lending bank cannot accept the subsidy less than G, the only solution is "abandon" or "no rescue." Second, when $J \geq G$, the government would be worse off with "no rescue" if the government offers any subsidy that is greater than G. Therefore, the government would offer the least required subsidy, G, and the lending bank would accept it. In summary, for a failing project, if its justifiable subsidy is more than the least required subsidy, the negotiated subsidy will be G; otherwise, the equilibrium will be "no rescue."

Compared with the solution in equation (5.3), our refined equilibrium solutions are more specific and insightful. In addition, it is easier to estimate the common knowledge needed to solve the game, G and J. In the following chapters, we will integrate the refined equilibria with the real options model in analyzing the valuation, financing, and tendering policies of BOT project investments.

5.4 Refined Nash Equilibria and Real Options Analysis

5.4.1 "Rescue" Nash Equilibrium

Holliday et al. (1991) argued that because of the scale and complexity of BOT projects, very often they are *developer-led*, and it is extremely difficult to identify a *clear client-contractor relationship* at the heart of the project. Therefore, it is possible that the developer is able to manipulate the situations, such as the clauses in the construction contract, in order to justify the future subsidy should significant cost overrun or project value depreciation occur. In other words, it is not very difficult to manipulate a large justifiable subsidy, J, so that no matter what happens in the future the condition $J \geq G$ can be satisfied. When $J \geq G$, the BOT game's Nash equilibrium solution will be to "rescue" the project until it is completed. As a result, unless the government imposes some specific policies to discourage government's future rescuing actions, the developer can continue to construct the project until the project is completed. There exists an implicit loan guarantee from government that enables the developer to assume that the project can obtain subsidy if any critical adverse events occur.

At the completion of the project, if the project value is still less than the outstanding debt, the lending bank will either request that the government buys back the project and fulfills the debt guarantee, if there is any, or sell the project to other developers. If the project value is greater than the outstanding debt, the developer will continue to manage the project. Therefore, in the "rescue" equilibrium, we will model the payoffs on the basis of the bankruptcy condition. The condition is given in equations (5.17) and (5.18).

1. The developer will continue to manage the BOT project if and only if

$$V_T - D_T(K_T) \geq 0 \tag{5.17}$$

where V_T and K_T are the project value and total construction cost at completion time, T, respectively. We will formally define V and K in Chapter 6.

2. The developer will sell the project to the government or other parties if and only if

$$V_T - D_T(K_T) < 0 \tag{5.18}$$

According to the analysis in Chapter 4, the BOT investment valuation is the valuation of the developer's profit structures. The implications due to the "rescue" Nash equilibrium are critical to the valuation of the profit structures. For example, equations (5.17) and (5.18) are critical to the derivation of the terminal payoff functions for each component in the profit structures. The payoff function is one of the key elements in the real options valuation model. We will derive these payoff functions in Chapter 6.

5.4.2 "No Rescue" Nash Equilibrium

Another possible Nash equilibrium of the BOT game is to "abandon" the project, or the "no rescue" equilibrium. As argued above, unless the government imposes some policies, the "rescue" equilibrium will be a dominant solution. Suppose the government wants to restrict itself from any future rescue actions. To achieve this goal, according to equation (5.16), the government should make the future justifiable amount, J, as small as possible. One way to accomplish this task is to have a legal system that makes any government's rescue actions illegal. This would make $J=0$ in any situations. The impact of this kind of policy toward the developer's investment evaluation process will be discussed in the following chapters. Although sometimes the cost overrun or project value depreciation is caused by the developer's inefficient management, in this case, J would be very small as well.

The payoff functions of the "no rescue" equilibrium are different from that of the "rescue" equilibrium, since the abandonment typically occurs before the completion of the project. The payoff functions needed under "no rescue" equilibrium will be based on the "bankruptcy" condition in equation (5.8). Similar to American options, when the project is abandoned or "exercised," one needs to determine the payoff at the time of abandonment, which can be any time t. We may call the payoff functions at any time t as the "time t"

payoff functions as opposed to the terminal payoff functions in the previous section. The time t payoff functions will be derived in Chapter 6.

CHAPTER 6 REAL OPTIONS AND GAME THEORETIC VALUATION OF BOT INVESTMENTS

This chapter integrates the results of game theoretic analysis with the real options analysis. The valuation model derived in this chapter is the implementations of the "*BOT Profit Structures Valuation Framework*" shown in Fig. 4.3. Therefore, the valuation model plays a crucial role in the BOT model. This chapter starts by introducing a very powerful numerical method in option pricing theory, the binomial option pricing model. Section 6.2 further investigates the numerical models that are suitable for BOT investment valuation. The characteristics of the BOT project future cash flow and cost are explicitly considered in section 6.3. Section 6.4 discusses how the BOT profit structures are evaluated through the numerical method derived. The evaluation criteria of a BOT investment is developed in section 6.5. Last, an illustrative example shows the implementation of the BOT profit structures valuation framework.

6.1 Binomial Option Pricing Model

Cox et al. (1979) developed a "binomial tree" approach, a numerical model, for solving options prices. As mentioned in section 2.5, there are numerical schemes other than binomial model. Because of the binomial model's advantages of simplicity and flexibility, the real options analysis of BOT investments will be based on this type of model. To solve the option price that is consistent with the capital market, the basic concept used is the "no-arbitrage" argument or principle in economic theories. That is, if the asset is priced too high or too low, the arbitrage transactions will adjust the prices until the market equilibrium is reached and there exists no arbitrage opportunity.

6.1.1 One-Step Binomial Tree and Replicating Portfolio

The one-step binomial tree is usually used to illustrate the use of the "no-arbitrage" concept in option pricing. In the binomial model, the stock price is assumed to follow a binomial process over discrete periods. Then a *replicating portfolio* of an option is created. A replicating portfolio comprises some specific shares of stocks and bonds that will replicate the option by generating identical monetary payoffs. Therefore, the price of the replicating portfolio would equal the price of the option being replicated.

Here, a numerical example is used to illustrate the concept of the replicating portfolio and its implication on option pricing. Take the IBM stock call option mentioned in section 2.4.2. The 3-month IBM stock call option has an exercise price of \$120. Suppose that the current IBM stock price is \$110 and the risk-free interest is 5%. If at the end of 3 months, the stock price will be either \$130 with probability 50% or \$100 with probability 50%. What would be the option price? To solve this problem, a replicating portfolio will be created. Suppose this portfolio consists of Δ units of IBM stocks and the dollar amount of B in riskless bond. Figure 6.1 shows the tree representations of IBM stock and bond price's movement. Then we can use Fig. 6.2 to show the replicating portfolio's movement. Consider the option price at exercise day. If the stock price is \$130, the option price is \$10, and if the stock price is \$100, the option price is \$0. The option price movement is shown in Fig. 6.3. Comparing Fig. 6.2 and 6.3, if the end-of-period values of both trees are the same, then the current value of the portfolio must be equal to the current value of the option. Otherwise, there would exist arbitrage opportunities in the market. This portfolio is the so-called "replicating portfolio" and it must satisfy

$$\Delta\, 130 + B\,(1+5\%*3/12) = 10$$

$$\Delta\, 100 + B\,(1+5\%*3/12) = 0$$

Solving these two equations, we have $\Delta = 0.333$ and $B = -32.922$. As a result, the current option value is the value of the portfolio, that is,

$$0.333*\$110 - \$32.922 = \$3.708.$$

It is worth noting that the binomial approach is a discrete-time model and its precision depends on how small the time step is. When the time step approaches zero, the discrete-time solution would approach Black and Scholes' continuous-time analytical solution.

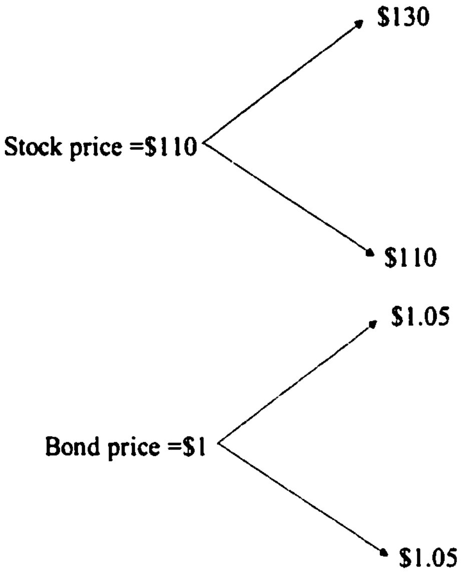

Fig. 6. 1 Tree Representations of Stock and Bond Prices

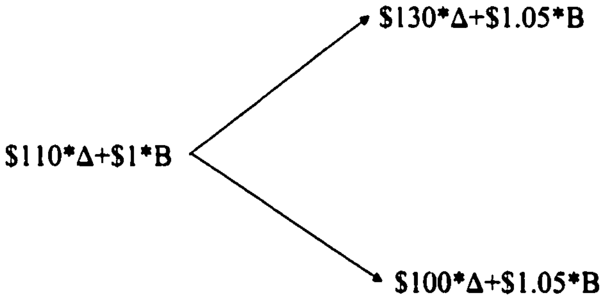

Fig. 6. 2 A Replicating Portfolio

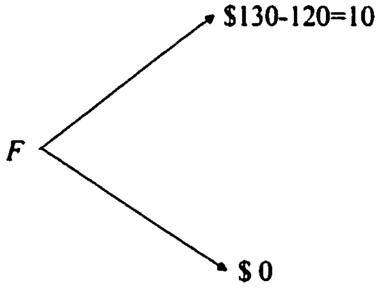

Fig. 6. 3 Tree Representation of the Option Price

6.1.2 n-Step Binomial Tree

On the basis of principles used in the one-step binomial tree, Cox et al. (1979) derived a generalized binomial model for pricing the options. The first step is to construct a binomial tree of the stock price movements. Given a specific distribution of the asset price in the future, one can transform the distribution into a binomial tree, as shown in Fig. 6.4 where S is the stock price. For illustration purposes, we divided the option's duration into only three periods. In an actual implementation, the total duration should be divided into a large number of periods so that each period is short enough to ensure the computation accuracy. After each period, the stock price can either go up by certain percentage, u, for example, 120%, or go down by $d=1/u=1/(120\%) = 83.3\%$ with the probability q and $1-q$, respectively. Here, S_{uud} represents that S goes up at period 1 and 2, and down at period 3.

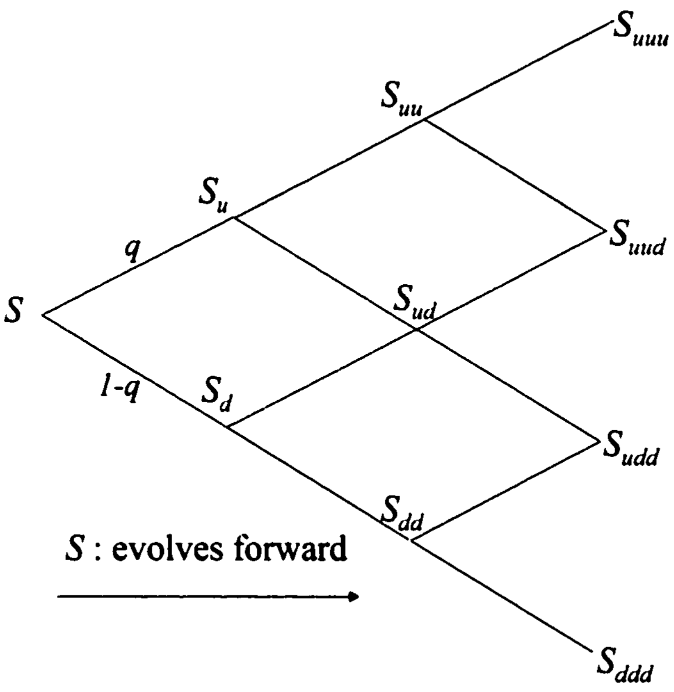

Fig. 6. 4 Dynamics of the Stock Prices

The second step is to construct an option valuation tree and perform the calculation as shown in Fig. 6.5, where F is the value of the option. Figure 6.5 shows that we solve the option price backward recursively from the maturity date. Upon maturity, the option price is $Max(0, S - X)$, where S is the exercise price. For example, $F_{uuu} = Max(0, S_{uuu} - X)$. At period 2, the option value at each node is obtained by computing the discounted expected period 3 option value. For example, F_{uu} can be computed by the following equation:

$$F_{uu} = \frac{1}{R}[qF_{uuu} + (1-q)F_{uud}], \tag{6.1}$$

where R is the discount factor and equal to $e^{r\Delta t}$, r is risk-free interest rate, Δt is the length of a time step, and $q \equiv \frac{R-d}{u-d}$. F_{ud} may also be computed in the same manner. Solving backward, F can be obtained by

$$F = \frac{1}{R}[qF_{uu} + (1-q)F_{ud}] \tag{6.2}$$

It is worth noting that one advantage of the model is that it can solve options with early exercise feature, such as American options. In American options, one needs to perform the calculation shown in Fig. 6.5 as well. The period 3 calculation is the same as the European options. However, the period 2 calculation involves the *comparison* between the discounted period 3 option price's expectation and the early exercise profit at period 2. If the early exercise profit $S_{period2}$-X is greater than $F_{period2}$, then one should exercise/invest at period 2; otherwise, one should keep the option alive and *wait* until period 3. For instance, as shown in Fig. 6.5, the option price when $S = S_{uu}$ is

$F_{uu} = Max(S_{uu} - X, \frac{1}{R}[qF_{uuu} + (1-q)F_{uud}])$. This is the so-called optimal stopping decision in stochastic dynamic programming. The tree in Fig. 6.5 for American options becomes a decision and valuation tree.

For the n steps binomial tree, the solution is solved backward recursively as shown above. The only question remaining in using the binomial model is how to decide the values for u and d. This question will be answered in the following sections.

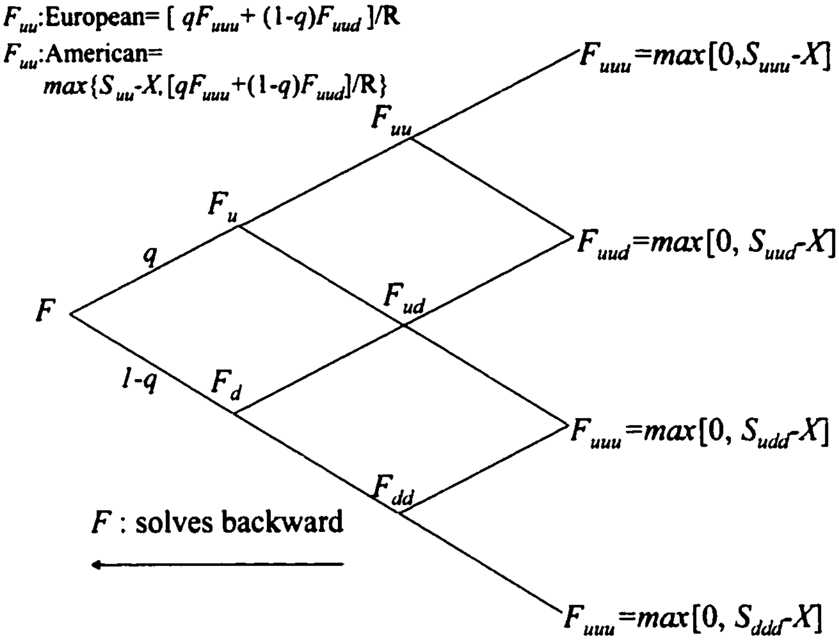

Fig. 6. 5 Binomial Tree for Option Valuation

6.1.3 Binomial Model and Risk-Neutral Valuation

Equation (6.1) or (6.2) has a simple interpretation consistent with the risk-neutral valuation introduced in section 2.4: by pretending the world is risk neutral, the option price can be computed by taking the expectation of the contingent payoff and discounting the expectation at a risk-free rate. This option price will be valid for the world of risk averse. To see how equation (6.2) pretends the world is risk neutral, let us use the probability q instead of p to compute the expected price of the stock after a time increment, Δt, where q is obtained by no-arbitrage argument and p is the stock's true probability. The expected stock price under probability q would be

$$E(S_{t+\Delta t}) = qS_u + (1-q)S_d = \frac{R-d}{u-d}uS + \frac{u-R}{u-d}dS = \frac{RS(u-d)}{u-d} = e^{r\Delta t}S \qquad (6.3)$$

Note that equation (6.3) shows that by using q, the expected rate of return of the stock is a risk-free rate; that is, the stock behaves as if it were in the risk-neutral world. In other words, using q for stock dynamics is equivalent to pretending the world is risk neutral. This is why q is also named the "risk-neutral probability."

6.1.4 Risk-Neutral Stock Dynamics and Values of *u* and *d*

Under the risk-neutral valuation framework, u and d can be obtained to compute $q \equiv \frac{R-d}{u-d}$. Suitable values for u, d, and q could be found by matching both the mean and variance of the logarithm of a price change (Luenberger, 1998). Assuming that $S_0=1$, the matching gives us

$$E(\ln S_{t+\Delta t}) = q\ln u + (1-q)\ln d \tag{6.4}$$

$$\operatorname{var}(\ln S_{t+\Delta t}) = q(\ln u)^2 + (1-q)(\ln d)^2 - [q\ln u + (1-q)\ln d]^2 = q(1-q)(\ln u - \ln d)^2 \tag{6.5}$$

According to risk-neutral valuation, assume that the stock dynamics is as shown in equation (2.31): $\frac{dS}{S} = rdt + \sigma dz$. Applying Itô 's lemma introduced in section 2.4 to the stock dynamics, equation (6.6) can be obtained

$$d\ln S = (r - \frac{1}{2}\sigma^2)dt + \sigma dz \tag{6.6}$$

Equation (6.6) implies that $\ln S$ is normally distributed with mean, $(r - \frac{1}{2}\sigma^2)dt$, and variance, $\sigma^2 dt$. Converting the mean and variance to their discrete-time forms, equations (6.4) and (6.5) can be rewritten as

$$qU + (1-q)D = (r - \frac{1}{2}\sigma^2)\Delta t \tag{6.7}$$

$$q(1-q)(U-D)^2 = \sigma^2 \Delta t \tag{6.8}$$

where $U = \ln u$ and $D = \ln d$.

By imposing another condition for convenience, $u = \frac{1}{d}$, that is, $U = -D$, equations (6.7) and (6.8) can be solved:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{6.9}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{6.10}$$

where terms of higher order than Δt are ignored. Note that although it is possible to find an expression of q by solving equations (6.7) and (6.8), the expression will have the identical value of $q = \frac{R-d}{u-d}$. Therefore, we shall continue to express $q = \frac{R-d}{u-d}$.

Alternatively, u and d may be solved by imposing risk-neutral probability $q = 0.5$ in equations (6.7) and (6.8) (Hull, 1997). In this case, u and d are given as

$$u = e^{(r-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}} \tag{6.11}$$

$$d = e^{(r-\frac{1}{2}\sigma^2)\Delta t-\sigma\sqrt{\Delta t}} \tag{6.12}$$

Hull (1997) argued that this alternative set of u, d, and q has the advantage that it eliminates the possibility that q may become negative. In the model of Cox et al. (1979), the risk-neutral probability, q, may become negative when $\sigma < \left|(r-q)\sqrt{\Delta t}\right|$, and this negative probability could cause problems in the model's interpretation and numerical computation.

6.1.5 Binomial Model for Dividend-Paying Stock

The binomial model can be used to price options on dividend-paying stocks. In principle, a stock's price will fall by the amount of dividend paid at the date of payment, or ex-dividend date. Thus, dividend paying can be modeled as stock price reduction. Option price will be affected by the stock price reduction. Dividends can be modeled in the form of dollar amount or of the dividend yield, a proportion of the stock price. In the BOT profit structures valuation model and most real options analysis, the dividend yield will be used for convenience. Suppose that the stock pays annual dividend yield, δ, continuously, the stock's risk-neutral dynamics can be remodeled as

$$\frac{dS}{S} = (r-\delta)dt + \sigma dz \tag{6.13}$$

As a result, the formula of *u* and *d* will become:

$$u = e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t+\sigma\sqrt{\Delta t}} \tag{6.14}$$

$$d = e^{(r-\delta-\frac{1}{2}\sigma^2)\Delta t-\sigma\sqrt{\Delta t}} \tag{6.15}$$

Other computations remain the same as in the nondividend paying stock's options. Note that this dividend-paying feature is crucial in the real options analysis. More explanations will be given in section 6.3.

6.2 Valuation of BOT Profit Structures: Binomial Models with Two State Variables

In BOT investments, the major uncertainties/risks underlying the investment are the value of the project, V_t, and the construction cost, K_t. The subscript *t* represents the value of *V* and *K* if the project is completed at time *t*. The value of the BOT profit structures is *contingent* on the values of *V* and *K*. From this perspective, the value of the BOT profit structures is a contingent claim or real option, and the underlying assets of the real option are *V* and *K*. Solving the valuation problem is equivalent to solving option pricing problems with two underlying assets/securities. Various models that deal with multi-state variables will be discussed in this section. A binomial pyramid model will be derived and implemented for the purpose of BOT profit structures valuation.

6.2.1 Valuation Models for Options on Two or More Assets

There are various models for options on two or more underlying assets. Margrabe (1978) derived an analytical solution for the *European exchange option* that gives its owner the right to exchange one risky asset for another. McDonald and Siegel (1985) valued the European exchange option on dividend paying underlying assets in an analytical form. The consideration of the dividend-paying feature is critical in the real options analysis. In the

American style option, McDonald and Siegel (1986) found an analytical solution for perpetual American exchange options. A perpetual option is an option without an expiration date. Carr (1988) solved the sequential exchange option that gives the owner the right to exchange a risky asset for another potential for further exchange. He derived a method of valuing nonperpetual American exchange options. These analytical solutions can be applied in the real options analysis only when the forms of the real option's terminal conditions match the solution's specific form. This restricts the use of the analytical solutions in the real options analysis.

As mentioned in section 2.4, many numerical methods are available in solving option pricing problems. Although it is recognized that the Monte Carlo simulation is more efficient in pricing options on more than two state variables, simulation method cannot solve the options with early exercise feature (Hull, 1997). The binomial model has the advantages of simplicity, transparency, and flexibility over other models, especially when the early exercise feature is involved. The tree-like representations clearly demonstrate the dynamics of state variables and possible decisions or payoff involved in the real options. The binomial model is flexible enough to consider the early exercise feature, complex payoff functions, and decision timing and choices. The drawback of the binomial model is that the computational burden becomes heavy when there are three or more state variables.

Evnine (1983) extended Cox et al.'s model to consider options on more than one state variable. Herbelot (1992) modified Evnine's model to manage dividend-paying feature. He applied his two state variable binomial model to evaluate various pollution control investments. Boyle's (1988) model considered two state variables and regarded the tree construction as a numerical problem by matching a three-jump discrete process to the state variable's continuous distribution. His model became a five-jump lattice model for a two state variable option. However, the selection procedure of the jump amplitudes in his model involves some trial and errors. Boyle, Evnine, and Gibbs (1989) developed a more generalized model that can easily consider more than two state variables. The basic idea is similar to Boyle's (1988) model; yet, Boyle et al.'s (1989) model does not need trial and error in determining jump amplitudes. Recently, simpler approaches were developed for options on two state variables. Rubinstein (1994) constructed a 3-D "binomial pyramid" for two correlated underlying assets by arranging the jump sizes so that the movement to each node has a fixed probability of 0.25. Hull and White (1994) used a slightly different binomial pyramid in their paper that has the most similarity with the original Cox et al.'s binomial tree. Nevertheless, it is not clear how to extend this model to consider more than two state variables. We shall apply Hull and White's binomial pyramid for the valuation of BOT profit structures.

6.2.2 Hull and White's Method for a Binomial Pyramid

Hull and White (1994) worked out the trinomial pyramid in a three-jump process for each state variable. This section shows how to derive a similar binomial pyramid that has a regular two-jump process for each state variable. The first step is to assume that the two state variables have no correlation. The second step is to adjust the probabilities at each node to reflect the correlation. The adjusted probabilities are presented in Hull (1997). First, assume that there is no correlation between the two state variables. An alternative binomial tree mentioned in 6.1.4 for each state variable is constructed by fixing the probability of moving up or down to 0.5. At the end of the first time increment, the probabilities of arriving at the four possible nodes, namely, (S_1u_1, S_2u_2), (S_1u_1, S_2d_2), (S_1d_1, S_2u_2) and (S_1d_1, S_2d_2), are equal to 0.5*0.5=0.25, where S_1 and S_2 are the two state variables, and u and d are defined in equations (6.11) and (6.12), respectively. If the dividend-paying feature is considered, u and d will be defined in equations (6.14) and (6.15), respectively. Figure 6.6 shows the general description of the movement in each time increment. If there is no correlation between two state variables, the building block of the binomial pyramid has been established. If there is correlation between the two state variables, it is necessary to proceed to the second step to adjust the probabilities.

	S_1- move	
S_2- move	Down	Up
Up	0.5x0.5= 0.25	0.25
Down	0.25	0.25

Fig. 6. 6 Jump Probabilities

Second, adjust the probabilities for each node according to the correlation. Hull and White (1994) argued that the probabilities can be adjusted by first assuming the correlation, $\rho = 1$, and computing the corresponding probabilities, and then interpolating the probabilities according to the real correlation where ρ is nonzero. When $\rho = 1$, the probability of reaching each node is shown in Fig. 6.7. As a result, the probability matrix can be adjusted as shown in Fig. 6.6 to Fig. 6.7 by solving the variables in Fig. 6.8 such that the matrices in Fig. 6.7 and 6.8 are equal. That is, through solving Fig. 6.8, probabilities are adjusted from $\rho = 0$ to $\rho = 1$. Here we obtain

a = -0.25, b = 0.25, c = 0.25, and d = -0.25.

The question is how to adjust the probabilities from $\rho = 0$ to any ρ that ρ is nonzero. Hull and White created another adjusted probability matrix in Fig. 6.9 according to Fig. 6.8. Then they argued that the adjustments should not change the mean and standard deviations of the unconditional movements of the state variables. The adjustments from a, b, c, and d have the effect of inducing a correlation of

$$(|a|+|b|+|c|+|d|)\varepsilon = A\varepsilon \tag{6.16}$$

where ε is the interpolation parameter. ε is solved by setting

$$A\varepsilon = \rho \tag{6.17}$$

Therefore, the value of ε is: ρ / A. Hull and White used this probability interpolation method in a trinomial pyramid. In a two-jump binomial pyramid, according to equation

(6.16): $A = 0.25+0.25+0.25+0.25 = 1$. By (6.17), $\varepsilon = \rho/1 = \rho$. Substitute $\varepsilon = \rho$ into the matrix in Fig. 6.9, the correlation adjusted probabilities are obtained and shown in Fig. 6.10.

	S_1- move	
S_2- move	Down	Up
Up	0	0.5
Down	0.5	0

Fig. 6. 7 Jump Probabilities when $\rho = 1$

	S_1- move	
S_2- move	Down	Up
Up	$0.25+a$	$0.25+b$
Down	$0.25+c$	$0.25+d$

Fig. 6. 8 Jump Probabilities with Adjustment Factors (1)

	S_1- move	
S_2- move	Down	Up
Up	$0.25+a\xi$	$0.25+b\xi$
Down	$0.25+c\xi$	$0.25+d\xi$

Fig. 6. 9 Jump Probabilities with Adjustment Factors (2)

	S_1- move	
S_2- move	Down	Up
Up	$0.25(1-\rho)$	$0.25(1+\rho)$
Down	$0.25(1+\rho)$	$0.25(1-\rho)$

Fig. 6. 10 Jump Probabilities Considering Correlations

6.2.3 Implementation of the Binomial Pyramid Model

In this research, a binomial pyramid model for the valuation of BOT investments shall be implemented. To implement the binomial pyramid, first imagine a reverse pyramid with the sharp end on the bottom and rectangular end on the top. The model starts with the node at the sharp end from the bottom. The point represents the current value of the state variables, (S_1, S_2). After each time increment, four branches will emanate from each single node. For example, Fig. 6.11 shows the possible price movements in the second level of the pyramid after one period of time increment. Figure 6.12 shows the dynamics of the two state variables by three levels of the binomial pyramid. Each node in the second level will generate another four nodes in the third level. Figure 6.12 shows the four nodes generated by the second level node, (S_1u_1, S_2d_2). Note that although different branches may join to one node, the probability attached on each branch should be differentiated. For example, the middle node of the third (top) level in Fig. 6.12, $(S_1u_1d_1, S_2d_2u_2)$, is joined by four branches from the four nodes of the second level. Each branch that reaches the middle node will have a different probability, and these probabilities will be used later in the backward computation of the option value. To obtain the option price, one needs to compute the expected payoff discounted at risk-free rate backward recursively from the top of the reverse pyramid to its bottom. The computation process is similar to the one introduced in section 6.1.2.

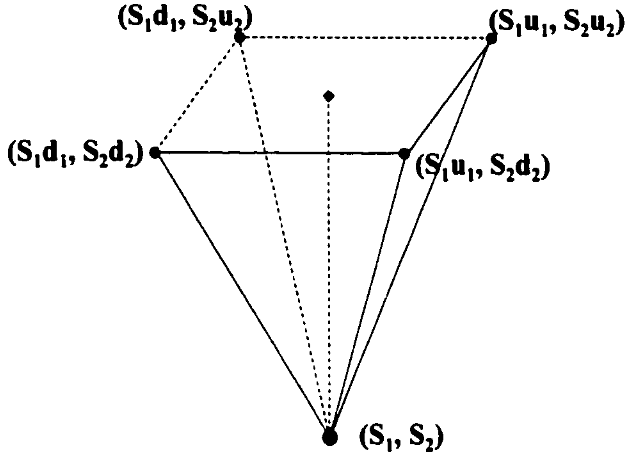

Fig. 6. 11 One Step Jumps for Two State Variables

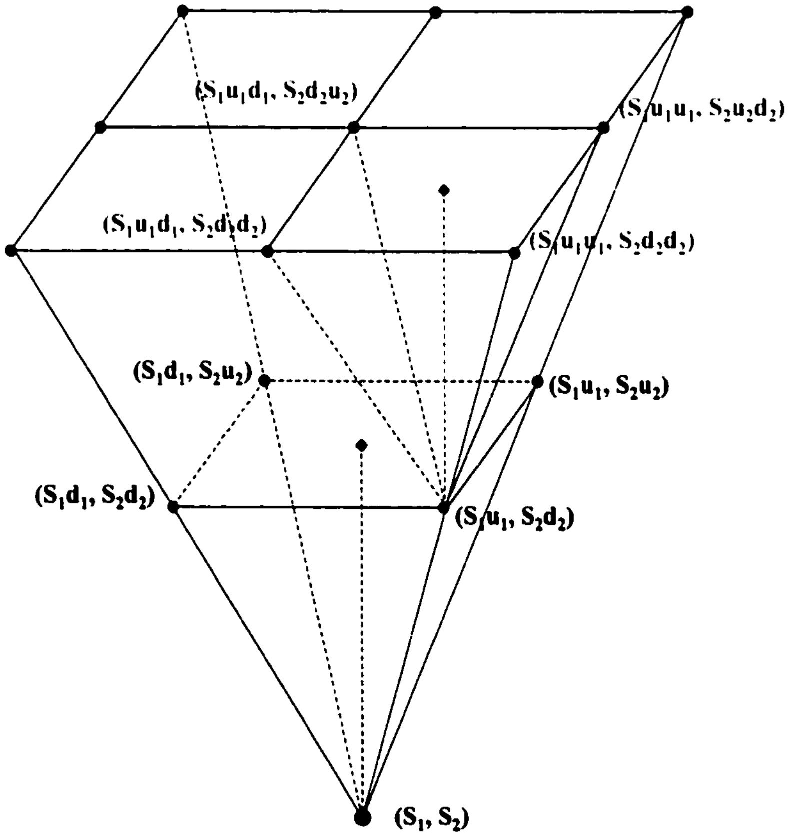

Fig. 6. 12 Reverse Binomial Pyramid (Two Steps)

A computer program, written in C++, of the reverse binomial pyramid was developed in this research. The performance of this program is compared with Boyle's (1988) results. Suppose the two state variables are S_1 and S_2. Boyle assumed that $S_1 = \$40$, $S_2 = \$40$, $\sigma_1 = 0.2$, $\sigma_2 = 0.3$, $\rho = 0.5$, $r = 0.048790$ compounded continuously, and $T = 7$ months. Table 6.1 shows the options prices computed by Boyle's method and the computer program derived in the research. Accurate values are given for comparison as well.

Table 6. 1 Comparison of Boyle's Method and the Implementation in This Research

Exercise Price	Boyle's, 50 steps	This research, 50 steps	This research, 300 steps	Accurate Values
European Call Option on the Maximum of the Two Assets				
\$35	\$9.419	\$9.420	\$9.420	\$9.420
40	5.483	5.487	5.487	5.488
45	2.792	2.800	2.797	2.795
European Put Option on the Minimum of the Two Assets				
\$35	\$1.392	\$1.386	\$1.388	\$1.387
40	3.795	3.797	3.797	3.798
45	7.499	7.505	7.500	7.500

6.3 The Dynamics of the BOT Project Value and Construction Cost, and Asset's Rate of Return Shortfall

6.3.1 The Dynamics of the BOT Project Value

Beidleman (1990) argued that the expectation concerning cash flow is a key to any financing scheme; especially in project finance, the forecasted cash flow is the principal credit support of the capital needed. To a degree, "*lenders look primarily to forecasted cash flow rather to project assets as collateral for the loan*" (Beidleman, 1990). Therefore, BOT project value will be defined on the basis of its forecasted cash flow instead of the physical assets' value. The major source of the future cash flow is the operating profit from the project's operation. For example, the tolls collected from a BOT highway as the project's major cash inflows. In many cases, additional sources of cash flow may come from other project related business activities that are granted to the BOT firm. For example, the BOT firm may be granted to develop shopping malls or hotels next to or inside a high-speed railway station. This type of privilege typically exists when the project itself is not able to pay off the construction cost or may not be able to create a sufficient amount of profit to compensate for the risks involved.

Here the project value of a BOT investment is defined as the project's expected cash flow in the operating phase discounted at risk-adjusted discount rate.[ii] As explained in the next section, 6.3.2, it is assumed that the project is always completed on schedule by transforming the schedule differences into cost differences. So, the future cash flow will be discounted back to the *scheduled* operating commencement. As a result, the project value is based on the future cash flow of the *full* concession/operating period. More specifically, the project value is the market value of a completed project. In real options

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[ii] In a more complete model, the value of the cash flow will interact with the managerial options such as options to shut down, and the value of these options should be included in the calculation of project value. For simplicity, this research did not consider this complication.

analysis, the market value of a completed project can be considered as the payoff received upon project completion (Majd and Pindyck, 1987).

In this research, the BOT investment's valuation and financing problems are focused on the developer's negotiation option during the construction process. Therefore, the major concern will be the stochastic nature of the market value of the completed project, although the project has not been completed during the construction period. For example, if the government agrees that the developer may operate a BOT highway for 30 years after the highway is completed, the developer may compute the project's value as mentioned above according to its discounted expected future cash flows. Suppose that the estimated market value of the 30-year cash flow is \$30 million. The \$30 million is the completed project value, and this value will fluctuate throughout the construction period. One possible case is that after 1 year of construction, a competing high-speed railway is being constructed on a similar route. Suppose the high-speed railway will reduce the value of the highway to \$15 million. As a result, the market will reflect the future competition from the high-speed railway and lower the market's valuation toward the highway project by approximately \$15 million. This example illustrates that the project value is subject to change/fluctuate during the construction period. The purpose of this section is to model and explain the dynamics of the project value during its construction period.

Assumptions adopted by Majd and Pindyck (1987), Dixit and Pindyck (1994), and Schwartz and Moon (2000) toward the project value's dynamics shall be applied in this research. They assumed that the project evolves according to lognormal process, or geometric Brownian motion:

$$\frac{dV}{V} = (\mu_V - \delta_V)dt + \sigma_V dz_V \tag{6.18}$$

where μ_V is the market required expected return of a completed and traded project, σ_V represents the project value's volatility, δ_V is the rate of return shortfall of a nontraded project, and dz_V is an increment to a standard Wiener process. The meaning and determination of μ_V and δ_V will be discussed in 6.3.3. In general, equation (6.18) implies that during the construction phase, in an infinitely short time interval, the logarithm of the project value is normally distributed with mean, $(\mu_V - \delta_V - \frac{1}{2}\sigma_V{}^2)dt$, and variance, $\sigma_V{}^2 dt$. Mathematically, it is equivalent to say that the project value will follow a lognormal distribution. Equation (6.18) shows that the degree of uncertainty of the project value depends on how far into the future one looks. That is, the longer the construction period is, the higher the degree of uncertainty is. It is critical to note that it is not appropriate to use equation (6.18) to model the project value during the operating phase. As Majd and Pindyck (1987) argued, the project value will "decay faster and at a time-varying rate," instead of a fixed rate, $\mu_V - \delta_V$. From this perspective, Trigeorgis' (1991) modeling of the project value during the operating phase might not be appropriate.

Note that the notation V_t will be used frequently later in the analysis. From the discussions above, it is worth pointing out that V_t represents the market value of the completed project at time t by presuming that the project is completed at t and being operated from t, whether the project is truly completed or not. Using the previous BOT highway example, at the beginning of the construction, we have $V_{t=0}$ equals \$30 million, and after 1 year when the competing high-speed railway starts its construction, $V_{t=1}$ equals \$15 millions.

A more complicated situation, which is not considered in this research, is that when the concession agreement specifies that if the project is completed earlier, the project's operation period will be longer by the time saved, and if the project is delayed, the project's operation period will be shorter. That is, the total duration of the concession period is fixed, and the concession period includes the construction and operation phases. In this case, to speed up or delay the construction process will have trade-offs on the profit of operation. In real options analysis, this complication may need to consider options with the early exercise feature. Also the dynamics in equation (6.18) will not be suitable for this trade-off situation, since the project value's rate of change will not be a constant rate under this complication.

From the complicated situation above, we may contrast our analysis with the complicated situation and obtain some deeper understandings of the assumptions toward the project value's dynamics. The analysis in this research does not involve the optimization between completion date and project value. This means that in the valuation process the project value is assessed as if the completion date could always be achieved. This "*as if*" assumption can be justified by the construction scheduling and planning process. A construction schedule cannot be unreasonably short because of limitations, such as resources availability, job site congestion, and weather conditions. To schedule beyond these limitations may incur significant cost that outweighs the additional benefit from the short schedule. An optimal schedule should consider multi-objectives under uncertainties, balance between all conflicting matters, and maximize the developer's profit. In other words, a good BOT schedule should consider the trade-offs in the complicated situation above. Therefore, for simplicity it is assumed that a good BOT schedule should be a goal for the developer to achieve. That is, once the construction begins, the developer should make every effort to achieve the construction schedule. If there are any random events that may affect the scheduled completion, either early or late, it is assumed that the developer is able to complete the project on schedule by adjusting the construction costs. The adjustment will then contribute part of the construction cost's uncertainty. This is also why V_t can be considered as the value of a completed project with the same operating duration no matter when t is. More specifically, this research assumed that the project could be completed on schedule with cost adjustment. This cost adjustment assumption coincides with the liquidated damage punishment in practice. The purpose of the liquidated damage punishment is to compensate the loss from being unable to operate on time due to construction delay, to a degree that after the compensation the overall payoff is just the same as if the project were completed on time. Therefore, the fact that the project can rarely be completed exactly on schedule will not affect the validity of our analysis. Since the dynamics of both project value and construction cost are considered in the analysis, the assumption of completing on time may be reasonably justified toward the approximation of the realistic situation.

6.3.2 The Dynamics of the BOT Project's Construction Cost

For simplicity, it is assumed that any possible construction delays can be either eliminated by incurring extra cost or compensated by liquidated damage punishment. Similarly, if the construction is ahead of schedule, we may assume that the construction is on schedule but with lower cost than the budget or that the project will finish early with rewards due to operating early. In other words, the completion schedule differences are transformed to construction cost differences, so that we may focus on the uncertainties of project value and construction cost only. In the real options analysis, it is not necessarily advantageous if the model considers all uncertain variables. In fact, the number of possible state variables can be infinite since nothing is certain in the long run. A good analysis should be based on only a few relevant state variables, and try to bundle similar risks together (Amram and Kulatilaka, 1999). Here the construction cost fluctuations due to the schedule uncertainty and non-schedule related uncertainty are bundled together as one state variable, construction cost. As a result, the concept of the construction cost here may not perfectly coincide with the common construction cost concept. The impact of the risk bundling is that one needs to carefully estimate the parameters in the dynamics of the state variables according to the formations of the state variables. For example, when the parameters are estimated from the contractors' past construction cost data, the data should be reorganized so that the construction cost matches the definition of the bundled state variable

Construction cost = Construction related cost – reward of early completion, or

= Construction related cost + liquidated damage punishment.

If the past contracts did not have any reward schemes for early completion, one should estimate an appropriate reward for the early completion.

Similar to the project value, the total construction cost, K_t, is defined as the market's valuation/estimation of the total construction cost if the project is completed at t. Similarly, K_t will fluctuate throughout the construction period. By treating the total construction cost as another asset in the market, the total construction cost may be modeled as

$$\frac{dK}{K} = (\mu_K - \delta_K)dt + \sigma_K dz_K \tag{6.19}$$

where K is the market valuation of the expected total construction cost, and other variables are defined similarly as those in (6.18). Note that if V and K are correlated, then

$$Cov(\frac{dV}{V}, \frac{dK}{K}) = \rho_{VK}\sigma_V\sigma_K dt \tag{6.20}$$

where ρ_{VK} is the correlation coefficient. It is worth noting that in this research, K is computed by discounting the cost incurred during the construction duration. Figures 6.13 and 6.14 illustrate the discounting process. Figure 6.13 shows that a construction project needs to perform four activities, I-IV. If all activities can be performed at the beginning of the construction process, and the cost for each activity is spent at time 0, the total construction cost would be \$400 million. However, it is assumed that all activities of a project can be done at once. Typically, due to the resources or activity relationship's constraints, project activities need to be scheduled in a time frame. The duration of the time frame is the construction schedule. Suppose that the activities in Fig. 6.13 are scheduled as shown in Fig. 6.14, and the cost associated with each activity is incurred at the beginning of each year. Note that \$106 at the beginning of year 2 is activity II's real cost, \$100, plus its time value, \$6, assuming that the time value adjusting factor is 6%. The 6% is the discount/inflation rate used in discounting the construction cost. The discount rate will be different subject to each developer and project according to the developer's discounting policy. The time value discount rate shall be denoted as r_c in this research.

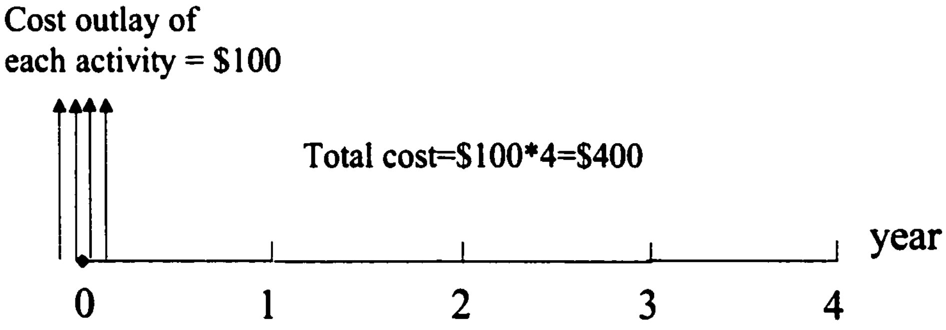

Fig. 6. 13 Construction Cost at Time 0

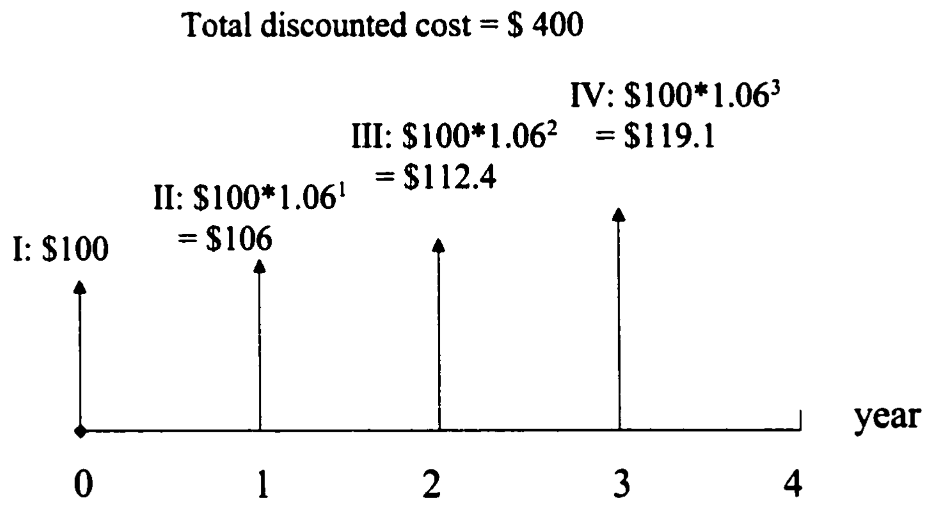

Fig. 6. 14 Construction Cost Schedule

Assume that there exist some financial assets that have similar risks to the assumed physical asset. Here the asset is the "delivery" of the project. Before the completion time, T, the physical asset is considered as *unfinished/undelivered* and has an appreciation rate of μ_{Kr}. However, according to its risk characteristics, the market's equilibrium/required rate of return on this asset is μ_K and μ_K will be evaluated according to capital market disciplines. At any time t before the completion, K will be updated according to state information. Therefore, K_t is defined as the estimated total construction cost at time t prices and this definition is critical when the early exercise is possible. At any time t, K must be updated and converted to time t prices. Also at time t, a cost schedule as shown in Fig. 6.14 can always be specified or estimated. More details regarding μ_K will be given in next section.

A very good model of the construction cost's dynamics was proposed by Majd and Pindyck (1987). However, it is not appropriate for our analysis. The reasons are given as follows. Majd and Pindyck (1987) modeled the dynamics of the remaining construction cost as

$$dK_R = -Idt + v(IK_R)^{1/2}dx + \gamma K_R dz_{K_R} \tag{6.21}$$

where dx and dz are the increments of uncorrelated Wiener processes, dx is uncorrelated with the economy and the stock market, I is the rate of cost incurred, and v and γ are parameters. Note that K_R is the construction cost remained to completion. Majd and Pindyck used (6.21) in analyzing the investment that takes time to build, consists of multistage capital outlays, and has an option to stop or wait during the construction or before the final stage. In this type of investment, the investor needs to balance the remaining cost and the project value, and maximize the overall payoff under uncertainty through the opinions to stop or wait. In BOT construction, the option to wait for a good timing to start construction is not available to the developer. The developer needs to commence the construction as soon as the project is awarded. Similarly, the developer has no option to stop or wait for the uncertainties to unfold during the construction. The main reason that the developer does not have the "stop" or "wait" options is that the BOT project is public related and supervised by government. If the schedule is delayed without compelling reasons, the government will intervene the project's construction. However, during the construction, the developer does have the option to either abandon the project permanently or request and negotiate government subsidies should adverse events occur. As argued in section 5.3, although the developer always prefers to request for subsidy, the lending bank may not be willing to continue a project if the project's market value of expected future cash flow is not enough to repay the loan. Therefore, the lending bank needs to balance between the project value and the total construction cost as shown in equation (5.8).

6.3.3 Market Required Rate of Return and the Capital Asset Pricing Model

In equations (6.18) and (6.19), the dynamics of the project value and cost, the discussions of parameters μ and δ are postponed to this and next section, since the concepts of these two parameters are rather complicated.

First concept is the market required rate of return, μ. In the Capital Asset Pricing Model[iii] (CAPM), an asset i's expected rate of return in an efficient market is given by

$$\mu_i = r + \beta_i (r_M - r) \tag{6.22}$$

where

$$\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} \tag{6.23}$$

Here, r is the market's risk-free interest rate, r_M is the capital market's expected rate of return, ex., the expected return on S&P 500 stock index, σ_M^2 is the variance of the market's rate of return, and σ_{iM} is the covariance of the return on asset i and the return capital market. Equation (6.22)'s intuitive interpretation is that the market's opinion of the asset i's return, μ_i, is equal to the risk-free interest rate, r, plus the risk premium, $\beta(r_M - r)$. Generally, $r_M - r$ is a positive number. Therefore, the larger β_i is, the higher the asset's market equilibrium return. A larger β_i means the asset is riskier. Equation (6.23) shows how the term risk is measured through β_i. The risk is measured by the covariance of the asset return and the market return. One extreme case is that when the correlation of the asset return and market return is zero, β_i will be zero and the asset's market equilibrium return only amounts to risk-free interest rate no matter how high the asset's variance is. CAPM changed the concept of risk from asset variance to asset's covariance with the market. The main reason is that in an efficient market an investor can always diversify away those risks that are uncorrelated with the market via a well-diversified portfolio, but those risks that are correlated with the market will stay. Therefore, from the capital market's perspective, the risks that are not correlated with the market will not be rewarded with risk premium, and vice versa. In real options analysis, if a state variable is uncorrelated with the market, the risk due to this state variable can be called as "private risk" (Dixit and Pindyck, 1994). For example, in an oil exploration project, the uncertainty of the oil reserve size is a private risk. On the other hand, the market correlated and un-diversifiable risk can be called as "market-priced risk" (Amram and Kulatilaka, 1999).

[iii] See Copeland and Weston (1988) or Luenberger (1998) for the introduction and derivation of CAPM.

Note that equations (6.22) and (6.23) can be reorganized as:

$$\mu_i = r + \lambda \rho_{iM} \sigma_i \tag{6.24}$$

where

$$\lambda = \frac{r_M - r}{\sigma_M} \tag{6.25}$$

Here $\rho_{iM}\sigma_i$ represents the total amount of risk, and λ is the risk premium per unit of risk. As a result, the total risk premium is

$$\mu_i - r = \lambda \rho_{iM} \sigma_i . \tag{6.26}$$

It is worth noting that all the parameters on the right-hand side of equation (6.24) can be reasonably estimated from observable data in the market if the asset is publicly traded, such as an IBM stock. If the asset is not publicly traded, then the parameters in equation (6.24) can be estimated by other assets with similar business or risk characteristics that are publicly traded. However, the concept is more sophisticated for the rate of return of a project that is not publicly traded, and will be discussed in the next section.

In BOT investments, there are two rates of return related with the project. They are μ_V and μ_K in equations (6.18) and (6.19), respectively. According to equation (6.24), they can be estimated by

$$\mu_V = r + \lambda \rho_{VM} \sigma_V \tag{6.27}$$

$$\mu_K = r + \lambda \rho_{KM} \sigma_K \tag{6.28}$$

6.3.4 Rate of Return Shortfall and Asset's Present Value

Equation (6.24) indicates the publicly traded financial asset's rate of return required by market's investors. There are other assets that are not publicly traded physical assets such investment projects. These non-publicly traded assets could be called as "real assets" as opposed to the "financial assets." If equation (6.24) is used to compute a real asset's return, the return has an interpretation: if the market has a traded security issued by a firm that is identical to the real asset, then the real asset's market required rate of return will be equal to the security's market required rate of return. As Majd and Pindyck (1987) argued, the use of the real asset's μ does not assume that the shares in identical/twin firm are actually publicly traded. It is only assumed that "we could calculate the value that would prevail if such shares were traded..." (Majd and Pindyck, 1987). As a result, the real asset's μ is, conceptually, different from the asset's actual return rate. Although there exists a market equilibrium return rate for a non-traded asset, the asset's actual return or growing rate is often different from the market equilibrium return rate. In many cases, we may find that the non-traded asset's growing rate is lower than the market equilibrium return rate. We may denote the real asset's appreciation rate as μ_r and the difference between μ and μ_r as δ. The mathematical relation is given as

$$\delta = \mu - \mu_r \tag{6.29}$$

δ can be called as "rate of return shortfall" when the real asset is a physical asset, firm, or project. δ is also referred as "convenience yield" when the real asset is a traded commodity such as crude oil. When the real asset is a project waiting to be constructed, the rate of return shortfall may represent "the opportunity cost of delaying construction of the project" (Dixit and Pindyck, 1994). Thus, when the real asset is an unfinished project, the rate of return shortfall may represent the opportunity cost of delaying completion of the project. Dixit and Pindyck (1994) argued that if the holder of the underlying asset has a constant exogenous rate of return shortfall rate, then μ_r "must change to preserve the equilibrium" shown in equation (6.29). They further stated that if μ_r is exogenously fixed, then the return shortfall rate, δ, "must change to take up the slack." McDonald and Siegel (1984) incorporated the effect of rate of return shortfall into the valuation of options on real assets. Trigeorgis (1993, 1996) and Dixit and Pindyck (1994) had detailed discussions on the rate of return shortfall/convenience yield.

In real options analysis, the consideration of δ is crucial, since most of the underlying assets considered are non-publicly traded real assets. δ must be appropriately considered in order to correctly evaluate the real options. In the commodity market, δ can be implied from the prices of the commodity's future contracts (Brennan and Schwartz, 1985). However, for real assets other than the commodities, it may be necessary to estimate δ by estimating the values of μ and μ_r. μ can be estimated according to equation (6.24) and μ_r can be estimated by the real asset's growing or appreciation rate. In some cases, the real asset's appreciation rate, μ_r, can be inferred by analyzing the competition from competitors. McDonald and Siegel (1986) argued that when a project is facing a lagged entry from other competing projects, the project's appreciation rate will be negative, that is, $\mu_r < 0$. Schwartz and Moon (2000) argued that μ_r would reflect the characteristics of a particular project; for example, negative μ_r may represent a shrinking market for the project, and vice versa. Combining equations (6.24) and (6.29), the rate of return shortfall of asset *i* is given as

$$\delta_i = r + \lambda\rho_{iM}\sigma_i - \mu_{ir} \tag{6.30}$$

where μ_{ir} is asset *i*'s appreciation rate.

6.3.5 Rate of Return Shortfalls in BOT Investment Analysis

In BOT investments, the rate of return shortfalls of the project's value and construction cost are denoted as δ_V and δ_K, respectively. The estimation of δ_V and δ_K will be needed for computing the jump amplitudes *u* and *d* as given in equations (6.14) and (6.15). Equation (6.29) may be used to estimate δ_V and δ_K. To estimate δ_V, we have

$$\delta_V = r + \lambda\rho_{VM}\sigma_V - \mu_{Vr} \tag{6.31}$$

Note that to estimate ρ_{VM} and σ_V may not be an easy task if the market does not have traded securities on similar projects. However, if we broaden the capital market to global/international market, due to recent trends of BOT's global financing (Finnerty, 1996), it may not be as difficult as it seems to estimate ρ_{VM} and σ_V from the global capital market perspective. This is also why Amram and Kulatilaka (1999) argued that "the financial market innovations blur the lines between real and financial assets." The other important variable in equation (6.31) is the BOT project value's appreciation rate,

μ_{V_r}. As argued in section 6.3.1, since the project value will be estimated according to the fully planned operation period, the project value will not be affected by the delaying or speeding of the completion. Plus, it is very common in a BOT project that the government may guarantee the developer that no competing project/facility will be built within certain years or the concession period. As a result, it is reasonable to assume that $\mu_{V_r} = 0$ in real terms in most cases unless it is believed that either the market for the project is growing or shrinking during the construction phase. In nominal terms, μ_{V_r} may be equal to the average inflation rate on V. The assumption that $\mu_{V_r} = 0$ will be applied to the illustrative examples later. However, in Chapter 8, the case study, nominal terms will be used and therefore μ_{V_r} will be proportional to the inflation rate.

Similar to equation (6.31), to estimate δ_K, we will have

$$\delta_K = r + \lambda\rho_{KM}\sigma_K - \mu_{Kr} \qquad (6.32)$$

Note that since there is no financial security on the "project cost," it is not possible to estimate ρ_{KM} and σ_K from similar traded securities. It may be necessary to estimate ρ_{KM} and σ_K by historical construction cost data from similar projects of a developer or government. To estimate μ_{K_r}, it is critical to analyze whether the discounted expected cost will grow or shrink as time passes. Usually, it is reasonable to assume that $\mu_{K_r} = 0$ in real terms. However, in nominal terms, it can be shown that μ_{K_r} is equal to the assumed construction cost inflation rate, r_c.

6.4 Valuation of the Profit Structures: Five-Step Profit Structures Valuation Framework

The definitions of the components of the profit structures are given in section 4.3, and will be applied for the purpose of the profit structures valuation. Results from BOT game theoretic analysis in Chapter 5 and the real options analysis framework for BOT project proposed in sections 6.1-6.3 will be incorporated to evaluate the profit structures. Through this integration, the value of a BOT investment will be consistent with the participants' rational economic behaviors and the capital market disciplines. This section will focus on the modeling of the valuation framework for each profit structure component here. The computation will be performed in next section through an example.

As argued in section 5.3, the refined Nash equilibrium of the BOT game is:
First, the "rescue" equilibrium under the condition in equation (5.15), when the justifiable subsidy, *J*, is greater than the project's least required subsidy, *G*, the Nash equilibrium will be to "rescue" and "continue the construction of the project until it is completed." Second, the "no rescue" equilibrium under the condition in equation (5.16), if the justifiable subsidy, *J*, is less than the project's least required subsidy, *G*, the Nash equilibrium will be to "abandon" the project before its completion. 6.4.1 will discuss the valuation process of a BOT investment's profit structures under the "rescue" Nash equilibrium. The valuation of the profit structures under the "no rescue" equilibrium will be discussed in section 6.4.2.

6.4.1 Valuation of the Profit Structures with "Rescue" Equilibrium

Step 1: Select the state variables and determine their dynamics and current values. As argued in sections 4.5 and 6.3, the state variables are the BOT project value and total construction cost. The dynamics of the project value is given in equation (6.18):

$$\frac{dV}{V} = (\mu_V - \delta_V)dt + \sigma_V dz_V$$

and the dynamics of the total construction cost is given in equation (6.19):

$$\frac{dK}{K} = (\mu_K - \delta_K)dt + \sigma_K dz_K$$

Step 2: Align the dynamics of the state variables with the capital market and project characteristics. This is a critical step in real options analysis. This step is to calibrate/estimate those parameters in equations (6.18) and (6.19) that can describe the project characteristics and capital market. This includes how to determine δ_V and δ_K that are consistent with the capital market. Combining the CAPM and the theory of rate of return short fall, δ_V and δ_K are determined by capital market's observable data, i.e., the equilibrium rates of return, and the project's characteristics, i.e., the state variable's appreciation rates. δ_V is given in equation (6.31)

$$\delta_V = r + \lambda\rho_{VM}\sigma_V - \mu_{Vr}$$

and δ_K is given in (6.32)

$$\delta_K = r + \lambda\rho_{KM}\sigma_K - \mu_{Kr}$$

Expertise in corporate finance and investment theory is needed to perform the tasks in this step.

Step 3: Construct a reverse binomial pyramid model for the two-state-variable problems. For a two-state-variable problem, a reverse binomial pyramid can be constructed. The most important parameters for the binomial pyramid are the jump probabilities and jump amplitudes. If the correlation of the two state variables is known, then the jump probabilities are given in Fig. 6.10. According to equations (6.14) and (6.15), the jump amplitudes for *V* are

$$u_V = e^{(r-\delta_V-\frac{1}{2}\sigma_V^2)\Delta t+\sigma_V\sqrt{\Delta t}} \tag{6.33}$$

$$d_V = e^{(r-\delta_V-\frac{1}{2}\sigma_V^2)\Delta t-\sigma_V\sqrt{\Delta t}}, \tag{6.34}$$

and the jump amplitudes for *K* are

$$u_K = e^{(r-\delta_K-\frac{1}{2}\sigma_K^2)\Delta t+\sigma_K\sqrt{\Delta t}} \tag{6.35}$$

$$d_K = e^{(r-\delta_K-\frac{1}{2}\sigma_K^2)\Delta t-\sigma_K\sqrt{\Delta t}} \tag{6.36}$$

Step 4: Determine the *terminal* payoff functions. Under the "rescue" equilibrium, it is assumed that the project can justify all necessary subsidies from the government so that the project can be completed at scheduled time, *T*. The terminal payoff functions are derived in the following.

- For equity value: Equity value is the value of equity investment. Following the bankruptcy condition at time T given in equations (5.17) and (5.18), the terminal/time T payoff function of the developer's *equity investment* at completion is

$$Max[0, V_T - D_T(K_T)] \tag{6.37}$$

At T, if the project value is greater than the outstanding debt, the payoff to the equity holder is $V_T - D_T(K)$; otherwise, the payoff to the equity holder is zero due to the bankruptcy. Applying equation (6.37) in the reverse binomial pyramid, equity value may be obtained.

Note that in equation (6.37), D_T is computed by imposing loan interests to the total discounted cost, K_T. A critical assumption in this research is that the loan interest rate, r_d, will be determined *exogenously*. In other words, the loan interest rate is not solved endogenously by the models; instead, it is given as a constraint/parameter of a financing alternative. There are two reasons for this assumption. First, the model derived is meant for the developer to evaluate an investment, and is not meant for the bank to determine the lending rate. Even if the developer can solve for a fair loan interest rate, the rate may still be different from the bank's actual rate. At the end, the developer still has to use the lending bank's interest rate. Thus, it is reasonable to assume that the rate is given exogenously. Second, although by the same principle of solving equity value, the model in this research may be used to solve for debt value; however, since the debt principal is uncertain at time T due to the cost uncertainty, it is difficult to infer an interest rate between the debt value and debt principal. Here it is assumed that given the amount of initial equity investment and other conditions, the loan interest rate, r_d, will be given by lending bank and known by the developer.

The problem now is how to compute D_T given r_d. One needs to make some assumptions regarding the construction cost in order to compute D_T. Note that D_T is by no means the exact amount of debt outstanding; instead, it is an approximation. The major reason is that the cost is incurred throughout the whole construction period. Note that here K_t is defined as the discounted construction cost at *time t prices*.

Therefore, K_T is the construction cost at time T prices, that is, the future value of the construction cost. However, given a total discounted construction cost, K_T, it could mean infinite amount of possible combinations of cost spending pattern throughout the construction period. Each possible combination will yield a specific and different amount of debt outstanding. Therefore, the debt outstanding cannot be assessed without making certain assumptions regarding how the cost is spent throughout the construction period. The cost-spending pattern shall be referred to as the "cost schedule." For example, in construction, a typical cost schedule could be similar to the schedule shown in Fig. 6.15.

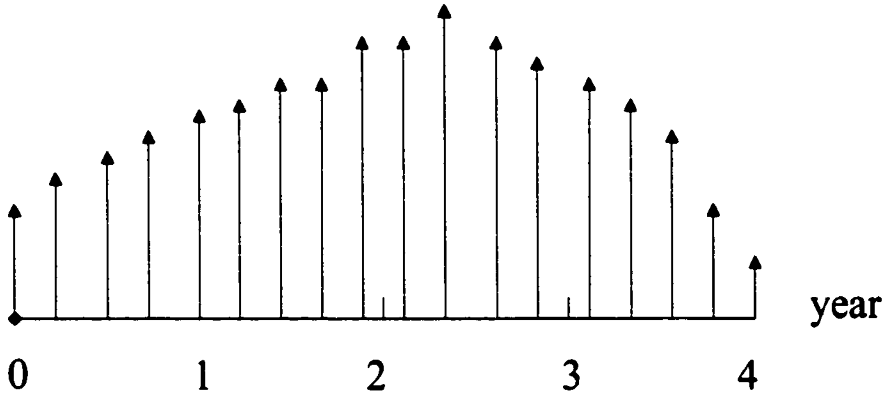

Fig. 6. 15 Typical Construction Cost Schedule

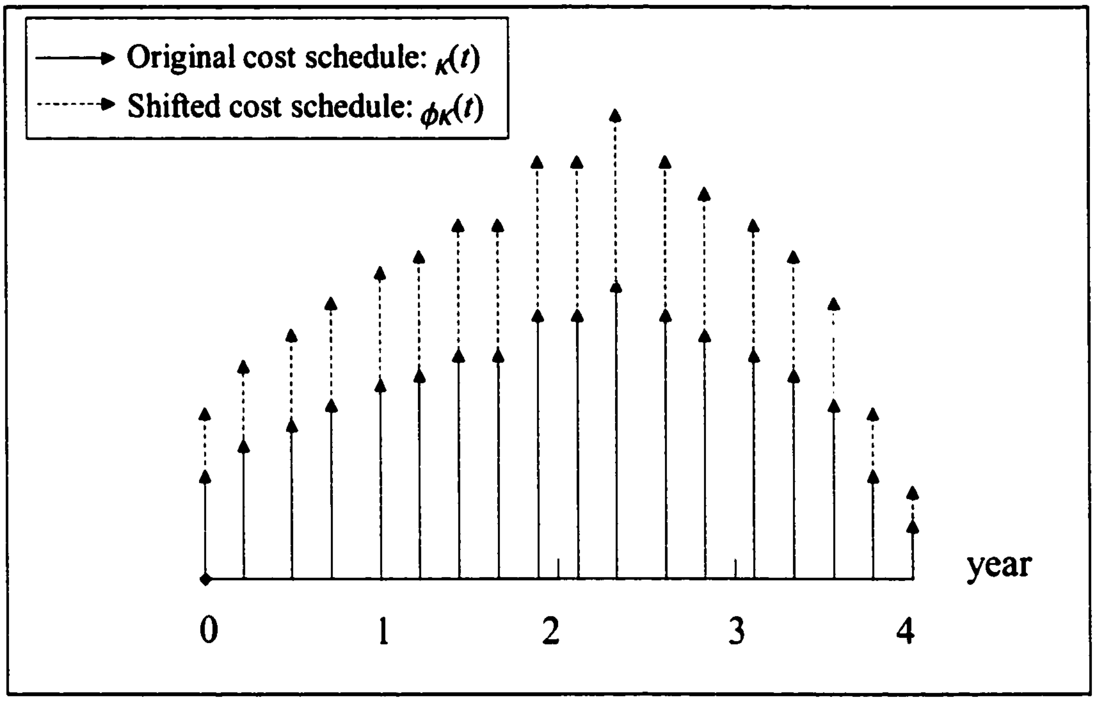

Fig. 6. 16 Shifted Cost Schedule Representing Cost Adjustment

Let $\kappa(t)$ denotes the project's cost schedule as shown in Fig. 6.15. First assume that the cost fluctuation is distributed along the construction period by shifting $\kappa(t)$ upward or downward as shown in Fig. 6.16. For simplicity, the shifting ratio is defined as

$$\phi_T = \frac{K_T e^{-r_c T}}{K_0}, \tag{6.38}$$

where r_c is the construction cost inflation rate when the cost is considered in nominal term, and $K_T e^{-r_c T}$ is the K_T's present value at *time 0*. As a result, given any K_T and discount rate, one may obtain a shifting ratio, ϕ_T. The shifted schedule as shown in Fig. 6.16, $\phi_T \kappa(t)$, shown in Fig. 3 may be obtained accordingly. Note that if the construction cost is considered in real term, then r_c will be equal to zero. Suppose that the initial equity investment amount, *I*, is used before debt and the starting date of using debt is t_d. t_d will be determined by I and $\phi_T \kappa(t)$. If the interest is compounded continuously, the debt outstanding at T can be expressed as

$$D_T(K_T) = \int_{t_d}^{T} \frac{K_T e^{-r_c T}}{K_0} \kappa(\hat{t}) e^{r_d(T-\hat{t})} d\hat{t} \tag{6.39}$$

where t_d is obtained by solving $I = \int_0^{t_d} \phi_T \kappa(\hat{t}) e^{-r_c \hat{t}} d\hat{t}$. Equation (6.39) can be used to approximate any type of cost schedule. For simplicity, in the valuation of profit structures, it is assumed that the discounted cost schedule is uniform, that is, $\kappa(t) = \frac{K_0}{T} e^{r_c t}$. In this case, a simple formula for the debt outstanding may be obtained:

$$D_T(K_T) = \frac{K_T e^{-r_c T}}{(r_d - r_c)T} e^{r_d(T-\hat{t}) + r_c \hat{t}} \Bigg|_{\hat{t}=T}^{\hat{t}=t_d} \tag{6.40}$$

where

$$t_d = \frac{I}{\left(\dfrac{K_T e^{-r_c T}}{T}\right)} \tag{6.41}$$

Note that equations (6.40) and (6.41) are by no means the only way to approximate the debt outstanding in "rescue" equilibrium. Different assumptions will yield different approximations.

- For construction contract value: Each construction contract is unique due to the uniqueness of the project. However, the construction contracts are generally categorized as two basic types. The first type is characterized by that the construction cost/price is fixed. The contractors will take the overall construction risks in this type of contract. For example, the lump sum contract and guaranteed maximum price (GMP) contract are of the first type. These contracts are used very often in the competitive bidding. The second type of contract is characterized by that the construction cost/price is not fixed and is determined by actual cost incurred plus additional fees or profit sharing. The fees can be fixed or be a certain percentage. The risks of construction cost are shared between the owner and contractors. For example, the target cost contract and the "cost plus fixed percentage fees" contract are of this type. Due to the risks of bankruptcy, the construction contract may be terminated before the completion date. As a result, it would be extremely difficult to evaluate the value of the construction contracts by traditional discounting method such as NPV method, which assumes that the cash flows continue throughout the whole construction period. The payoff function of the construction contract depends on the type of the contract. Therefore, there may be infinite amount of payoff functions for various construction contracts. For illustrative purposes, a generic form is used to represent a contract. The payoff of the contract basically consists of two parts: the profit for the cost incurred that is within the budget, and the profit or penalty for the cost incurred that is beyond the budget. The generic terminal payoff function of the *construction contract* can be formulated as

$$\alpha\int_0^T \phi_T \kappa(\hat{t}) e^{r_c(T-\hat{t})} d\hat{t}, \qquad \text{if} \quad K_T e^{-r_c T} \le K_0 \tag{6.42}$$

$$\alpha\int_0^{t_p} \phi_T \kappa(\hat{t}) e^{r_c(T-\hat{t})} d\hat{t} + \beta\int_{t_p}^T \phi_T \kappa(\hat{t}) e^{r_c(T-\hat{t})} d\hat{t}, \qquad \text{if} \quad K_T e^{-r_c T} > K_0 \tag{6.43}$$

where t_p is the time when the cost incurred reaches the budgeted cost, and can be obtained by solving

$$K_0 = \int_0^{t_p} \phi_T \kappa(\hat{t}) e^{-r_c \hat{t}} d\hat{t}, \tag{6.44}$$

α is defined as the profit markup over the construction cost that is within the original estimated/planned cost, K_0, and β is defined as the profit markup over the construction cost that is beyond the planned cost. β should be less than or equal to α. As argued in section 4.2, the construction contract value includes the budgeted profit and over-budget profit. Equation (6.42) represents the time *T*'s future value of the budgeted profit if the cost is within budget. Equation (6.43) represents the future value of the contract profit that includes the budgeted profit and the over-budgeted profit.

Note that under normal construction industry conventions, the markup profit is included in the bid price, or estimated construction cost. Therefore, the amount of contract profit is included in K_T and K_0. Equations (6.42) and (6.43) simplify the actual pattern of construction contract profit. To facilitate using the option pricing framework, it is assumed here that the payoff is received only at the end of construction, T. Since K_T and K_0 are *discounted* expected costs, it is necessary to adjust the contract profit so that the payoff amount in equations (6.42) and (6.43) matches the assumed payoff pattern; that is, the payoff is received only at T.

As mentioned above, for simplicity, it is assumed that $\kappa(t) = \frac{K_0}{T} e^{r_c t}$ and $\phi_T = \frac{K_T e^{-r_c T}}{K_0}$ in the analysis. Given the assumptions, the payoff functions in equations (6.42) to (6.44) can be reorganized as equations (6.45) to (6.47):

$$\alpha K_T, \qquad \text{if } K_T e^{-r_c T} \le K_0 \tag{6.45}$$

$$\alpha K_T \frac{t_p}{T} + \beta K_T \frac{T - t_p}{T}, \qquad \text{if } K_T e^{-r_c T} > K_0 \tag{6.46}$$

where t_p is obtained by solving $K_0 = \frac{K_T e^{-r_c T}}{T} t_p$, and is given as

$$t_p = \frac{K_0}{\left(\frac{K_T e^{-r_c T}}{T} \right)} \tag{6.47}$$

By substituting equation (6.47) into equation (6.46), equation (6.46) can be rewritten as

$$\alpha K_0 e^{r_c T} + \beta (K_T - K_0 e^{r_c T}), \qquad \text{if } K_T e^{-r_c T} > K_0 \tag{6.48}$$

Note that equation (6.48) gives a more intuitive expression of the payoff function in equation (6.46) when there is cost overrun.

- For operating related contract value: For the operating related contract, its terminal payoff function is given in equation (6.49)

$$\begin{cases} W, & \text{if} \quad V_T - D_T(K) \ge 0 \\ 0, & \text{if} \quad V_T - D_T(K) < 0 \end{cases} \tag{6.49}$$

where W is the profit from operating related contracts *contingent* on the continuing operating and managing of the project. If at completion time T, the project is being sold to the government or other parties, the contingent profit, W, will not be realized and the payoff will be zero.

Step 5: Plug the terminal payoff functions into the reverse binomial pyramid and compute the value of the profit structures. As in the binomial model, the profit structures can be computed as the option prices by working backward recursively from the terminal nodes of the binomial pyramid. The values of the terminal nodes are given by the terminal payoff functions obtained in step 4.

6.4.2 Valuation of Profit Structures with "No Rescue" Equilibrium

When the justifiable subsidy is less than the least required subsidy, the Nash equilibrium is to "abandon," since the government will not be able to provide any subsidy to the failing BOT project. In this case, the project may be bought back by the government or sold to other developers, that is, the project may be terminated before time T. The early termination of a project is similar to the early exercise feature of an American option. The difference is that in an American style option, the early exercise timing is determined simultaneously with the value of the options. That is, the exercise timing/condition is also a problem to be solved by valuation models. However, in BOT investments, the early exercise condition is determined in advance, and the option prices are obtained accordingly. More specifically, the exercise condition is not determined by the developer; instead, it is imposed by the lending bank. In other words, in American options, the early exercise timing is to determined by the maximization of the payoff or value of the options, whereas in the "no rescue" equilibrium, the early exercise timing is imposed against the developer for the protection of debt holders.

The valuation steps under "no rescue" equilibrium is similar to the steps under "rescue" equilibrium. The first three steps are identical. Steps 4 and 5 are modified as the following:

Step 4: Determine the "bankruptcy" condition, and "*time t*" payoff functions. Note that in the "no rescue" Nash equilibrium, at each time step of the binomial model, it is necessary to examine if the project meets the "abandon" or "exercise" condition. If the condition is met, the value will be calculated by the bankruptcy's payoff function at time t instead of the backward-discounted expected value. It is worth noting that the *terminal* payoff functions mentioned in section 6.4.1 are still needed in the "no rescue" equilibrium for the first step of backward computations at time T.

The bankruptcy condition is given in equation (5.19):

$$V_t - D_t(K)e^{-r_d(T-t)} < 0$$

For simplicity, suppose that the discounted cost schedule is uniform. The estimated total debt required at *time t prices*, $D_t(K_t)$, will be given as:

$$D_t(K_t) = \frac{K_t}{(r_d - r_c)T} e^{r_d(t-i)+r_c i} \Big|_{i=T}^{i=t'_d} \tag{6.50}$$

where

$$t'_d = \frac{I}{\left(\dfrac{K_t e^{-r_c t}}{T}\right)} \tag{6.51}$$

The time t payoff functions are the developer's payoff at any time t, and they are formulated as followed.

- For equity profit: Similar to "rescue" equilibrium above, the payoff function here is to calculate the value of the equity. If the project is bankrupted, the equity value will be zero. Note that the payoff upon project bankruptcy may be referred to as the "bankruptcy" payoff. If the bankruptcy condition is not met, one only needs to compute the backward-discounted expected equity value and there is no need to calculate the project's payoff. This can be expressed as

$$\begin{cases} \textit{Do not need payoff functions} & \textit{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} \geq 0 \\ 0, & \textit{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} < 0 \end{cases} \tag{6.52}$$

- For construction contract value: If the project is bankrupted, the payoff will be considered as

$$\alpha\int_0^t \phi_t \kappa(\hat{t}) e^{r_c(t-\hat{t})} d\hat{t}, \qquad \textit{if} \quad \int_0^t \phi_t \kappa(\hat{t}) e^{-r_c \hat{t}} d\hat{t} \le K_0 \tag{6.53}$$

$$\alpha\int_0^{t'_p} \phi_t \kappa(\hat{t}) e^{r_c(t-\hat{t})} d\hat{t} + \beta\int_{t'_p}^t \phi_t \kappa(\hat{t}) e^{r_c(t-\hat{t})} d\hat{t}, \qquad \textit{if} \quad \int_0^t \phi_t \kappa(\hat{t}) e^{-r_c \hat{t}} d\hat{t} > K_0 \tag{6.54}$$

where

$$\phi_t = \frac{K_t e^{-r_c t}}{K_0}, \tag{6.55}$$

t'_p is obtained by solving

$$\int_0^{t'_p} \phi_t \kappa(\hat{t}) e^{-r_c \hat{t}} d\hat{t} = K_0 \tag{6.56}$$

and $\int_0^t \phi_t \kappa(\hat{t}) e^{-r_c \hat{t}} d\hat{t}$ is the discounted value of the cumulative cost incurred up to time t. Note that $\phi_t \kappa(\hat{t})$ is time t's estimated cost schedule adjusted according to K_t, therefore, $\int_0^t \phi_t \kappa(\hat{t}) e^{-r_c \hat{t}} d\hat{t}$ is an approximation of the discounted value of the cumulative cost incurred up to time t. If $\int_0^t \phi_t \kappa(\hat{t}) e^{-r_c \hat{t}} d\hat{t} < K_0$, then the approximated cost incurred is less than the budgeted cost. Suppose that the *normal* markup profit, αK is included in the cost, equation (6.53) will give *time t*'s future value of the normal markup profit on the cost incurred up to *time t*. Equation (6.54) means that if at *time t*, the cost incurred is less than the budgeted cost, then the contract profit will be the normal markup profit imposed on the cost incurred. On the other hand, if at *time t*, the cost incurred is over the budgeted cost, then the contract profit will be the normal markup profit plus the over-budget markup profit as given in equation (6.54).

Again, for simplicity, it is assumed that $\phi_t = \dfrac{K_t e^{-r_c t}}{K_0}$ and $\kappa(t) = \dfrac{K_0}{T} e^{r_c t}$. The payoff functions in equations (6.53), (6.54) and (6.56) can be reorganized as

$$\alpha K_t \frac{t}{T} \qquad \text{if} \quad K_t e^{-r_c t} \frac{t}{T} \le K_0 \tag{6.57}$$

$$\alpha K_0 e^{r_c t} + \beta \left[K_t \frac{t}{T} - K_0 e^{r_c t} \right] \qquad \text{if} \quad K_t e^{-r_c t} \frac{t}{T} > K_0 \tag{6.58}$$

Note that the payoff functions of the construction contract profit in equations (6.53) to (6.56) or (6.57) to (6.58) are not the only formulations. One may adopt other schemes of construction contract payoff according to the characteristics and types of the construction contracts or BOT agreement.

- ☐ For operating related contract value: Since the profit from the operating related contracts is contingent on the continuing of the project, the payoff function will be zero if the project is bankrupted. The terminal payoff function is the same as the "rescue" equilibrium's payoff function given in equation (6.49). The *time t* payoff function is the same as equity value's *time t* payoff function in equation (6.52).

Step 5: Plug the payoff functions into the binomial pyramid model and compute the values of the profit structure components backward recursively. Note that in the first backward computation, one should use the terminal payoff functions and compute the value of the contingent claims as in "rescue" equilibrium. For the following backward computations, the "bankruptcy" condition in equation (5.19) has to be checked at each time step. If the condition is met at a specific node, the value at the node is determined by its bankruptcy payoff instead of its backward discounted expected value.

6.5 BOT Investment Decision Criterion: Full Net Preset Value

To evaluate a BOT investment, the developer will not only look into the equity investment, but also the associated construction contract and operating related contract value. Therefore, it is essential to evaluate the whole profit structures defined in this research. The judging criterion for BOT investments will be the "Full Net Present Value" (FNPV). The BOT's *FNPV* is defined as

$$FNPV \equiv E + C + O - I \tag{6.59}$$

where E, C, O are the equity value, construction contract value, and operating related contract value, respectively, and I is the amount of equity investment. Note that the E, C, O in equation (6.59) should be computed under the game and real options theoretic valuation framework. The FNPV is net/overall profit for the BOT developer. The term "full" means: first, the valuation process considers the values of managerial options, such as the negotiate for subsidy, and second, the model considers the profit structures as a whole.

According to the capital budgeting theory, the management should undertake an investment if its NPV is positive, and abandon an investment if the NPV is negative. Similarly, the first decision rule in a BOT investment is that if the BOT's FNPV is positive, the investment should be undertaken, and vice versa. Note that since the financing alternatives, namely, the amount of I, may affect the values of E, C, O, the FNPV will be different under different available financing alternatives. Typical capital budgeting theory may suggest that one should choose the alternative that gives the maximum FNPV. Nevertheless, some theories suggested that under limited capital, one should choose the investment alternatives according the ranking of capital return rate, $\frac{FNPV}{I}$, or try to maximize the overall capital return within the capital constraints (Copeland and Weston, 1988). In this research, the issue of how to allocate the limited capital optimally is not the concern of this research. We will focus on how to maximize the NPV of the developer's firm. In other words, we assume that the developer prefers the investment alternative with highest FNPV. The developer's maximization rationale is the core concept of the *BOT Investment Rationale and Decision Framework* shown in Fig. 4.2. Mathematically, this rationale can be described by

$$\underset{e}{Max}\, FNPV(e) * P(e) \tag{6.60}$$

where e is the variables that affect the FNPV and the probability of winning a BOT project, P. Equation (6.60) is the second decision rule in a BOT investment.

Equations (6.59) and (6.60) has crucial implications on the government's BOT policies. Good BOT policies must be based on the correct understandings of the BOT developers. We shall discuss these implications and the impacts on the BOT policies in Chapter 7.

6.6 An Illustrative Example

An example is used to illustrate the game and real options theoretic analysis of the BOT investment's valuation, financing, and procurement problems. The example is modified from a case study on the Toronto Airport discussed in Walker and Smith (1995). However, the numbers used in this example do not represent any actual figures in the Toronto Airport project.

6.6.1 Case Descriptions

SmartCorp Inc. is a joint venture of several leading construction firms and a major insurance company in Canada. The management is evaluating a BOT *airport terminal* investment. Suppose that SmartCorp will fund an independent BOT firm, SmartAir, to develop and operate the airport if SmartCorp wins the BOT bid. In the BOT airport terminal project, the BOT firm, SmartAir, will obtain a 40-year ground lease from the government to develop and operate the terminal. The development of the airport includes the terminal building with retail shops, restaurants, two airport hotels, and parking lots. SmartCorp's preliminary analysis shows:

- SmartCorp has only one alternative to finance the BOT project. The alternative is that SmartCorp invests $100 million as equity in forming SmartAir. The other source of funding for project cost is the bank loan. Assume that the loan interest rate is 10%, compounded continuously.
- The airport terminal's estimated construction cost is $400 million in year 0's prices, not including the net financing cost. The estimated cost inflation or growth rate is 4% per annum. The estimated cost volatility, σ_K, equals 0.2 per annum, which is moderate.

- The estimated optimal construction time is 4 years.
- The estimated future free cash flows from operating the airport terminal for 36 years is \$610 million in year 4's prices. The estimated project value volatility, σ_V, equals 0.4 per annum. The estimated growth rate of free cash flow is 5% per annum. The project value is uncorrelated with the project cost, that is, $\rho_{VK} = 0$.
- SmartCorp will be the major contractor for the construction contracts. The construction contracts will impose 10% markup profit in the estimated construction cost. For the excess cost over the budgeted cost, SmartCorp can only impose a 3% markup profit in the excess cost.
- SmartCorp will also be the airport's major insurance provider. SmartCorp charges SmartAir insurance premiums annually. The total profit from the insurance policies for 36 years will be \$30 million in year 4's prices.

The problem is: should SmartCorp invest in the BOT project? In the next section, we use the BOT model derived above to analyze this BOT investment. Additional assumptions or estimates will be needed to perform the analysis, and will be given during the analysis.

6.6.2 Implementation of Five Steps Profit Structures Valuation Framework under "Rescue" Equilibrium

In this section, it is assumed that SmartCorp's analysis shows that if any critical adverse events occur such that the project requires government subsidy to continue the project, SmartCorp can successfully justify the subsidy and complete the project on time. Therefore, we should evaluate the investment under "rescue" Nash equilibrium. The analysis will follow the five steps proposed.

Step 1: Select the state variables and determine their dynamics and current values. The BOT project value and total construction cost are selected as the state variables. The dynamics of the two state variables are the same as those in equations (6.18) and (6.19), that is,

$$\frac{dV}{V} = (\mu_V - \delta_V)dt + \sigma_V dz_V \text{, and}$$

$$\frac{dK}{K} = (\mu_K - \delta_K)dt + \sigma_K dz_K .$$

The current value of *V* can be computed by discounting \$610 million to year 0's prices. Note that according to the SmartCorp's preliminary analysis, we have $\mu_{V_r} = 0.05$ and $\mu_{K_r} = 0.04$. Thus, the current value of *V* equals $V_0 = 610e^{-\mu_{V_r}*4} = 610e^{-0.05*4} = \$\ 499.4$ million. The current value of *K* equals \$400 million as indicated in the preliminary analysis.

Step 2: Align the dynamics of the state variables with the capital market and project characteristics. First, μ_V and μ_K are estimated according to CAPM formula in equation (6.24) and (6.25). δ_V and δ_K are computed and estimated according to the concept of rate of return shortfall. Suppose that SmartCorp's further analysis showed that the expected market rate of return and volatility are 15% and 0.3 per annum, respectively, and that the correlations ρ_{VM} and ρ_{KM} are equal to 1.2 and 0, respectively. By equations (6.27) and (6.28), one can obtain that μ_V = 19.8% and μ_K = 7%. Second, the rates of return short fall, δ_V and δ_K, are determined by the equilibrium rates of return and the state variable's growth rates. By equations (6.31) and (6.32), one can obtain that δ_V = 14.8% and δ_K = 3% per annum.

Step 3: Construct a reverse binomial pyramid for the two state variable problem. The most important parameters in the binomial pyramid is the jump probabilities and jump amplitudes. The jump probabilities are given as in Fig. 6.10 where $\rho = \rho_{VK} = 0$. The jump amplitudes are computed by equations (6.33) to (6.36). Suppose the time steps are 300, the jump amplitudes are

$$u_V = 1.04393,\ d_V = 0.95181$$

$$u_K = 1.02309,\ d_K = 0.97691$$

Step 4: Determine the *terminal* payoff functions. Under "rescue" equilibrium, only the *terminal* payoff function is needed.

- For equity value: Combining equations (6.37), (6.40), and (6.41), the *terminal* payoff function is given as

$$Max\left[0, V_T - \left(\frac{K_T e^{-r_c T}}{(r_d - r_c)T} e^{r_d(T-i)+r_c i} \Bigg|_{i=T}^{i=t_d} \right)\right] \tag{6.61}$$

where $t_d = \dfrac{I}{\left(\dfrac{K_T e^{-r_c T}}{T}\right)}$, I = 100 million, T = 4, and r_d = 0.1.

- For construction contract value: The *terminal* payoff function is given in equations (6.45) and (6.48) with $\alpha = 0.10$, $\beta = 0.03$ and r_c=0.05.

- For operating related contract value: The *terminal* payoff function is given as

$$\begin{cases} W, & if \quad V_T - D_T(K_T) \geq 0 \\ 0, & if \quad V_T - D_T(K_T) < 0 \end{cases}$$

where W = \$30 millions and $D_T(K_T)$ is given in equation (6.40).

Step 5: Plug the *terminal* payoff functions into the binomial pyramid, and compute the value of the profit structures backward recursively. In this example, the time steps used are 300. The difference between the results of using 200 time steps and 300 time steps is greater than the difference between 300 time steps and 500 time steps. Also, the difference between the results of using 300 steps and 500 steps is of no significance. Therefore, the use of 300 time steps shall be adopted in the binomial model. The computation starts at time T by computing the *terminal* payoff function, and then the computation is repeated backward recursively until time 0. The values of the profit structures are computed as:

- Equity value: \$98.4 million.
- Construction contract value: \$34.9 million.
- Operating related contract value: \$7.2 million.

6.6.3 Five Steps Valuation under "No Rescue" Equilibrium

The other possible equilibrium that the airport terminal BOT project could have is the "no rescue" equilibrium. For example, suppose the government passes a law to *forbid* any post-awarding actions after the BOT agreement is signed, the opportunity cost of subsidy will be infinitely high for government such that the project can only be sold to the government or other private parties.

In profit structures valuation, the first three steps are identical to those under "rescue" equilibrium. In step 4, the *time t* payoff functions are needed in addition to the terminal payoff functions derived previously. Specifically,

- For equity value: the payoff function is given in equation (6.52)

$$\begin{cases} \mathit{Do\ not\ need\ payoff\ functions}, & \mathit{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} \ge 0 \\ 0, & \mathit{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} < 0 \end{cases}$$

- For construction contract value:

$$\begin{cases} \mathit{Do\ not\ need\ payoff\ functions}, & \mathit{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} \ge 0 \\ \mathit{Equations}\,(6.57) - (6.58), & \mathit{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} < 0 \end{cases}$$

- For operating related contract value:

$$\begin{cases} \mathit{Do\ not\ need\ payoff\ functions}, & \mathit{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} \ge 0 \\ 0, & \mathit{if} \quad V_t - D_t(K_t)e^{-r_d(T-t)} < 0 \end{cases}$$

Step 5: According to section 6.4.2, the values of the profit structures may be obtained:

- Equity value: \$76.1 million.
- Construction contract value: \$9.8 million.
- Operating related contract value: \$2.2 million.

6.6.4 Investment Evaluation

Consider the first financing alternative: I = \$100 million. Under the "rescue" equilibrium, the BOT investment's FNPV is:

$$FNPV = \$98.4+34.9+7.2-100 = \$40.5 \text{ million}$$

Since the FNPV is positive, SmartCorp should undertake this investment. It is worth noting that although the FNPV is positive, the profit of the equity investment alone is negative, that is, E-I = \$98.4-100= -\$1.6 million. In other words, from the perspective of the equity/share holder, the project is not viable. As a result, under rescue equilibrium the developer and government have different conclusions regarding the project undertaking.

Under the "no rescue" equilibrium, the FNPV is:

$$FNPV = \$76.1+9.8+2.2-100 = -\$11.9 \text{ million}$$

Since the FNPV is negative, SmartCorp should not invest in the BOT airport. It is worth pointing out that the loss on equity is one major cause that SmartCorp should not invest even with the construction and operating related contract profit. From the perspective of government, since the equity value is less than \$100 million, the project is not viable either. Therefore, for government, it is desirable that the developer obtains a negative FNPV when the project is not "viable" from the government's perspective. If the developer obtains a positive FNPV mainly because of large construction or operating related contracts, the government may suffer from the high possibility of project's failure in the future. Also note that the construction and operating contract values are roughly one fourth of those values in "rescue" equilibrium. The main reason is that in "no rescue" equilibrium, the project may be bankrupted before project completion and the early termination of project would significantly reduce the values of construction and operating related contracts.

6.6.5 Impacts of Financing Alternatives

Suppose that SmartCorp can arrange lower equity investment, $70 million, and higher debt ratio with 14% interest rate. As a result, another set of figures can be obtained:

The profit structures under "rescue" equilibrium are:

- Equity value: $81.1 million.
- Construction contract value: $34.9 million.
- Operating related contract value: $5.8 million.

In this case, the FNPV is $51.8 million. Compared with previous equity level *I*=$100m, this lower equity level would result in higher FNPV. Thus, if the government does not require high equity level, low equity level would be more attractive to the developer.

The profit structures under "no rescue" equilibrium are:

- Equity value: $61.2 million.
- Construction contract value: $4.2 million.
- Operating related contract value: $0.8 million.

The FNPV is -$3.8 million. The developer should not undertake the investment because of equity loss and small construction and operating related contract values.

CHAPTER 7 PROJECT DEVELOPING STRATEGIES AND TENDERING POLICIES

The third framework in the BOT model is the *BOT Tendering Framework*, the third element of the BOT model, as shown in Fig. 4.4. The tendering/procurement framework concerns the developer's bidding strategies and the government's proposal evaluation and procurement polices. The bidding strategies include the financing arrangement, information repackaging, and the proposal preparation. The BOT tendering framework was developed by applying game theory to further investigate how a developer determines financing strategies under different government tendering policies. The conventional wisdom regarding the developer's BOT equity level decision and the government BOT tendering policies is reexamined. New implications are given on developer's financing and bidding strategies and government's BOT policies based on the real options and game theoretic analysis.

7.1 Government Roles, BOT Policies, and Developer's Financial Decisions

As argued previously, the developer's maximization rationale should be applied to the investment's FNPV instead of the BOT firm's value. As a result, the developer will determine the financing package according to the FNPV maximization rationale. In this research, it is assumed that the BOT project is mainly financed by equity and bank loan/debt, and therefore the financing decision will be in terms of the equity investment level. In other words, the developer's optimal equity investment level should maximize the BOT investment's FNPV. However, if the government imposes certain procurement policies, the developer's optimal investment level will be complicated by the policies. For example, if the government is constrained by a law that forbids any forms of subsidy toward a failing BOT project, the developer should consider using the "no rescue" equilibrium in the valuation process of the profit structures. Otherwise, if the project is *developer-led* and it is difficult to identify a *clear client-contractor relationship*, the developer may assume that the BOT project has a "rescue" equilibrium and evaluate the investment accordingly. The impacts of government policies on the BOT procurement process and the developer's valuation process will be discussed in the following sections.

7.1.1 Government Roles and Policies

Smith (1999) argued that "*by the beginning of the twentieth century, the relationship between the private sector and government in infrastructure procurement had begun to reach some kind of maturity.*" He concluded that the relationship had made the government acting as the following roles today in various forms:

- ☐ regulator
- ☐ customer
- ☐ facilitator
- ☐ investor
- ☐ planner
- ☐ protector of the public interest
- ☐ defender of the realm
- ☐ guarantor
- ☐ an agent of economic change
- ☐ supporter of export trade

Smith (1999) further argued that a new role of government today is

- ☐ promoter of privately financed projects.

Note that depending on different countries and their economic and political developing stages, the emphases on the government's roles are different. Government policies will be formed according to the major roles. As a result, different roles expected upon the government by the people will result different policies, and different policies will have critical impacts on the BOT tendering or infrastructure privatization process. For example, a government that mainly acts as a planner and an agent of economic change may not have the policies against subsidizing a failing BOT project, since the government's major role is to provide the infrastructure and boost the economy. Thus, the fairness may not be as important as that in other developed countries. On the contrary, a government that mainly acts as a protector of public interest may have tighter regulations on the BOT project's tendering and management process in order to protect public interest from potential corruption, over-subsidization of the project, or project's unreasonably high profitability. In some infrastructure projects, the government may be the major consumer/buyer of the project outputs, and the government will act as a customer. To summarize, a good policy in one country may not be an appropriate policy for another. The judging criteria of the policies depend on the roles of the government.

7.1.2 Assumptions of Government Roles and Tendering Scheme

This research mainly focuses on the government roles as a facilitator and protector of the public interest. Under the two roles, the government's priority is to provide the infrastructure systems as planned while the public interest is safeguarded by ensuring that the tendering process is fair and capable of selecting the most qualified developer, and the rescue action is justifiable. In general, the competitive bidding/tendering scheme is recognized as the most suitable one to ensure procedural clarity and fairness, to promote competition, and to minimize the time and cost of developing BOT projects (UNIDO, 1996). Therefore, it is assumed that the competitive bidding scheme will be used by government in BOT projects.

7.1.3 Government BOT Policies and Developer's Financial Decisions

As mentioned in section 1.2.3, the awarding of a project under competitive scheme will be given to the proposal with the highest evaluation points instead of lowest bid. Thus, the evaluation criteria for assigning points to a proposal will be crucial to the developer's decision making. Therefore, a portion of the government's BOT policies will be transformed into the evaluation criteria. Among those critical success factors in winning a

BOT project, the "financial viability" was recognized as one of the most important factors (Tiong, 1996). In other words, the government typically will weigh the project's financial viability heavily in evaluating a proposal. In evaluating a project's financial viability, if the government considers equity investment level to be essential, the government may specify how the points are increased along with the level of equity. It is worth noting that the validity of this policy or evaluation criterion is subject to be examined later by the real options and game theoretic analysis. Other evaluation criteria regarding the financial viability may include the expected net profit, construction cost, financing packages, and operating plans.

Not all BOT policies can be transformed into the proposal evaluation criteria. Some BOT policies may not be included in the evaluation criteria, such as policies regarding the project procurement approach, and the prohibition or allowance of future subsidy. However, these types of policies may have material impacts on the developer's financial decisions. For example, the government's subsidy policy will affect the developer's assuming BOT game equilibrium, and therefore the developer's investment valuation process.

As a result, various forms of policies may have significant impacts on the developer's financial decisions. It is essential to understand the purpose of each particular BOT policy and its impacts on the developer's decision making. For example, suppose that in a BOT project, the competitive tendering scheme is applied and the equity level is an important evaluation criterion specified by the government. It may be easy to understand that the purpose of this evaluation criterion is to choose a developer who has the highest value of E-I, the equity value minus the equity investment. However, it is complicated to understand the impacts of the high equity ratio criterion on the developer's financial decisions. In other words, it is not clear whether the developer who uses the highest equity ratio will actually have the highest value of E-I or not. One of the main purposes of this research is to investigate the impacts of a specific evaluation criterion and the criterion's effectiveness.

7.2 Signaling Game and BOT Tendering

7.2.1 Signaling Games in BOT Projects

Signaling games are the most widely applied class of dynamic games of incomplete information. Usually there are two moves and two classes of players involved in a signaling game. The first move of the game is initiated by one class of player, "Sender." The Sender has private information regarding his type, for example, low productivity or high productivity on his job performance. If possible, the Sender will send certain message or "signal" to the other class of players, "Receiver," to convince the Receiver regarding the Sender's type. The "Receiver" will play in the second move depending on the signal received from the Sender. The main idea is that effective communication can occur if one type of player is willing to send a signal that would be too expensive for the other type of player to send (Gibbons,1992). The most effective communication is the "separating equilibrium," in which different types of "Senders" will send different signals and the "Receiver" will believe the signal and act accordingly. On the other hand, the "pooling equilibrium" is an ineffective communication. In pooling equilibrium, both types of Senders will send the same signal. As a result, the Receiver will not believe the signal received, and will not be able to differentiate effectively the Sender's type. Cho and Kreps (1987) argued that with more strict equilibrium requirements, the pooling equilibrium cannot exist. Thus, we will ignore the pooling equilibrium in this research.

In a BOT project, the developer is the Sender, and the developer will send signals to the government, the Receiver, in order to convince the government regarding the developer's type. The developer's type may be "have a financially viable project" or "not have a financially viable project." The signals may be the equity level of the project cost, the viability figures shown in the proposal, or the self-exclusion of being a construction contractor. In other words, *the proposal itself is a collection of signals*. The actions of the government after receiving the "signals" or proposals would be the proposal evaluation points, that is, the decision of project awarding.

Note that since the profit structures are different in each BOT project, the results from signaling game analysis may be different as well. For example, the equity level criterion may be an effective criterion in one project, but may not be effective in another one. Furthermore, there are so many choices of possible signals that it is impossible to analyze all possible signals. Therefore, only limited signals will be discussed. The BOT signaling game analysis is expected to provide both the developers and government with deeper insights toward the bidding/financing investment strategies and government procurement policies. More complete and rigorous analysis on broader signal categories can be performed in future research.

7.2.2 Job Market Signaling Game and Its Equilibrium

Spence (1973) was the first to show how signaling could be a solution to the asymmetric information problem. After Spence, much literature on signaling games emerged and was applied in many asymmetric problems. For example, in financial literature, Leland and Pyle (1977) and Myers and Majluf (1984) successfully applied signaling games in the analysis of capital structure and corporate financing decisions. This section will discuss the job market signaling game to illustrate the essence of the signaling game. Then the key elements of signaling game will be applied to the BOT project.

Figure 7.1 gives the extensive form (without payoff matrices) of a *simplified* job market game. First, nature determines the worker's productivity ability: high (H) or low (L), with probability p of high productivity. Here we name the high-ability workers as "worker H" and low-ability worker as "worker L." Second, the worker learns his ability and chooses his education level: high education (E_H) or low education (E_L). Here it is assumed that it costs \$0 for both types of workers to obtain low education, E_L, and costs C_H and C_L to obtain high education level for worker *H* and worker *L*, respectively. Third, the firm observes the worker's education level and offers the wage level: high wage (W_H) or low wage (W_L). The dot lined ellipses in Fig. 7.1 indicate that the firm has no knowledge of which node the firm is at, i.e., the type of the worker is private information.

There are two types of equilibria:

1. Pooling equilibrium:

Obviously, if the low-ability worker can obtain a high level of education as easily as the high-ability worker, the low-ability worker would want to obtain a high education level to convince the firm that he is high-ability worker. In this case, the firm cannot believe the signal sent by the workers, and therefore, will not offer a high wage level to the worker with a high education level.

2. Separating equilibrium:

The conditions for the signal to be effective must be that it is not in the low-ability worker's interest to imitate the high-ability worker. In this case, the firm will believe the signal regarding productivity ability and offer wages accordingly. These conditions can be expressed mathematically. First, in order for the worker *L* not to imitate the worker *H* on the education level, conditions must satisfy

$$W_H - C_L < W_L \tag{7.1}$$

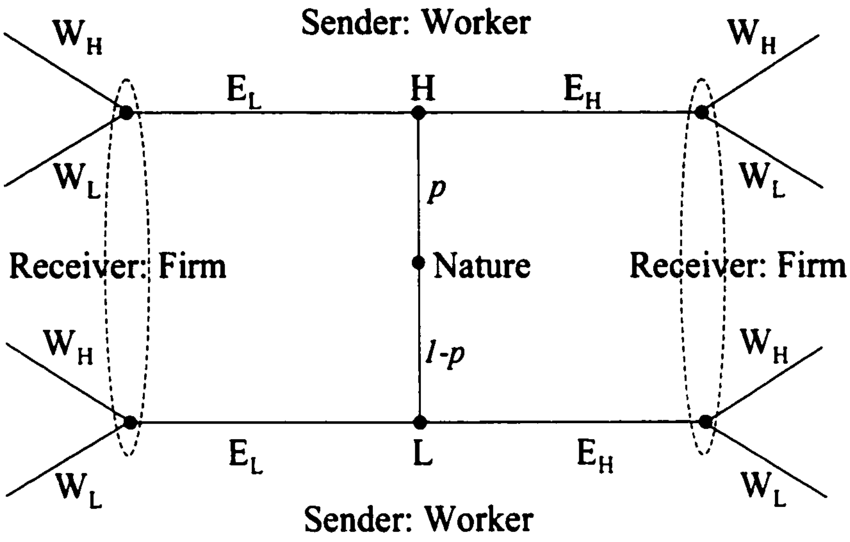

Fig. 7. 1 Extensive Form of a Simplified Job Market Game

Equation (7.1) is obtained on the basis of specifies worker *L*'s payoff maximization rationale. Under this rationale, it is better for worker *L* to have a low education level and receive wage W_L if equation (7.1) is satisfied. Second, to motivate the worker *H* to obtain high education to signify that he is a high-ability worker, conditions must satisfy

$$W_H - C_H > W_L \tag{7.2}$$

Combining equations (7.1) and (7.2), we may obtain

$$C_H < W_H - W_L < C_L \tag{7.3}$$

The interpretation of equation (7.3) is straightforward. From the perspective of the firm, the difference of the high-wage offer and low-wage offer must be large enough to compensate worker *H*'s extra education cost, but not large enough to compensate worker *L*'s high education cost. If the wage offer's difference is too large such that worker *L* is willing to incur extra education cost, the signal would become ineffective. From the perspective of the worker, worker *H*'s signaling cost, C_H, must be less than worker *L*'s signaling cost, C_L. If the difference of wage offer is fixed, then worker *H*'s signaling cost must be low enough and worker *L*'s signaling cost must be high enough so that equation

(7.3) is satisfied. If $C_H \geq C_L$, the signal will be ineffective. The separating equilibrium will be the only possible equilibrium because of the refinement of Cho and Krep (1987). Detailed and rigorous discussion regarding the job market signaling game can be found in Spence (1973,1974) and Cho and Krep (1987).

7.2.3 Implications on BOT Investments

In a BOT project, the developer's payoff is the FNPV. The FNPV depends on the type of the developer and the developer's type (*H* or *L*) is unknown to the government. Suppose that developer *H* is favored by the government over developer *L*. In a signaling game, developer *H* may try to send some costly signal to signify the developer's type. The cost of the signal is reflected on the reduction of the FNPV. As equation (7.3) shows, for the signal to be effective, the cost of developer *H*'s signal must be significantly lower than developer *L*'s cost for sending the same signal. The cost of the signal must also be fully compensated by government's corresponding action after the signal is received. The action by the government is based on the belief of the government toward the signals.

The government's main action toward a project bidding is to decide each developer's possibility of winning or the evaluation points of the proposal. The evaluation points or probability of winning will then determine the developer's expected payoff. An effective evaluation criterion is expected to give the developer *H* higher expected payoff, W_H, and the developer *L* lower expected payoff, W_L. The expected payoffs, W_H and W_L, should satisfy equation (7.3).

A very basic but critical question in any game theoretic analyses is that what defines the type *H* developer and the type *L* developer. This definition is determined by the roles and objectives of a government. As argued in section 6.6.4, it is assumed that the government's major concern is the project's viability to the shareholders, or more specifically, $E_A - I_A$, where E_A is the project's total equity value and I_A is the project's total equity investment. Here the subscript, A, indicates the total amount as opposed to the partial amount that belongs to the developer. Note that in the FNPV formula, equation (6.59), E and I are the fractions of E_A and I_A that belong to the developer. If the developer's equity owning ratio as defined in section 4.3.1, ψ_E, is equal to 1.0, i.e., there is no passive equity, then E will equal E_A and I will equal I_A. The assumption regarding government's major concern is based on the role of a government noted in section 7.1.2, the protector of public interests. Therefore, to the government, the type *H* developer means that the value of $E_A - I_A$ of the project is sufficiently high, and type *L* developer has a low value of $E_A - I_A$.

As a result, developer *H* will try to send effective signals to show that the project's $E_A - I_A$ is sufficiently high. For example, if developer *H*'s major profit is mainly from equity value instead of construction and operating related contract value, developer *H* may announce that the developer will not be the major contractor. Meanwhile, developer *L* may not be able to make the same announcement, since to forego the construction contracts, developer *L* may suffer from equity loss while receiving no compensation from the profit of the construction contracts. In this case, the announcement of not being the contractor may be an effective signal. Note that this signal may be effective only under the game equilibrium, that is, both players are playing the signaling game. For example, if the government does not recognize the effectiveness of the signal, then developer *H* may not be willing to convey the costly signal.

7.3 Screening Game and BOT Tendering

7.3.1 Screening Game Versus Signaling Game

"Screening" was used to refer to the market process identified by Rothchild and Stiglitz (1976). The information problem in the Rothchild and Stiglitz's (1976) insurance model was almost identical to that in Spence's (1973) job market model. However, in the screening game, the Receiver initiates the move and the Sender responds. An effective screening scheme design can induce the Sender to reveal private information or types. For example, in the job market analysis, the firm may make the first move by specifying a menu of contracts regarding the education levels and wage offers. The workers will then respond by selecting their preferred contracts. In the job market, an effective screening menu of contracts would induce the worker *H* to select the contract with a high education level and a high wage, and the worker *L* to select a low education level and a low wage contract. On the other hand, an ineffective screening scheme cannot induce the workers to reveal their types. For instance, if the difference of the wage level is too large, it may be optimal for worker *L* to invest more on education and obtain the high wage. If the difference of the wage level is too small, it may be optimal for worker *H* to select low education, thus eliminating the need to incur high education cost.

7.3.2 Implications on Government's BOT Policies

In BOT projects, the government can freely specify any combination of evaluation criteria and their weights. The evaluation criteria are similar to the education levels, and the criteria's weights or points are similar to the wage levels. If the government clearly specifies the criteria and weights in the Request for Proposals (RFPs), the RFPs are very similar to the menu of contracts in a screening game. An effective screening scheme may induce each developer to *self-select* into the contract or evaluation scheme so that the best developer will earn highest evaluation points. Nevertheless, building an effective evaluation/screening scheme for BOT projects is a difficult task. One major reason is that there are many evaluation criteria and there is no framework that can assess the impacts of each criterion on the developer's profit structures. If the government adopts an ineffective screening scheme and selects a developer accordingly, the developer may very well not be an optimal partner with the government. Therefore, the government must understand and even quantify the impacts of each evaluation criterion on the developer's profit structures. One of the most important contributions of this research is to develop a framework to evaluate the developer's profit structures. Built upon this contribution, the government can make effective screening policies based on clear understanding of the developer's profit structures.

RFPs are not always clear in their evaluation criteria, and thus, developers are unable to self-select into the evaluation scheme. Even under this circumstance, i.e., the screening game is unclear, the developer can still use the signaling game.

7.4 Examination of Some Typical Government BOT Policies

In light of the theory of dynamic game with incomplete information, an effective policy should be able to *screen* out unwanted developers or identify a good developer according to certain valid signals. In the real world, the developer's most likely estimated figures are unknown to the government, and the figures in the developer's proposal may be skewed/manipulated toward winning the BOT project. For example, if false representation about the developer's most likely estimated cost will not become or increase the developer's future liabilities, the developer will have the incentive to adopt the optimistic project cost estimates in order to enhance the financial viability "figures" and winning probability. Therefore, effective policies should consider the developer's incentives of intentional use of optimistic estimations in the proposal. In this section, the validity and effectiveness of some typical government BOT polices will be studied by using the BOT airport example in section 6.6. This section will also discuss under what conditions these specific policies are valid. Tiong (1996) listed some typical government evaluation criteria toward a BOT project's financial package:

- ☐ high equity level
- ☐ low construction cost
- ☐ acceptable tolls/tariff levels
- ☐ short concession period.

The effectiveness of some typical BOT policies and proposal evaluation criteria will be discussed.

7.4.1 High Equity Level Evaluation Criterion

According to Tiong (1995a), equity level or equity-to-debt ratio requirements are commonly specified in RFPs. Some RFPs may further state that the level of equity is an important criterion in selecting proposals. Tiong (1995a) argued that the rationales behind this criterion are (1) high equity will reduce the project's debt burden; (2) it signifies the developer's faith in the project's viability; and (3) it may motivate the developer to complete the project on time and on budget. From the perspective of game theory, these rationales mean that high equity may either be a valid *signal* to signify the developer's private information, such as project's viability, or be able to *screen out* unqualified developers. The BOT airport example in Chapter 6 will be used to illustrate the validity of the high equity level criterion.

Table 7.1 summarizes the SmartCorp's profit structures on the BOT airport with respect to different equity levels under the "rescue" game equilibrium. Note that the government should judge the project's financial viability from the perspective of the equity holders instead of the developer. The reason is that during the concession period, the project is the BOT firm's sole asset; therefore, low equity value is equivalent to the project's low viability. Therefore, from the perspective of either equity holders or government, the project is viable when I = \$70 million since \$81.1-70 > 0, whereas the project is not viable with higher equity level. Note that although the higher equity level, \$100 million, gives lower FNPV because of the value loss in the equity, both equity levels yield positive FNPV. Furthermore, if the probability of winning the project can be significantly increased by higher equity level, the difference in FNPV, \$51.8-40.5 = \$11.3 million, may be counterbalanced by the winning probability difference. Therefore, in *signaling rationale*, the adoption of higher equity level cannot effectively signify higher project viability, since the loss due to low equity value has limited impact on the overall profit structures and the developer's expected investment payoff, $FNPV(e)*P(e)$. In *screening rationale*, higher equity level requirement cannot effectively screen out the developers who have equity loss. Therefore, in this specific BOT airport case, the equity level is not an effective evaluation criterion.

Analyzing the numbers in Table 7.1, one may find that the construction contract value contributes a major portion of the FNPV. Suppose that the construction contract value is reduced to zero by the government's policy that forbids the developer to become the project's contractor. In this case, higher equity level will reduce the FNPV to $5.6 million, whereas low equity level will give $16.9 million. As a result, even if the higher equity level has higher winning probability, it may be more difficult to counterbalance the loss of FNPV. Therefore, in signaling rationale, the developer will not use high equity level unless the project is viable. Also in screening rationale, the requirement of high equity level may screen out those developers who do not have a viable project.

To summarize the results from the BOT airport case, the high equity level criterion is not effective if the developer can significantly benefit from the construction and operating related contracts. On the other hand, if the developer is required to be excluded from being the project contractor, the high equity level may be an effective criterion.

Table 7. 1 FNPV under "Rescue" Equilibrium

Profit Structures:	Equity: I = $100m	Equity: I = $70m
Equity value	$98.4 m	$81.1 m
Construction Contract Value	34.9 m	34.9 m
Operating Related Contract Value	7.2 m	5.8 m
Full Net Present Value	***$40.5 m***	***$51.8 m***

7.4.2 "No Rescue" Policy

Another BOT game equilibrium is the "no rescue" equilibrium. Under the "no rescue" policy, the government cannot and will not provide any forms of subsidies should any critical adverse events occur. No subsidy to the project after the project awarding can be justified. In the "no rescue" equilibrium, the BOT firm will be bankrupted or taken over whenever severe adverse events occur. The developer cannot continue to benefit from the construction contract, nor can the developer obtain subsidy from the government to continue the project and gain the possibility of future changes that favor the project development. Therefore, the profit structures may be reduced significantly.

Table 7.2 summarizes the SmartCorp's profit structures on the BOT airport under the "no rescue" equilibrium. Note that the FNPVs for both equity levels are negative. In this case, the developer will abandon the investment opportunity. One of the major reasons for the low FNPV is that the project does not have sufficient future net profit to support its construction cost. Therefore, if the developer has to take all the risks, the developer may require higher future tolls/tariffs, longer concession period, or other before-awarding subsidy to enhance the project's financial viability. However, it is possible that to another developer, the project may become financially viable. For example, there may be some developer who has the specialties to effectively reduce the construction cost or manage the future operation so that the project itself may become viable. In this case, this superior developer will have the positive FNPV and outperform those who have negative FNPV.

Table 7. 2 FNPV under "No Rescue" Equilibrium

Profit Structures:	Equity: I = \$100m	Equity: I = \$70m
Equity value	\$76.1 m	\$61.2 m
Construction Contract Value	9.8 m	4.2 m
Operating related Contract Value	2.2m	0.8 m
Full Net Present Value	***- \$11.9 m***	***- \$3.8 m***

Note that from the analysis above, the ***non-confusion*** of the BOT policy is no less important than the policy itself. For example, suppose that the equity investment is $I =$ \$100 million. If the government does not specify the policy regarding the post-awarding subsidy, there may be two possible consequences. First, some prudent and responsible developers may assume that there will be no post-awarding subsidy and evaluate the investment accordingly to obtain FNPV= -\$11.9 million. Second, some aggressive developers may assume that the BOT project will have a "rescue" equilibrium and obtain the investment's FNPV= \$40.5 million as shown in Table 7.1. As a result, in the proposal selection, the government may favor the aggressive developer's proposal since the proposal's figures are based on the "rescue" equilibrium and appear to be more viable. In this case, the government will fail in selecting a responsible developer. On the contrary, if the government clearly specifies the conditions of post-awarding subsidies or formally prohibits any post-awarding subsidies, all developers will explicitly evaluate the value of the subsidy on the same ground, that is, under "no rescue" equilibrium. In this case, the government will have a better chance in awarding the project to the best developer.

7.4.3 "Low Project Cost" Evaluation Criterion

It is reasonable that the government prefers those proposals that suggest lower project cost. However, when the project cost becomes an evaluation criterion, the developer may have incentives to understate the figure in order to enhance winning probability *unless* understating the project cost will significantly reduce the developer's FNPV. To evaluate the investment's profit structures with the underestimated cost, it is necessary to obtain new payoff functions in the valuation process.

First, consider the "rescue" equilibrium:

- For equity value: Since the bankruptcy condition is determined at completion time, the underestimation of the cost will not affect the actual outstanding debt D_T. Therefore, the payoff function will be the same as in previous analysis.

- For construction contract value: Note that since the construction contract's payoff depends on whether there is a cost overrun, suggesting a lower project cost in the proposal will increase the probability of the cost overrun and therefore reduce the profit from the contract. Suppose that the project cost is $300 million, $100 million less than the most likely estimated cost, $400 million. The payoff function has the same form given in equations (6.45) and (6.48) except that the initial construction cost, K_0, is replaced by the underestimated cost, K_0^u. The new payoff function is given as

$$\alpha K_T, \qquad if\ K_T e^{-r_c T} \le K_0^u \tag{7.4}$$

$$\alpha K_0^u e^{r_c T} + \beta(K_T - K_0^u e^{r_c T}), \qquad if\ K_T e^{-r_c T} > K_0^u \tag{7.5}$$

- For operating related contract value: As with the equity value, the payoff function here will not be affected by the understated cost.

According to the new payoff functions, the new value of the profit structures may be obtained. Table 7.3 shows the investment's new profit structures with respect to the most likely estimated cost and the underestimated cost when the equity level is $100 million. Note that the developer's new FNPV is reduced by $4.4 million only. Therefore, if the low construction cost is an important evaluation criterion, the developer may have incentives to imitate the low-cost developers in order to enhance the proposal's winning probability. In this specific case, the low-cost criterion is not effective.

Second, we consider the "no rescue" equilibrium. Similarly to previous analysis, the payoff functions of the profit structures need to be modified. The only modification on the payoff functions is for the construction contract value. The only change in the construction contract value's payoff function is to replace K_0 with K_0^u. To recompute the construction contract value, the construction contract value is $9.4 million. The under-stating of the project cost reduces the construction contract value by only $0.4 million under the "no rescue" equilibrium. Therefore, when the low construction cost is an important evaluation criterion, the developer has even stronger incentives to suggest the optimistic project cost in the proposal in order to enhance the project's winning probability. In this specific case, the low-cost criterion is not effective.

Table 7. 3 FNPV under the Most Likely and Optimistic Estimations

Profit Structures:	**Suggested Cost: $400m** Equity: I = $100m	**Suggested Cost: $300m** Equity: I = $100m
Equity value	$98.4 m	$98.4 m
Construction Contract Value	34.9 m	30.5 m
Operating Related Contract Value	7.2 m	7.2 m
Full Net Present Value	***$40.5 m***	***$36.1 m***

CHAPTER 8 CASE STUDY - THE CHANNEL TUNNEL

This case study is used to demonstrate how the BOT model derived in this research can be applied in real-world situations, and to validate the model. The case study was based on the available data and information collected from the Channel Tunnel project. Some financial data or projections/estimations required in the real options and game theoretic model were not available. One reason was that the conventional valuation scheme did not use these data or estimations. Another reason was that these data might be confidential so that the developer would not release them unless required by the government. A more serious problem is that even when the data were released, the figures might not have been the "most likely" estimations used by the developer during the in-house evaluation. This is the so-called "information asymmetry" problem. This is why the game theory was adopted to study how the government can resolve the information asymmetry problems. In dealing with the data limitation problems, certain assumptions regarding these private information will be made. This chapter will begin with a general introduction and discussion. The results from conventional analysis performed by the Channel Tunnel's BOT firm will be presented. Then further analyses according to the BOT model derived in the research will show how the model can be applied in real-world projects and how well the model can explain certain situations or data observed.

8.1 The Channel Tunnel Project - General Discussion and Conventional Analysis

8.1.1 General Background

As a result of expenditures cut on public projects and the depression of the construction market in the U.K. during the late 1970s and early 1980s, the development of the Channel fixed-link and its financing scheme once more became an important issue in the U.K. during the early 1980s. The British government decided that the project should be financed by private parties and developed in BOT scheme. In April 1985, developers were solicited to submit proposals for a fixed-link channel crossing. In January 1986, the project was awarded to CTG/FM, the former name of the current Channel Tunnel owner, Eurotunnel. The selected fixed-link crossing system incorporates three tunnels: two rail tunnels and a service tunnel, each nearly 31 miles long. The tunnel system is called the Channel Tunnel or Chunnel. The Channel Tunnel is used by shuttle trains, operated by Eurotunnel, carrying road vehicles between the terminals, and by through trains, run by the railway. In the proposal, the estimated construction cost, not including financing cost, inflation provision, and other overhead, was about £2.3 billion. The formal estimated construction cost shown in the 1987 shareholder prospectus, *Offer for Sale* (1987), was £2.8 billion, or £4.8 billion including financing and other indirect costs. The actual overall construction cost on Channel Tunnel's completion was about £10.5 billion, nearly twice the estimate in the *Offer for Sale*, and the project was completed on May 1994, 1 year after the scheduled completion date. In 1995, approximately £8 billion was in bank loans, and the interest burden was about £2 million a day. Moreover, the Channel Tunnel faced strong competition and potential price wars from ferry and airline companies. In September 1995, Eurotunnel unilaterally stopped the interest payments on the bank loans. A set of financial restructuring actions, such as debt-equity swap, was developed to improve Eurotunnel's financial situation. As of 30 June 2000, the total debt was about £7.2 billion (Eurotunnel Interim Report, 2000) after a series of financial restructuring. The bright side was that from 1995 to 1999 the Channel Tunnel continued to increase sales.

8.1.2 Financing

According to the *Offer for Sale* (1987), the total financing requirement up to the scheduled completion date was £4,874 million, including the construction, overhead costs, and financing costs. Table 8.1 shows the estimated capital requirement.

Table 8. 1 Channel Tunnel's Estimated Total Financing Requirement

[Source: *Offer for Sale, 1987*]

	£ million
Construction costs (at July 1987 prices)	2,788
Corporate and other costs (at July 1987 prices)	642
Provision for inflation	469
Net financing costs	975
Total	4,874

To meet the capital requirement, £1,023 million was raised as equity, and £5,000 million was from the credit facilities or bank loans. The total financing amount of £6,023 million provided a safety margin of approximately £1,150 million for unexpected cost overrun. Table 8.2 shows the details of the financing arrangement.

Table 8. 2 Channel Tunnel's Financing Arrangement

[Source: *Offer for Sale, 1987*]

	£ million	£ million
Equity -- Private placement		253
-- Public placement		770
Total equity		1,023
Credit facilities -- main	4,000	
-- standby	1,000	
		5,000
Total financing		6,023

Therefore, the equity to total financing requirement ratio (defined as equity ratio here) was approximately 17% considering the safety margin. In this research, the equity ratio will be measured considering the margin. Note that the privately placed equity to total financing requirement ratio is 4.2% only. These ratios will be used later in the analysis of government tendering policy. Note that the privately placed equity, £253 million, was the sum of the first equity financing, £47 million, by the ten major contractors, and the second equity financing, £206 million, by the major founding banks of Eurotunnel. The third equity financing, £770 million, was publicly placed on November 1987.

8.1.3 Procurement Process and Government Policies

As shown in Fig. 1.1, the main members of most BOT developers/promoters are the construction firms that may eventually participate in the construction contracts. The situation was the same in the Channel Tunnel project. The Channel Tunnel Group (CTG) was the major promoter of the project and was composed of Britain's five largest contractors: Balfour Beatty, Taylor Woodrow, Costain, Tarmac, and Wimpey. CTG's French partner, France Manche (FM), was also composed of another five major construction firms in France. This joint venture was known as CTG-FM. The two governments were aware of the possible problem of having one private entity representing itself as both owner and contractor. The governments made it known that they were not keen on having this owner/client relationship. This assertion is considered as one of the governments' major BOT tendering policies later in the game theoretic analysis. Note that, in responding to this policy, the ten founding contractors withdrew from the CTG/FM and formed another joint venture, Transmanche-Link (TML), in July 1985, approximately 3 months before the proposal submission deadline, 31 October 1985. CTG/FM later became Eurotunnel representing the owner side of the project, and TML was the builder of the Channel Tunnel. Ideally, these two entities should have been working in a normal owner and client relationship. However, as Fetherston (1997) pointed out, "in fact, it [the Eurotunnel] was a curious and frail vessel its crew a gaggle of people seconded from the ten contracting companies." For example, the top management of the contractors was still managing Eurotunnel. Reeve, the top management of TML, acknowledged that "a number of staff oscillated between the two [Eurotunnel and TML] for several months" (Fetherston, 1997). Fetherston (1997) argued that it was impossible for Eurotunnel to prepare a highly complicated proposal in 3 months without help and "the only ones who could help were…the contracting companies who had created CTG/FM, whose executives still sat on and controlled its board." Fetherston concluded that "it meant that the contractors would be signing contracts with an entity they had created and still dominated." The only balancing factor was that the CTG/FM hired an independent expert auditor to review the proposal before the submittal. Levy (1996) argued that this decision proved to be important and had a significant influence on the award of the concession agreement.

A construction contract proposal draft by TML was presented to the CTG/FM (Eurotunnel) in mid July 1985, shortly after TML was formed, and had three major conclusions:

1. The contract will be on target costs basis.
2. The construction cost, not including overhead and financing cost, is £2.77 billion.
3. The construction needs 8 years.

One the other hand, the CTG/FM insisted that

1. There must be some lump sum construction contracts so that the contractor will bear the cost overrun risks on these lump sum contracts.
2. The construction cost should be reduced to £2.33 billion.
3. The construction duration cannot be more than 7 years.

After series of negotiations, a construction contract protocol, Stansted Protocol, was signed by CTG/FM and TML on 30 October 1985, the night before the proposal deadline (Fetherston, 1997). This protocol later became the basis of the construction contracts between Eurotunnel and TML. This protocol was also considered as being strongly in favor of the contractor or TML's interests (Fetherston, 1997).

8.1.4 Credit Agreement and Construction Contracts

According to the *Offer for Sale* (1987), the purpose of the Channel Tunnel project's credit agreement was to specify the loan drawdown conditions so that the banks could monitor the progress of the project and the expected cash flows and to exercise control over the project in the event of significant adverse developments. According to the credit agreement, Eurotunnel had to spend more than £700 million raised by equity placement before the loan drawdowns can begin. In other words, the banks required that Eurotunnel committed available capital obtained from equity first. Here, the £700 million constitutes approximately 70% of total equity raised. Furthermore, the banks' judging criteria, such as cover ratios and specified progress, for the continuing loan drawdowns during construction would be prepared and revised at least semi-annually by lending banks either with or without consulting Eurotunnel. The banks' assumptions on the cost and future cash flows would be more conservative than Eurotunnel's. If the cover ratios cannot be satisfied, the banks could reject further loan drawdowns. In critical adverse developments, the banks could bankrupt the developing firm and take over the project. As mentioned in section 5.3.1, in BOT projects the default/bankruptcy conditions will be very different from those in the conventional bank loans. Conventional bank loans' default conditions focus on the non-payment of interest or principal. In BOT projects, the default conditions mainly focus on the non-satisfaction of some specific cover ratios that are determined or prepared by the banks periodically. Therefore, when there is a significant cost overrun or reduction of future revenues, the project can be bankrupted and taken over by banks. The *Offer for Sale* (1987) listed some possible conditions that could possibly trigger the project's bankruptcy:

1. a delay in opening of more than 14 months
2. a reduction in revenues of more than 7% below the banks' estimation
3. an increase of banks' estimated construction costs by more than £375 million.

Note that banks' estimation of revenues will be more conservative or much lower than Eurotunnel's estimation. Therefore, the reduction in Eurotunnel's revenues must be more than 7% so that bankruptcy condition 2 can be met.

Table 8. 3 Breakdown of Channel Tunnel's Construction Costs

[Source: *Offer for Sale, 1987*]

	£ million
Tunnels, tunnel linings and other underground structures (target works)	1,367
Terminals, fixed equipment, tunnel cooling and automatic control system (mainly lump sum items)	1,169
Shuttles, locomotives and other procurement items	252
Total	2,788

Another important contract that Eurotunnel negotiated and signed was the construction contract. Table 8.3 shows the estimated major items of construction expenditure as of 1987. The total construction cost shown in Table 8.3 has been listed in Table 8.1 as part of the total project cost.

Note that the dividing of the works shown in Table 8.3 was based on the Stansted Protocol. Stansted Protocol was signed under pressure from the French contractors, which stipulated that if no agreement emerged at Stansted Airport, the contractors would not participate in the next day's Submission to Governments (Fetherston, 1997). The protocol guaranteed good profits, limited liability, and promised bonuses if the work was finished within 7 years (Fetherston, 1997). According to the *Offer for Sale* (1987), the construction works were divided into three categories (see Table 8.3). Some important details regarding each category include:

1. Target works: The target works account for nearly 50% of total construction cost. The contractor will be reimbursed for actual costs incurred plus a fixed fee of 12.36% of the target cost. If there is a cost saving, TML will receive a bonus equal to 50% of the saving. If there is a cost overrun, TML will be responsible for 30% of the cost overrun with an upper limit of 6% of total target cost. In other words, if there is a serious cost

overrun in this contract, TML still has revenue of 12.36%-6% = 6.36% of total target cost and this revenue is virtually guaranteed.

2. Lump sum works: The lump sum works account for approximately 40% of the construction cost. Any cost overruns will be borne by TML except that they relate to variations or additions to Eurotunnel requirements or are caused by inflation.
3. Procurement items: TML will be responsible for the preparation, management and supervision of the procurement subcontracts in order to ensure the adequacy and fitness of the procurement items. TML will be reimbursed for the procurement cost plus a fee of 11.5% of the procurement cost.

Moreover, both target cost and lump sum contracts are subject to price adjustment if there are "justifiable" reasons such as unforeseeable ground conditions, changes requested by the governments or Eurotunnel, and inflation. Liquidated damages are at the rate from £354,000 to £536,000 per day, depending on the delay of schedule, and the maximum total penalty is £165 million.

According to Holliday et al. (1991) and Levy (1997), incomplete documents and the "fast-track" design contributed many significant claims over the *lump sum* contract prices. These disputes and claims caused tension between Eurotunnel and TML, the productivity decrease, and the skyrocketing construction cost. At the completion of the project, the overall cost was approximately £10.5 billion.

8.1.5 Eurotunnel's Traffic Forecast and Revenue Estimation Scheme

The traffic forecast was performed by Eurotunnel's traffic and revenue consultants and was based on the traffic volume data from 1964 to 1985. Computer models were developed to estimate the future cross-Channel market, Eurotunnel's market share, tariffs, and Eurotunnel's revenues.

- In the computer models, the size of future market depends on the gross domestic product (GDP). Various GDP growth rates were assumed for different future time periods so that the size of the market could be estimated. Eurotunnel's market share and tariffs needed to be estimated after the estimation of future market size so that the revenue could be inferred.

- To estimate/forecast the market share, the traveler's choice of method and route of travel was analyzed (*Offer for Sale*, 1987). The consultants assumed that in deciding the method and route of travel, travelers would be minimizing the combined travel *time* and *cost*, and that Eurotunnel would attract traffic from other competitors according to the Channel Tunnel's traveling time and cost characteristics.
- To estimate the tariff, the consultants analyzed the cost structure and competition intensity, and concluded the Channel Tunnel's competitors' estimated tariffs.

Table 8. 4 Projected Revenues of the Channel Tunnel

[Source: Adapted from *Offer for Sale, 1987*]

1993-1998

	1993**	1994	1995	1996	1997	1998
	£ million	£ million	£ million	£ million	£ million	£ million
Revenues* (Nominal)	488	762	835	908	986	1,072
Revenues (in 1987 Prices)	357	526	543	557	571	586

Later years

	2003	2013	2023	2033	2041
	£ million	£ million	£ million	£ million	£ million
Revenues (Nominal)	1,586	3,236	6,184	11,356	17,824
Revenues (in 1987 Prices)	647	739	789	808	796

* Inflation rate assumption: 4.0%, 4.5%, 5.0% and 5.5% for 1987, 88, 89 and 90, respectively, and 6% for 1991 and thereafter.

** The figures for 1993 represent trading only from May to December.

- Finally, the revenue forecast was obtained simply by multiplying market size by market share, and by tariffs. Note that the projections showed that in the expected Chunnel opening year, 1993, the sysmtes would carry 44% of cross-Channel passenger traffic and 17% of cross-Channel freight traffic. Table 8.4 shows the Eurotunnel's revenue projections in nominal and real terms. The projections can be adjusted back to 1987 prices to show Eurotunnel's projected revenue-growing tendency.

8.1.6 Eurotunnel's Project Valuation and Risk Assessment

Table 8.5 shows Eurotunnel's profit projections over the operating period after inflation adjustment. The profit was defined as the dividends distributed to the shareholders. However, in the prospectus, Eurotunnel pointed out that the financial projections including revenue, profit, and returns for the period to the end of the BOT concession do not constitute a forecast, since the projections relate to periods up to 55 years. Eurotunnel further explained in the *Offer for Sale* (1987) that the projections will be *materially* affected by changes in *economic and other circumstances*, and therefore the projections are set out "for illustrative purposes" only.

The prospectus did not give any opinions or estimations regarding how volatile the projections may be. Without the volatility/reliability information, the effectiveness of the projections will be very limited in evaluating a project even if the projections are the expected or most reasonable or "most likely" future profits. The *Offer for Sale* used sensitivity analysis as its approach to assess the project risks and their impacts on project returns. The sensitivity analysis was conducted with respect to the projections and assumptions on the revenue, inflation rate, real interest rate, construction cost, and completion time.

Table 8. 5 Shareholder's Expected Profits

[Source: Adapted from *Offer for Sale, 1987*]

1993-1998

Year	1993	1994	1995	1996	1997	1998
	£ million	£ million	£ million	£ million	£ million	£ million
Dividends* (Nominal)		149	169	217	277	328
Dividends* (in 1987 Prices)		103	110	133	160	179

Later years

Year	2003	2013	2023	2033	2041
	£ million	£ million	£ million	£ million	£ million
Dividends* (Nominal)	566	1,476	2,986	5,605	8,880
Dividends* (in 1987 Prices)	231	337	381	399	397

Inflation rate assumption: same as in revenue projections

Table 8.6 shows the varying levels of projections and assumptions used in the sensitivity analysis. In the prospectus, the sensitivity analysis showed that the project's return would be positive even under the worst scenario, implying that in the "pessimistic" case, the project would still have positive returns. It is argued that the sensitivity analysis was either too optimistic or misleading. Recall that the Eurotunnel pointed out that the projections cannot constitute a forecast and will be *materially* affected by economic and other circumstances. Thus, a 10% change in revenue and construction cost, 1% change in inflation, and 1.5% change in interest rate are not commensurate with the imprecision of the projections.

Holliday, Marcou, and Vickerman are researchers in political science, law, and transport economics, respectively, in the U.K. Vickerman was the Director and Holliday a Research Fellow of the Channel Tunnel Research Unit at University of Kent at Canterbury. Holliday et al. (1991) argued that because of the complexity of Channel Tunnel traffic and competition characteristics and data availability, "Eurotunnel's revenue forecasts are always likely to be suspect, and continually subject to revision." They pointed out that previous studies on Channel Tunnel projects did conclude broadly consistent rates of return of between 12% and 15%. From these earlier studies, Holliday et al. (1991) argued, "The Eurotunnel prospectus and some of the early enthusiasts, who forecast financial rates of return of well over 20%, must be seen as too optimistic." Table 8.7 shows the projected revenues and the actual revenues from 1995 to 1999. Table 8.7 indicates that the projected revenues by Eurotunnel were overestimated by approximately 47% or 51% of the projected amount in average. That is, the actual revenues from 1994 to 1999 were only 53% or 49% of the projected amount in average. Holliday et al. (1991) concluded, "this perhaps calls into question the appropriateness of a conventional rate of return analysis for a project of this type, where an enormous capital outlay is incurred well before any revenues are earned."

Table 8. 6 Varying Levels of Projections Used in Sensitivity Analysis

[Source: Adapted from *Offer for Sale, 1987*]

Projections/ Assumptions	Varying levels
Revenue:	+ - 10% in revenues & + - 5% in operating costs
Inflation:	+ - 1% from 1993 + - 0.5% in 1988 and 1% from 1988-1992
Real interest rate:	+ - 1.5%
Construction cost:	+10%
Delay:	6 months of delay & an increase of £ 30 million overhead cost

Table 8. 7 Projected Revenues and Actual Revenues From 1995 to 1999

Compared by actual year **Year**	1995 £ million	1996 £ million	1997 £ million	1998 £ million	1999 £ million	Average
Turnover/Revenues						
From 1987 Projections	835	908	986	1,072	1,158	
From Actual Operations	299	448	456	618	626	
Over-estimate (in £ million)	*536*	*460*	*530*	*454*	*532*	
(in percentage of 1987 projections)	*64%*	*51%*	*54%*	*42%*	*46%*	51%
Compared by the years of operation **Year**	Year 2 £ million	Year 3 £ million	Year 4 £ million	Year 5 £ million	Year 6 £ million	Average
From 1987 Projections	762	835	908	986	1,072	
From Actual Operations	299	448	456	618	626	
Over-estimate (in £ million)	*463*	*387*	*452*	*368*	*446*	
(in percentage of 1987 projections)	*61%*	*46%*	*50%*	*37%*	*42%*	47%

8.2 The Channel Tunnel Project - Implementations of Five-Step Profit Structures Valuation Model

8.2.1 Basic Assumptions

Copeland et al. (1996) argued that in practice it is more appropriate to use nominal dollars, interest rate, and discount rate during the project valuation process. Therefore, nominal terms shall be used for all the parameters and estimations.

- Risk free interest rate, r:

Since the Channel Tunnel's concession period is 55 years, the risk-free interest rate from the government long term bond shall be estimated. According to the 1986 monthly average U.K. government long-term bond yield (Datastream, 2000; bond code: UKHCPYLD), r can be approximated by 9.8% per annum or 9.35% compounded continuously.

- Market rate of return, r_M, and risk premium, λ:

According to 221 observations from 1975 to 1993 by Ferson and Harvey (1993), the mean equity return rate, r_M, is approximately 22% per annum or 19.9% compounded continuously, while the variance, σ_M, is about 27% per annum. λ can be computed by equation (6.25), $\lambda = \frac{r_M - r}{\sigma_M} = 0.39$.

- Discount rate, k, for future free cash flow:

The weighted average cost of capital (WACC), k, is the appropriate discount rate for discounting the future cash flows. To estimate the WACC, k, one needs the cost of equity, k_S, and cost of debt, k_D. The cost of equity is computed according to the CAPM shown in equation (6.24). One may express k_S as

$$k_S = r + \lambda \rho_{SM} \sigma_S \tag{8.1}$$

k_S is the market equilibrium/required rate of return on Eurotunnel equity/stock. For simplicity, Datastream's (2000) industrial sector index for rail, road, and freight (Datastream code: RROADUK) from November 1980 to November 1986 was used to calibrate the risk of the Channel Tunnel's operating cash flows. To compute the correlation, ρ_{SM}, Datastream's (2000) United Kingdom capital market index (Datastream code: TOTMKUK) was used for the consistency with the use of sector index. One obtains $\sigma_S = 0.215$ and $\rho_{SM} = 0.653$ according to the observed market data from Datastream (2000). By using equation (8.1), one obtains k_S = 14.83%.

According to the *Offer for Sale*, the 1987 estimated loan interest rate for the concession period was 9%, based on the banks' cost of funds in 1987. However, the credit agreement specified that the loan interest rate would be "based on reference rates designed to reflect the lenders' cost of funds, which will differ according to the currency and market of drawing, plus margins..." Thus, 9% may not be a good estimate for the expected interest rate throughout the concession period. For simplicity, it is assumed that the cost of debt is approximately the expected risk-free interest rate, 9.8%, plus the margin after the completion, 1%, specified in the prospectus, that is, 10.8% interest rate. The 10.8% interest rate is equal to $k_D = r_d = 10.26\%$, compounded continuously. It is worth noting that when there is government debt guarantee, the loan interest rate, r_d or k_D, will be equal to the risk-free interest rate, 9.35%. Having values for k_S and k_D, one can compute k by

$$k = w_S k_S + (1 - w_S)(1 - \tau) k_D \tag{8.2}$$

where w_S is the target equity to firm/project value ratio, that is, equity ratio, and τ is the tax rate. By equation (8.4), $w_S = 0.17$ at 1993, the year of project completion. However, since the loan will be paid off over the concession period, the equity ratio will be changing over time. According to the *Offer for Sale*, the principal repayment schedule is somewhat even; for simplicity, it is assumed that the principal will be repaid evenly. In year 2041, 1 year before the concession ends, w_S will be equal to 1. Therefore, it can be assumed that w_S will be increased by 0.01729 each year.

From the prospectus' profit projections, the tax rate, τ, will be 10% for the first year, 20 % for the next 5 years, and 40% for the rest of concession period. Therefore, the free cash flow discount rate, k, will be:

$$k = \begin{cases} w_{S,t} * 14.83\% + (1 - w_{S,t}) * (1 - 0.1) * 10.26\% & t = 1 \\ w_{S,t} * 14.83\% + (1 - w_{S,t}) * (1 - 0.2) * 10.26\% & t = 2 \sim 6 \\ w_{S,t} * 14.83\% + (1 - w_{S,t}) * (1 - 0.4) * 10.26\% & t = 6 \sim 49 \end{cases} \tag{8.3}$$

where

$$\begin{cases} w_{S,t} = 0.17 & t = 1 \\ w_{S,t} = 0.17 + 0.01729 * (t - 1) & t = 2 \sim 49 \end{cases} \tag{8.4}$$

For example, in year 6 of the operation, $w_{S,6}$ is equal to 0.2565 and thus k is equal to 9.91%, while in year 20, k will be equal to 10.48%.

Table 8. 8 Expected Construction Cost

[Source: *Finnerty, 1996*]

	1986	1987	1988	1989	1990	1991	1992	1993	Total
	£ million	£ million	£ million	£ million	£ million	£ million	£ million	£ million	£ million
Construction	14	168	504	575	671	507	300	22	2,761
Owning group costs	37	103	81	74	70	66	73	61	565
Inflation	0	3	30	68	118	130	110	30	489
Net financing costs	8	49	29	95	160	245	327	111	1,024
Total expected cost	59	323	644	812	1,019	948	810	224	4,839

- Construction cost's inflation rate, r_c:

Table 8.8 shows Eurotunnel's estimate of the overall cost per annum during the construction period (Finnerty, 1996). Although the numbers in Table 8.8 were not the formal figures that appeared in the prospectus, the "total" of each cost item in Table 8.8 was already very close to the figures in the prospectus. According to Table 8.8, one may infer the average inflation rate for construction cost assumed by Eurotunnel and obtain $r_c =$ 3.6% per annum compounded continuously.

8.2.2 Step One: Select State Variables and Determine their Dynamics and Current Values

As argued above, the project value, V, and the construction cost, K, are chosen as the state variables in the Channel Tunnel project. The dynamics of the project are given in equations (6.18) to (6.20).

The next task is to determine the current values of the two state variables.

1. Current Project Value, $V_{t=0}$:

Note that the project value is defined as the project's profitability characterized by the future operating cash flows. The present value of the cumulated future cash flows will be compared with the total debt outstanding to determine a project's financial viability. Here the project value is computed by discounting the estimated future *free cash flows*. The free cash flows are equal to the after-tax operating earnings of the company, plus non-cash charges such as depreciation cost, less investments in operating working capital, property, plant and equipment, and other assets. The taxes are the cash taxes on the "earning before interests and taxes" (EBIT). Table 8.9 shows the calculations of the free cash flows according to the projections in the prospectus. By interpolating relevant estimates between two distant years, for example, 2013 to 2023, one may obtain each year's free cash flows.

To discount the free cash flows, the discount rate for each year can be calculated by equations (8.3) and (8.4). Since each year's discount rate is different, the free cash flow may be discounted by the corresponding year's discount rate and then the discounted cash flow is added to its previous year's free cash flow for further discounting. This procedure is performed backward from year 2041 recursively as shown in Table 8.10. Table 8.10 shows that the project value at year 1993 is £10,649 million, that is, $V_{t=7}$ = £10,649 million. The value of $V_{t=0}$ can be determined after μ_{Vr} is obtained in next section.

Table 8. 9 Computations of Free Cash Flows

[Source: Adapted from *Offer for Sale,* 1987]

1993-1998

	1993** £ million	1994 £ million	1995 £ million	1996 £ million	1997 £ million	1998 £ million
Turnover/Revenues	488	762	835	908	986	1,072
Operating costs	(86)	(145)	(155)	(168)	(183)	(206)
Earning before interest and taxes	402	617	680	740	803	866
Cash taxes on EBIT	(40)	(123)	(136)	(148)	(161)	(346)
Depreciation adjustment	103	158	159	160	162	167
Capital expenditure	(262)	(37)	0	0	0	(39)
Working capital investments	(3)	(7)	(11)	(11)	(10)	(12)
Free cash flows	200	608	692	741	794	636

Later years

	2003 £ million	2013 £ million	2023 £ million	2033 £ million	2041 £ million
Turnover/Revenues	1,586	3,236	6,184	11,356	17,824
Operating costs	(304)	(631)	(1,027)	(2,246)	(3,604)
Earning before interest and taxes	1,282	2,605	5,157	9,110	14,220
Cash taxes on EBIT	(513)	(1,042)	(2,063)	(3,644)	(5,688)
Depreciation adjustment	184	234	271	328	383
Capital expenditure	(108)	(227)	(1,042)	(650)	0
Working capital investments	(20)	(24)	(40)	(49)	(50)
Free cash flows	825	1,546	2,283	5,095	8,865

** The figures for 1993 represent trading only from May to December.

Table 8. 10 Computation of the Value of Future Cash Flows

1993-1998	1993*	1,994	1,995	1,996	1,997	1,998
	£ million	£ million	£ million	£ million	£ million	£ million
Free cash flows	200	608	692	741	794	636
equity ratio	0.170	0.187	0.205	0.222	0.239	0.256
Discount rate	10.19%	9.45%	9.56%	9.68%	9.79%	9.91%
Discounted value	10,649	11,197	11,699	12,181	12,678	13,188

Later years	2003	2013	2023	2033	2041
	£ million	£ million	£ million	£ million	£ million
Free cash flows	825	1,546	2,283	5,095	8,865
equity ratio	0.343	0.516	0.689	0.862	1.0
Discount rate	9.13%	10.63%	12.13%	13.63%	14.83%
Discounted value	16,161	22,970	30,430	32,572	7,643

* In 1993, it is only counted as 8/12=0.667 year.

2. Current Project cost, $K_{t=0}$:

Note that the construction cost here will be considered as the value of a physical asset. Before the completion time T, the physical asset is considered as unfinished and has an appreciation rate of μ_{K_r}. At any time t before the completion, K will be updated according to state information. Therefore, K_t is defined as the estimated construction cost at *time t prices* and this definition is critical when the early exercise is possible. At any time t, K must be updated and converted to time t prices. At time t, one can always specify/estimate a *cost schedule* as shown in Fig. 6.16. This cost schedule can be converted to arbitrary time t' prices given the cost inflation or escalation rate, r_c. The construction cost $K_{t=0}$ can be obtained by discounting the cost schedule back to time 0. Another equivalent way to compute $K_{t=0}$ is to ignore the inflation provision as shown in Table 8.1, and one may obtain $K_{t=0} = 2{,}788+642 =$ £3,450 million.

8.2.3 Step Two: Align the Dynamics of the State Variables with the Capital Market and Project Characteristics.

1. Project Value's Volatility, σ_V, *Market Correlation,* ρ_{VM}, *and Rate of Return Shortfall,* δ_V

- Project value's volatility and market correlation:

For simplicity, the estimated volatility and market correlation of the rail, road, and freight industry sector was used to represent the project value's volatility during the construction period, 1980-1986. The estimations of σ_V, ρ_{VM} and μ_V are $\sigma_V = 0.215$, $\rho_{VM} = 0.653$, and $\mu_V = 0.1483$.

- Rate of return shortfall:

According to equation (6.31)

$$\delta_V = \mu_r - \mu_{Vr} = r + \lambda \rho_{VM} \sigma_V - \mu_{Vr}$$

Dixit and Pindyck (1994) argued that if the holder of the underlying asset has an constant exogenous rate of return shortfall rate, then μ_{Vr} "must change to preserve equilibrium," $\delta_V = \mu_r - \mu_{Vr}$. They further argued that if μ_{Vr} is exogenously fixed, then the return shortfall rate, δ_V, "must change to take up the slack." For commodity, it is relatively easy to compute δ_V from its future contract. However, when the underlying asset is not a commodity, it may be more feasible to exogenously determine the physical asset's growth rate, μ_{Vr}. In the Channel Tunnel project, according to Table 8.9, the free cash flow increases by approximately 5.7% per annum by solving

$$608 * e^{\mu_{Vr} * 47} = 8{,}865$$

or 5.7% per annum by taking the average of yearly growth rate over the concession period. Thus it is assumed that $\mu_{Vr} = 5.7\ \%$. As a result,

$$\delta_V = 0.1683 - 0.057 = 0.0913$$

Recall that in previous section, $V_{t=0}$ was left to be determined. Here one may compute $V_{t=0}$ by:

$$V_{t=0} = V_{t=7} * e^{-\mu_{Vr}*7} = \text{£}7{,}145 \text{ million} \tag{8.5}$$

2. Project Cost Volatility, σ_K, Market Correlation, ρ_{KM}, and Rate of Return Shortfall, δ_K

- Project cost's volatility and market correlation:

It is very difficult to estimate the cost volatility and its market correlation from observing the financial market, since currently there may not exist any trading company whose value solely relies on the construction cost of a single project. As a result, the cost volatility and correlation needs to be estimated by the contractor's historical cost data on similar projects. However, Eurotunnel did not release any statistics regarding the construction cost's volatility and correlation estimation. For convenience, in this study the cost volatility will be estimated based on some comments found in the prospectus regarding the construction cost and some personal conjectures. In the prospectus, the geological and geometrical conditions were considered to be favorable, and the techniques for tunneling were considered well proven and employed. The estimate of tunneling rate was considered conservative for the expected ground conditions. One may conclude that Eurotunnel's estimate of the construction difficulties/risks was limited if the prospectus fully revealed Eurotunnel's evaluation. Under this circumstance, it is reasonable to assume that the volatility is mild and assign $\sigma_K = 0.2$. Since the technical difficulties/risks may reduce the correlation between the construction cost and the market as a whole, it is assumed that the correlation is low, specifically, $\rho_{KM} = 0.25$.

- Rate of return shortfall:

Given these the cost volatility and market correlation, one can compute μ_K by equation (4):

$$\mu_K = r + \lambda \rho_{KM} \sigma_K = 0.0935 + 0.39*0.25*0.2 = 11.30\%.$$

Similar to the project value, one can obtain the rate of return shortfall by

$$\delta_K = \mu_K - \mu_{Kr} = r + \lambda \rho_{KM} \sigma_K - \mu_{Kr}$$

Note that before the completion time, *T*, the physical asset is considered as unfinished and has an appreciation rate of μ_{Kr}. This cost schedule can be converted to arbitrary time t' prices given the cost inflation or escalation rate, r_c. One may also convert the estimated cost schedule in Table 8.8 to *t=7* prices and obtain $K_{t=7}$ = £4,288 million. Here μ_{Kr} can be estimated by solving:

$$K_{t=0} * e^{u_{Kr}*7} = K_{t=7} \tag{8.6}$$

and one can obtain $\mu_{Kr} = 0.036$. Note that it can be proved that μ_{Kr} is, in fact, equal to the inflation rate, r_c. As a result, the rate of shortfall of the project cost is given by

$$\delta_K = \mu_K - \mu_{Kr} = 0.077.$$

- Correlation between Project Value and Cost, ρ_{VK}

Because of the data limitation, it is assumed that the correlation between construction cost and project value is minimal, specifically, $\rho_{VK} = 0.08$.

8.2.4 Step Three: Construct a Reverse Binomial Pyramid for the Two-State-Variable Problems

Here a reverse binomial pyramid will be constructed. The jump probabilities are given as in Fig. 6.10 where $\rho = \rho_{VK} = 0.08$. The jump amplitudes are computed by equations (6.33) to (6.36). Supposing that the time steps are 300, the jump amplitudes for *V* and *K* are:

$$u_V = 1.04393,\ d_V = 0.95181$$

$$u_K = 1.02309,\ d_K = 0.97691$$

The values of the parameters from step one and two are summarized in Table 8.11.

Table 8. 11 Estimated Parameters for the Channel Tunnel Project

Items	**State Variables Assumptions/Estimations**
V_0 : Project Value	£ 7,145 million
σ_V : Volatility	0.215/year
μ_V : Market Required Return	0.1483/year
μ_{Vr} : Project Value Growth Rate	0.057/year
δ_V : Rate of Return Shortfall	0.0913/year
K_0 : Project Cost	£ 3,450 million
σ_K : Volatility	0.2/year
μ_K : Market Required Return	0.113/year
μ_{Kr} : Project Cost Growth Rate	0.036/year
δ_K : Rate of Return Shortfall	0.077/year
ρ_{VK} :	0.08
	Financial and Market Assumptions
Initial Equity Funding	£ 1,023 million
Risk-free Interest rate, r	9.35%/year ; Reference date: 1986
Market Rate of Return, r_m	0.199 /year; Reference period: 1975-1993
Market Volatility, σ_m	0.27/year
Fixed bank loan interest rate, r_d	10.26%/year; Reference date: 1986
Construction Cost Escalation Rate	3.6%/year

8.2.5 Step Four: Determine the Terminal and *Time t* Payoff Functions

Three sets of payoff functions for the profit structures shall be derived, including the payoff functions for the equity value, construction contracts value, and operating related contract value. Note again that the payoff functions for the project under the "rescue" equilibrium are identical to the payoff functions for the project under the government debt guarantee. The only difference that will occur during the implementation is that the loan interest rate, r_d, is equal to 10.26% for the project under the "rescue" equilibrium, and is equal to risk-free interest rate, r=9.35% for the project under the government debt guarantee.

- For Equity Value:

The *terminal* and *time t* payoff functions are given in equations (6.37) and (6.52), respectively. Note that according to the analysis performed in step one to three above, r_c and r_d in equation (6.52) are 3.6% and 10.26%, respectively, when there is no debt guarantee. With the debt guarantee, r_d will equal risk-free rate, r. Since the expected construction period is approximately 7 years, T will be equal to 7. For simplicity, if it is assumed that the discounted cost schedule is uniform, the $D_T(K_T)$ will be given by equation (6.40) and $D_t(K_t)$ will be given by equation (6.50).

- For Construction Contract Value:

In the Channel Tunnel project, three major construction contracts included the lump sum contract, the target cost contract, and the procurement based contract. The details of each type of contract will be given in next section. The payoff functions of each contract above shall be derived according to each contract's characteristics. The derivations are discussed in the followings.

1. Lump sum contract:

The payoff functions of the lump sum construction contracts can be formulated as shown in equations (8.7) and (8.8). Note that equation (8.7) is the terminal condition and is applied to the first run of backward computations. Equation (8.8) is the *time t* payoff function needed when there are default risks.

$$\alpha_L K_{L,0} e^{r_c T} + (K_{L,0} e^{r_c T} - K_{L,T}) \qquad \text{when } t=T \tag{8.7}$$

$$\begin{cases} \textit{Do not need payoff functions} & \textit{if} \quad V_t - D_t(K_t) e^{-r_d (T-t)} \geq 0 \\ [\alpha_L K_{L,0} e^{r_c t} + (K_{L,0} e^{r_c t} - K_{L,t})] R(t), & \textit{if} \quad V_t - D_t(K_t) e^{-r_d (T-t)} < 0 \end{cases} \tag{8.8}$$

Here α_L is the contractor's own profit markup ratio and $K_{L,0}$ represents the lump sum construction cost amount at time 0 prices. $R(t)$ is defined as the cost spending ratio up to time t, that is, the total cost spent up to t (at time t prices) divided by the *time t*'s estimated total cost. The formulation of $R(t)$ will be discussed in the next section. The first term of equation (8.7) is the normal profit of the lump sum contract, and the second term represents the profit due to the cost saving or the loss due to the cost overrun. The second row of equation (8.8) is similar to equation (8.7) except that the profit needs to be multiplied by the cost spending ratio, $R(t)$, due to the construction contract's early termination. Note that if the project is either under "rescue" equilibrium or government debt guarantee, only the terminal payoff function (8.7) is needed.

Note that in a lump sum contract, the contractor will take all cost overrun risks. In this case, the total cost, K_t, should exclude the effects of the cost overrun of a lump sum contract. As a result, the bankruptcy condition in equation (8.8) should be modified. The modification is necessary under the assumptions that the contractor and the developer are financially independent, and that the developer carefully prevents any claims resulted from the cost overrun. In the Channel Tunnel, since we assume that the contractor and the developer are closely related, we will not consider any modifications toward equation (8.8). In other words, we assume that in Channel Tunnel, from the bank's perspective, the cost overrun of a lump sum contract will also degrade the project's financial viability.

In general, the cost of each contract will be uncertain as well as the overall cost. As argued previously that it may not be advantageous and practical to consider every possible state variables. Instead, different types of contracts were grouped together as one state variable, K. Therefore, it is crucial to specify/assume certain relationship between each cost item such as $K_{L,t}$ and the total cost, K_t. For simplicity, it is assumed that the original proportion ratio, $\frac{K_{L,0}}{K_0}$, shall prevail. As a result, at any time t, $\frac{K_{L,t}}{K_t} = \frac{K_{L,0}}{K_0}$, that is,

$$K_{L,t} = \frac{K_{L,0}}{K_0} K_t \tag{8.9}$$

According to the Channel Tunnel's contract, the amount of lump sum contract is £1,169 million, that is, $K_{L,0}$ = £1,169 million. Also because it is impossible to obtain the information regarding the contractor's profit markup on the Channel Tunnel's lump sum contract, a ratio was assumed to perform the analysis. Suppose that the markup ratio, α_L, is equal to 12% of the total lump sum cost. As mentioned above, since it is assumed that the discounted cost schedule is uniform, one may further assume that the project's progress is uniform as well. As a result, the cost spending ratio can be approximated by

$$R(t) = \frac{t}{T} \tag{8.10}$$

Note that the cost spending ratio can be modified according to other assumptions regarding the cost schedule, $k(t)$. The value of $K_{L,t}$ will be determined at the end of this section along with the values of $K_{G,t}$ and $K_{P,t}$.

2. *Target cost contract*:

In the target cost contract, the risks are shared by the contractor and owner. The payoff functions are given in equations (8.11) and (8.12).

$$\alpha_G K_{G,0} e^{r_c T} + s_G (K_{G,0} e^{r_c T} - K_{G,T}) \qquad \text{when} \quad t=T \tag{8.11}$$

$$\begin{cases} \textit{Do not need payoff functions} & \textit{if} \quad V_t - D_t(K_t) e^{-r_d (T-t)} \geq 0 \\ [\alpha_G K_{G,0} e^{r_c t} + s_G (K_{G,0} e^{r_c t} - K_{G,t})] R(t), & \textit{if} \quad V_t - D_t(K_t) e^{-r_d (T-t)} < 0 \end{cases} \tag{8.12}$$

Here α_G is the fixed percentage for the fees and $K_{G,0}$ is the target construction cost amount at time 0 prices. Similar to the $K_{L,t}$, one may conclude that $K_{G,t} = \frac{K_{G,0}}{K_0} K_t$. The profit/loss sharing ratio, s_G, denotes the sharing scheme of cost saving or overrun. Equation (8.12) has interpretations similar to those of equation (8.8). Note that if the profit sharing ratio is different from the loss sharing ratio, or if there is an upper or lower limit on the cost sharing, the *second term* of equation (8.11) can be modified to (8.13).

$$\begin{cases} Min[L_s, s_s(K_{G,0}e^{r_c T} - K_{G,T})] & if\ K_{G,0}e^{r_c T} - K_{G,T} \geq 0 \\ Max[L_r, s_r(K_{G,0}e^{r_c T} - K_{G,T})] & if\ K_{G,0}e^{r_c T} - K_{G,T} < 0 \end{cases} \quad \text{when } t=T \tag{8.13}$$

where L_s and L_r are the upper limit of cost saving and overrun, respectively, and s_s and s_r are the profit sharing and loss sharing ratios, respectively. By the same token, the *cost sharing portion* in equation (8.12), $\alpha_G K_{G,0}e^{r_c t} + s_G(K_{G,0}e^{r_c t} - K_{G,t})$, can be modified to equation (8.14).

$$\begin{cases} Min[L_s, s_s(K_{G,0}e^{r_c t} - K_{G,t})] & if\ K_{G,0}e^{r_c t} - K_{G,t} \geq 0 \\ Max[L_r, s_r(K_{G,0}e^{r_c t} - K_{G,t})] & if\ K_{G,0}e^{r_c t} - K_{G,t} < 0 \end{cases} \tag{8.14}$$

According the Table 8.3, the amount of target cost contract is £1,367 million, that is, $K_{G,0}$ = £1,367 million. Since the project's contract specified that the fee was 12.36% of the target cost, α_G = 12.36% shall be assigned. However, since the profit sharing ratio is different from loss sharing ratio, and the cost overrun sharing has an upper limit, one needs to modify equations (8.11) and (8.12) to equations (8.13) and (8.14), respectively. According to the project's contract, the *cost saving* sharing ratio will be s_s=0.5 and the *cost overrun* sharing ratio will be s_r=0.3. Since the upper limit of cost overrun sharing was 6% of total target cost, therefore, $L_r = -0.06 * K_{G,0} * e^{r_c t}$. Also because there is no limit on cost saving sharing, one may assign $L_s = \infty$, that is, the $Min[\bullet]$ terms in equations (8.13) and (8.14) will become $s_s(K_{G,0}e^{r_c T} - K_{G,T})$ and $s_s(K_{G,0}e^{r_c t} - K_{G,t})$, respectively.

3. *Procurement contract*:

The procurement contract is similar to the target cost contract, except that the procurement contract does not have cost sharing schemes. The payoff functions are shown in equations (8.15) and (8.16).

$$\alpha_P K_{P,T} \qquad \text{when } t=T \tag{8.15}$$

$$\begin{cases} \textit{Do not need payoff functions}, & \text{if } V_t - D_t(K_t)e^{-r_d(T-t)} \geq 0 \\ (\alpha_P K_{P,t})R(t), & \text{if } V_t - D_t(K_t)e^{-r_d(T-t)} < 0 \end{cases} \tag{8.16}$$

where α_P is the fixed percentage for the fees and $K_{P,0}$ is the procurement contract cost at time 0 prices. Also, similar to equation (8.9), $K_{P,t} = \frac{K_{P,0}}{K_0} K_t$.

Note that equations (8.7) to (8.16) are by no means the only way to model the corresponding contract's payoff functions. The payoff functions can be modified to another form according to different clauses/agreements in the contract. Equations (8.7) to (8.16) are formulated by specifically considering the Channel Tunnel's construction contracts. According to Table 8.3, $K_{P,0}$= £252 million. The project's contract specified that TML will be reimbursed for the cost plus a fee of 11.5% of the procurement cost. Therefore, α_P= 11.5%.

By the definition of construction cost in step one, $K_{t=0}$ = 2,788+642 = £3,450 million. Therefore, $K_0 = K_{L,0} + K_{G,0} + K_{P,0} + 642 =$ £3,450 million. In the lump sum contract, according to previous assumption that the ratio $\frac{K_{L,t}}{K_t}$ will remain constant such that $\frac{K_{L,t}}{K_t} = \frac{K_{L,0}}{K_0}$, $K_{L,t} = \frac{K_{L,0}}{K_0} K_t = \frac{1{,}169}{3{,}450} K_t = 0.339 K_t$ is obtained. Similarly, in the target cost contract, $K_{G,t} = \frac{1{,}367}{3{,}450} K_t = 0.396 K_t$. In the procurement item contract, we have

$K_{P,t} = \frac{252}{3{,}450} K_t = 0.073 K_t$. These ratios or relationships will be used in the payoff functions for the computation of the time t cost of each contract.

- For Operating Related Contract Value:

Since it is assumed that there is no operating related contract in the Channel Tunnel project, there will be no need to derive the payoff functions for the operating related contract.

8.2.6 Step Five: Plug the Payoff Functions into the Reverse Binomial Pyramid and Compute the Equity Value or Construction Contract Value

In this case study, the time steps used were 300. Several rounds of test computations show that the difference between the results of using 200 time steps and 300 time steps is greater than the difference between 300 time steps and 500 time steps. Also, the difference between the results of using 300 steps and 500 steps is of no significance. Therefore, 300 time steps is applied in the binomial model. The equity value and construction contract value can be obtained in this step. In the following sections, further analyses will be conducted based on the implementations of the Channel Tunnel's five-step profit structures valuation discussed in section 8.2.

8.3 Equity Value: the First Element in the Profit Structures

The equity values both under the "rescue" and "no rescue" equilibria shall be computed in this section. Also, the equity value under the government debt guarantee will be computed to evaluate the value of the debt guarantee. Furthermore, the equity values will be evaluated with respect to various parameters such as the initial values and volatilities of the state variables. Since the initial equity investment in the Channel Tunnel project is £1,023 million, the financial viability of the project from the perspective of shareholders will depend on whether the equity value is greater than or equal to the equity investment, £1,023 million.

8.3.1 Effects of Initial Project Value and Cost on the Equity Value

The equity values under the default risk and government debt guarantee are £2,325 million and £2,641 million, respectively. The value of the debt guarantee can be obtained by the difference, £2,641-2,325 = £316 million. In other words, the debt guarantee is equivalent to a £316 million subsidy to the shareholders. The result of the five-step valuation model showed that the Channel Tunnel project is financially viable from the shareholder's perspective. The equity value, £2,325 million, is approximately 2.3 times of the original equity investment, £1,023 million.

However, the major concern of the *passive* shareholders and government tendering agent is the possibility and impact of the overestimation of project value, V_0, and the underestimation of project cost, K_0. It is critical to understand as to what degree of the over- or under-estimation on V_0 and K_0 the project is still financially viable. Figures 8.1 and 8.2 show the equity values with respect to the adjusted V_0 and K_0, respectively. Since the Channel Tunnel project does not involve any guarantee, the valuation of equity valuation will be under the default risk. Figures 8.1 and 8.2 show that the project will be financially unviable if the adjusted V_0 falls below 55% of projected V_0 or the adjusted K_0 grows beyond 180% of project K_0.

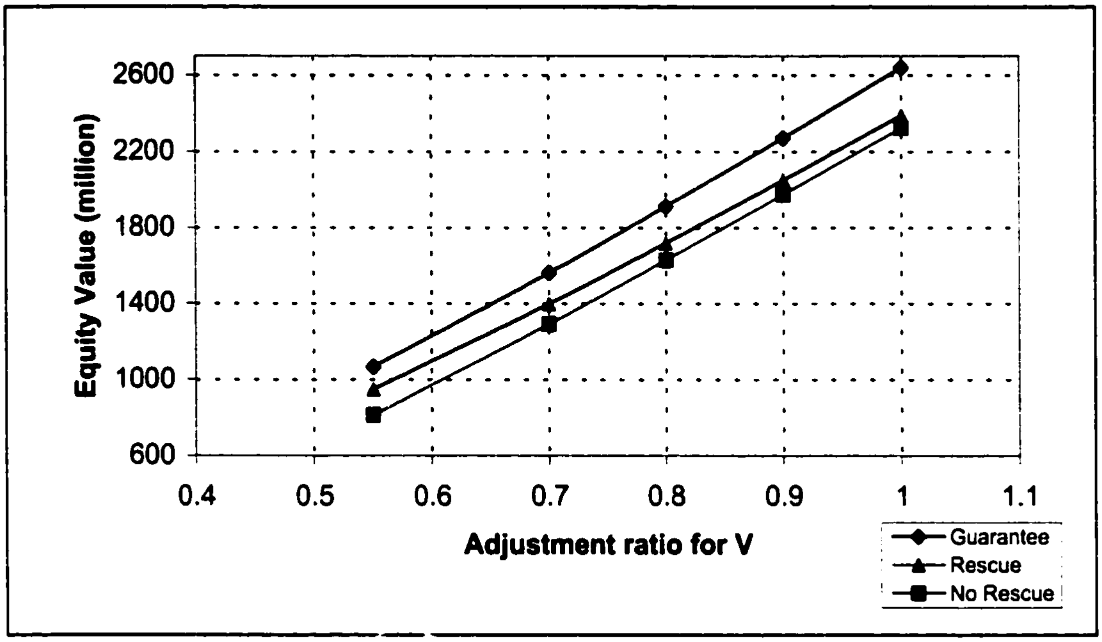

Fig. 8. 1 Equity Values w.r.t. Different Adjustments of V_0

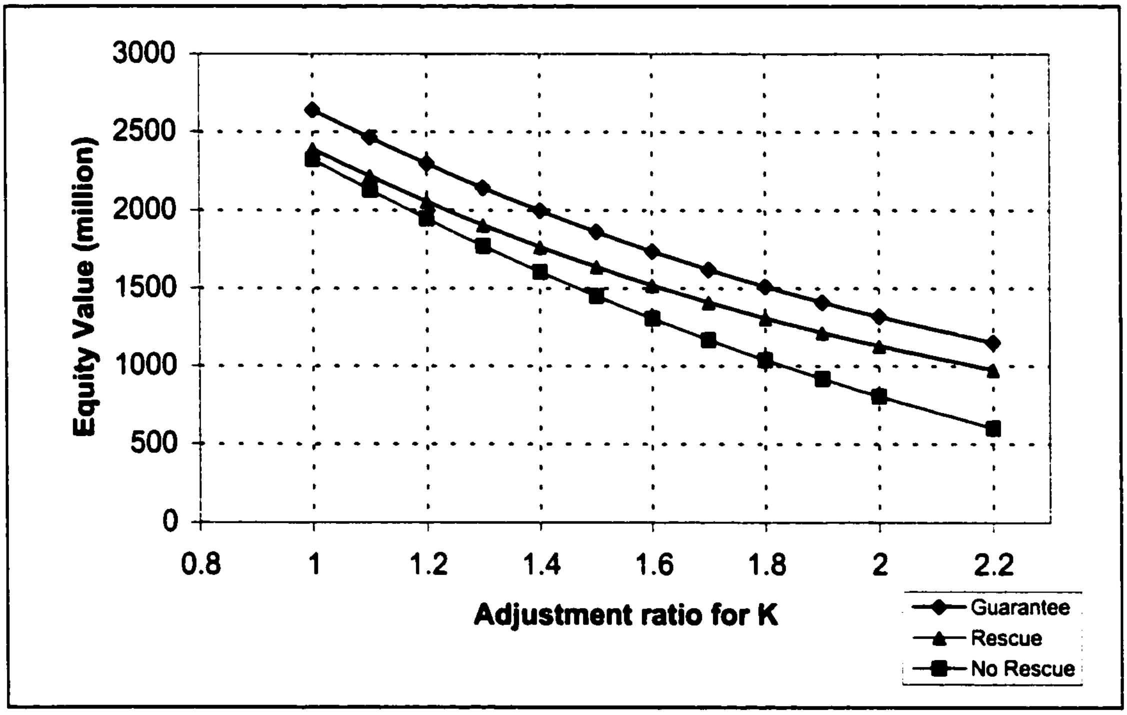

Fig. 8. 2 Equity Values w.r.t. Different Adjustments of K_0

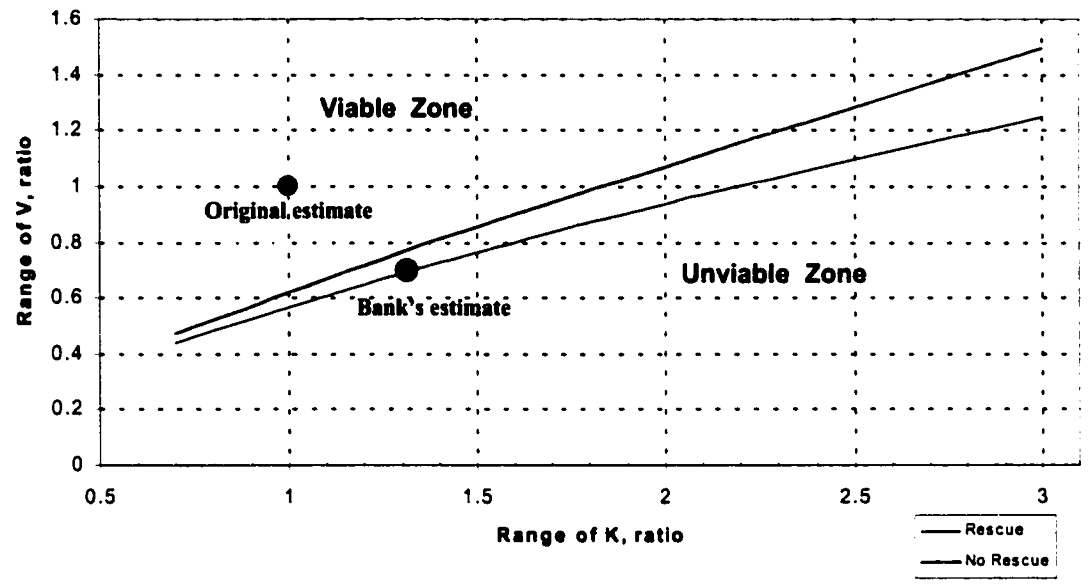

Fig. 8. 3 Project Viability Profile

Figure 8.1 is obtained by fixing the initial cost ratio to 1.0, and we may find the critical points that the project is financially viable, that is, equity value equals £1,023 million. If we vary the initial cost by ratios from 0.7 to 3.0 and find a critical point for each initial cost estimate, we may obtain a financial viability portfolio as shown in Fig. 8.3. The profile is divided by a "critical line," which is formed by a set of critical points. The two critical lines in Fig. 8.3 represent two different game solutions, the "no rescue" and "rescue" solutions. Projects that fall on the left-hand side of the critical line are considered financially viable, and those fall on the right-hand side of the critical line are considered financially unviable. This type of "viability profile" can help the project participants evaluate a project's financial viability when there are different opinions or estimates regarding the initial estimates. For example, suppose that lending bank believes that the developer's original estimate in Fig. 8.3 is too optimistic and that the project cost estimate should be 30% higher and the project value estimate should be 30% lower, the bank will obtain another point shown in Fig. 8.3. According to the profile, the bank's estimate shows that the project is financially unviable. In some cases, different estimates from different participants may fall in the same viability zone even when the participants have different opinions toward the estimates. Moreover, the viability profile can help the participants examine how sensitive the viability is with respect to initial estimates. The sensitivity analyses are very valuable in the project evaluation process.

8.3.2 Effects of Volatilities of V and K on the Equity Value

The equity values with respect to the state variable's volatility or uncertainty can be examined. If the volatilities of V and K vary while all other parameters remain the same, one may obtain the equity values regarding various volatilities in Fig. 8.4 and 8.5. Surprisingly, Fig. 8.4 and 8.5 show that higher volatilities of V or K will yield higher equity values, and that the impacts of the volatility are less obvious when the project is under default risk. If the Channel Tunnel's cost volatility becomes 0.5 instead of 0.2, the equity value will increase by £193 million and become £2,518 million. The major reason is that the equity's asymmetric payoff due to the equity's limited liability. Higher volatility increases the shareholder's maximum payoff but not the maximum loss. The maximum loss of the shareholders is always the equity investment amount, I = £1,023 million.

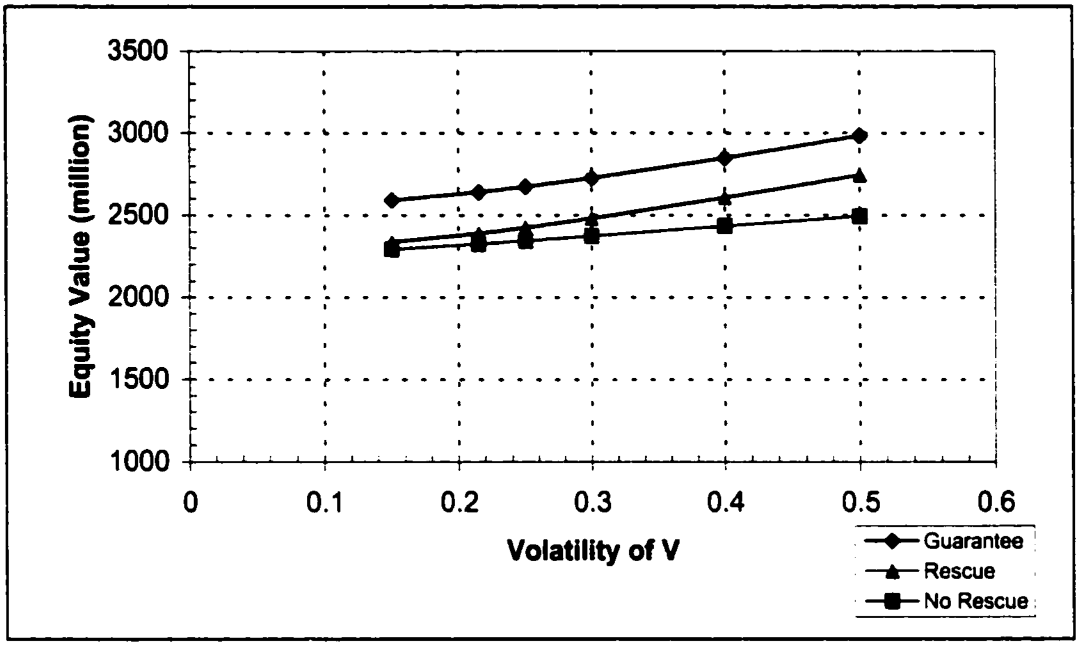

Fig. 8. 4 Equity Values w.r.t. Different Volatilities of V

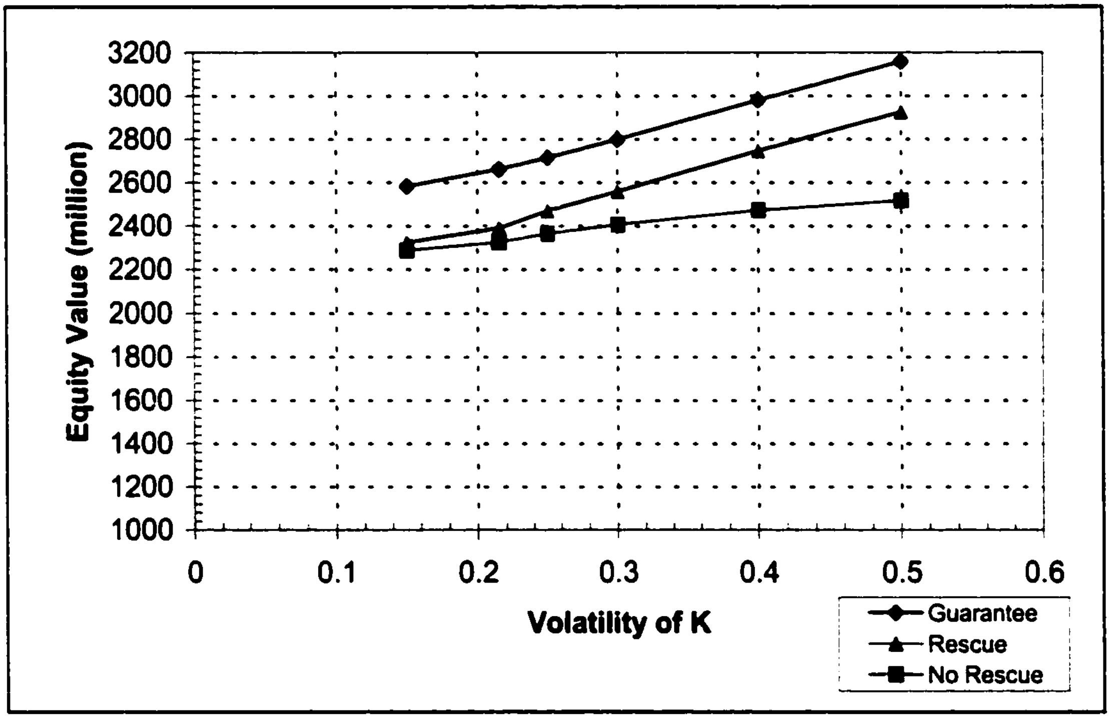

Fig. 8. 5 Equity Values w.r.t. Different Volatilities of K

8.3.3 Effects of Various Equity Ratios on the Equity Value

Different equity ratio requires different amount of equity investment, I. In classic financial theories, the optimal equity ratio is determined from the perspective of the shareholders. It is assumed that the shareholder's objective is to maximize the net profit from the equity investment instead of the return rate. This assumption is consistent with the economic theory's utility maximization (Copeland and Weston, 1988). In other words, the optimal equity ratio should yield highest equity investment profit, $E - I$, where E is the equity value. However, as argued in Chapter 1, equity investment profit is not an appropriate evaluation criterion for the developer. The developer should evaluate the profit structures in the BOT investment's decision making. Figure 8.6 shows the equity investment profits regarding different equity ratios ranging from 5% to 35%. Note that here it is assumed that the practically possible equity ratios that can be arranged are within 5% to 35%. Moreover, each equity ratio's corresponding loan interest rate will be given exogenously for the computation of equity investment profit. Table 8.12 shows each ratio's corresponding loan interest rate assumed in the analysis. Given these assumptions, in the Channel Tunnel project, the smallest equity investment profit equals £790 million when the equity ratio is 5%, and the maximum profit equals £1,330 million approximately when the equity ratio is close to 20%. Note that the Channel Tunnel project's actual equity ratio, 17%, is quite close to the 20% obtained here.

The interest rate in Table 8.12 does not apply in the case that the project is under debt guarantee. As argued above, under debt guarantee, the interest rate should be equal to risk-free interest, 9.35%. An important finding is that if the Channel Tunnel is under debt guarantee, the equity investment profit is maximized when the equity ratio is 5%, and the profit decreases as the equity ratio increases. The major reason is that the greater the default risk, the higher the value of the debt guarantee.

Table 8. 12 Different Equity Ratio's Corresponding Loan Interest Rate

Equity Ratio	Equity Investment Amount	Assumed Loan Interest Rate
5%	£300 million	11.80%
10%	£600 million	10.96%
17%*	£1,023 million	10.26%
25%	£1,500 million	9.76%
35%	£2,100 million	9.35%

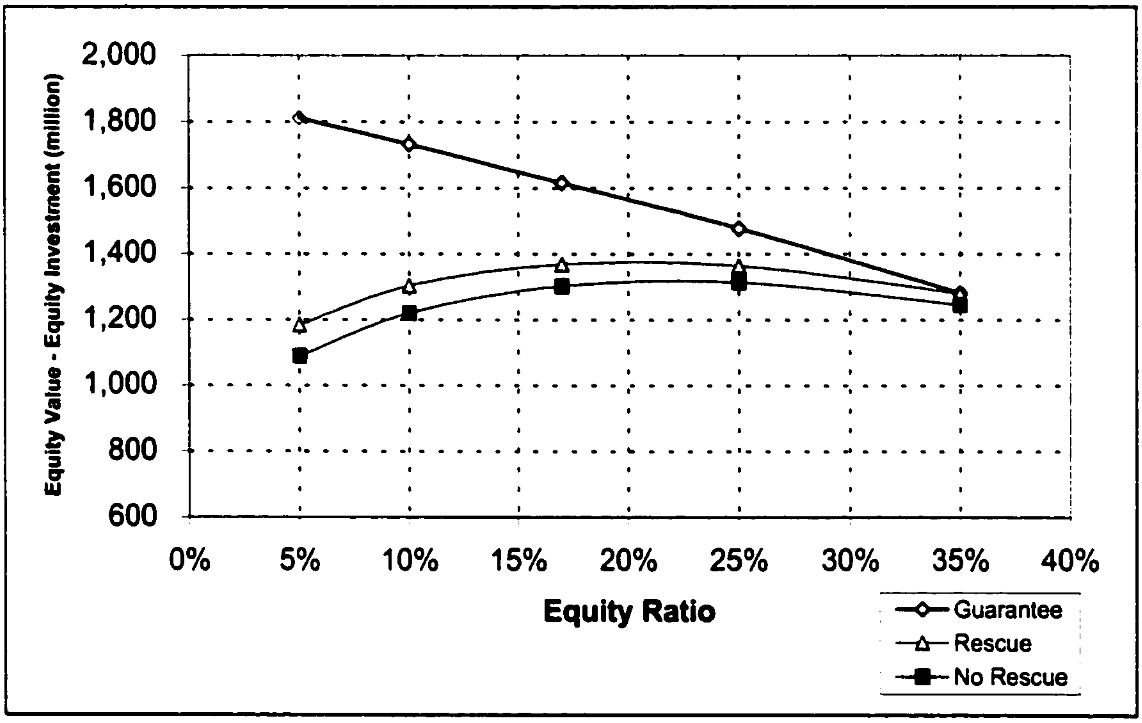

Fig. 8. 6 Equity Investment Profits w.r.t. Different Equity Ratios

8.4 The Construction Contract Value: the Second Element in the Profit Structures

This section solves for the values of the Channel Tunnel's construction contracts with respect to each type of contract. The contractor may also concern how the value of different type of contract relates to the project's characteristics. In the traditional evaluation method, it is very difficult to understand the complexity the dynamics of the construction contract value. This section will demonstrate how the five-step profit structures valuation model can be used to analyze the Channel Tunnel's construction contracts.

8.4.1 Effects of Volatilities of V and K on the Lump Sum Construction Contract Values

The Channel Tunnel's lump sum contract cost amount was £1,169 million. According to the payoff functions derived previously, one may obtain the value of the Channel Tunnel's lump sum contract, £198 million. Also note that when there's a debt guarantee or the project is under the "rescue" equilibrium, the contract's value will be equal to £193 million. In fact, as shown in Fig. 8.7, if there's a debt guarantee, the value of the contract will be independent from the project value's volatility, σ_V. However, if the project is under default risk, higher project value volatility will reduce the lump sum contract value. The major reason is that higher project value volatility will increase the chances of default, and hence increase the probability of early contract termination. The early contract termination caused by low project value will reduce the contract's profitability, since the construction contract could be forced into termination even when the project cost is within budget.

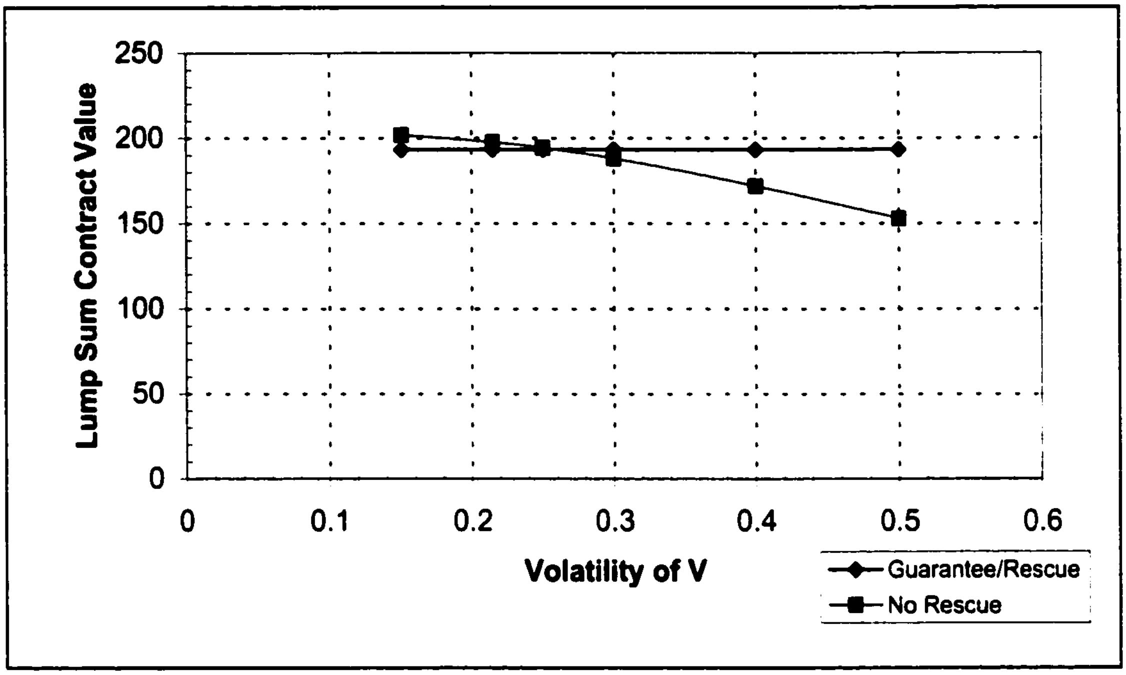

Fig. 8. 7 Lump Sum Construction Contract Values w.r.t. Different Volatilities of V

Surprisingly, higher project cost volatility will increase the contract value as shown in Fig. 8.8. The major reason is that the cost volatility will create asymmetric payoff in the lump sum contract. When there is cost saving, the contract will continue until the completion date, and the profit will be realized. However, when there is a serious cost overrun, the contract will be terminated early along with the project; as a result, part of the loss will not be realized.

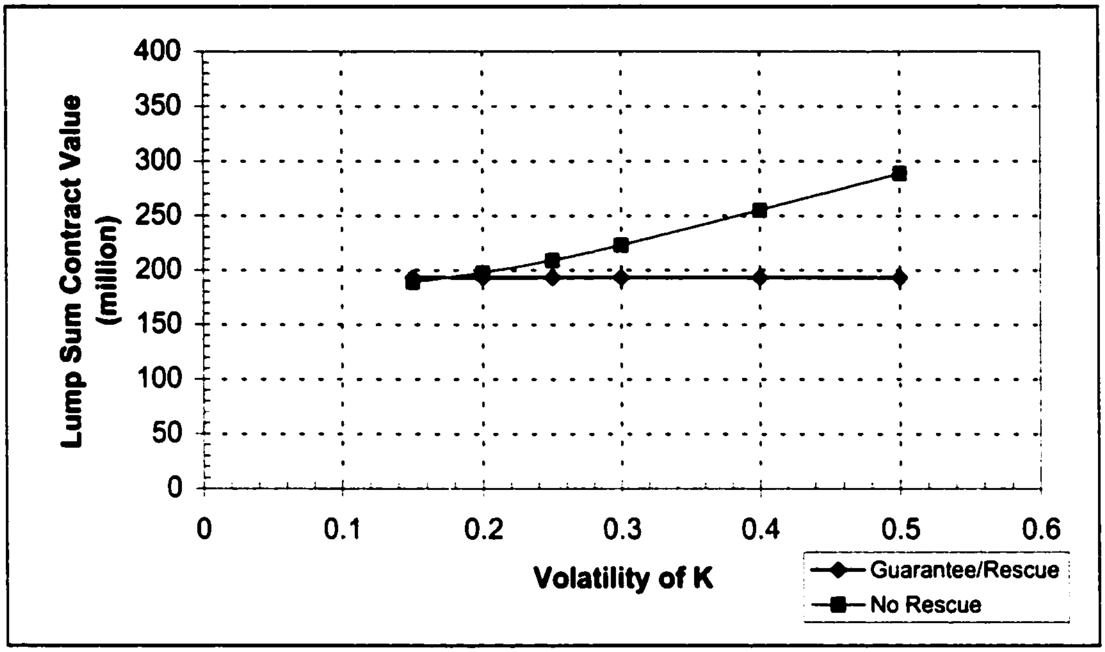

Fig. 8. 8 Lump Sum Construction Contract Values w.r.t. Different Volatilities of K

8.4.2 Effects of Equity Ratios on the Target Cost Construction Contract Values

The Channel Tunnel's target cost contract amount was £1,367 million, and the value of the target cost contract, £203 million, can be obtained. When the project is under the "rescue" equilibrium or debt guarantee, the contract value is £234 million. Figure 8.9 indicates how the equity ratio affects the value of the target cost contract value. The loan interest rates assumed in the analysis are listed in Table 8.12. The results in Fig. 8.9 show that higher equity ratio will yield higher contract value. Note that once the equity ratio passes beyond 17%, the Channel Tunnel's equity ratio, the contract value becomes less sensitive to the equity ratio.

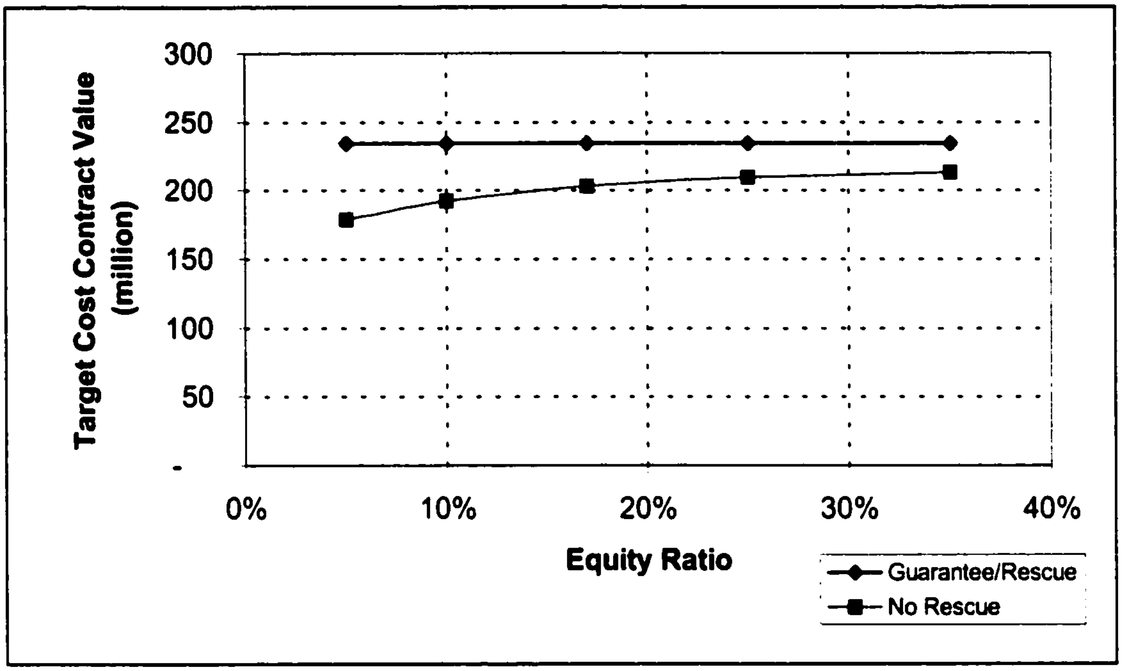

Fig. 8. 9 Target Cost Construction Contract Value w.r.t. Different Equity Ratios

8.4.3 Effects of Cost Volatilities on the Procurement Construction Contract Values

The values of the Channel Tunnel's procurement based contract may be obtained: £15 million under "no rescue" equilibrium, and £17 million under the "rescue" equilibrium or debt guarantee. Figure 8.10 shows that higher cost volatility will reduce the value of the procurement contract.

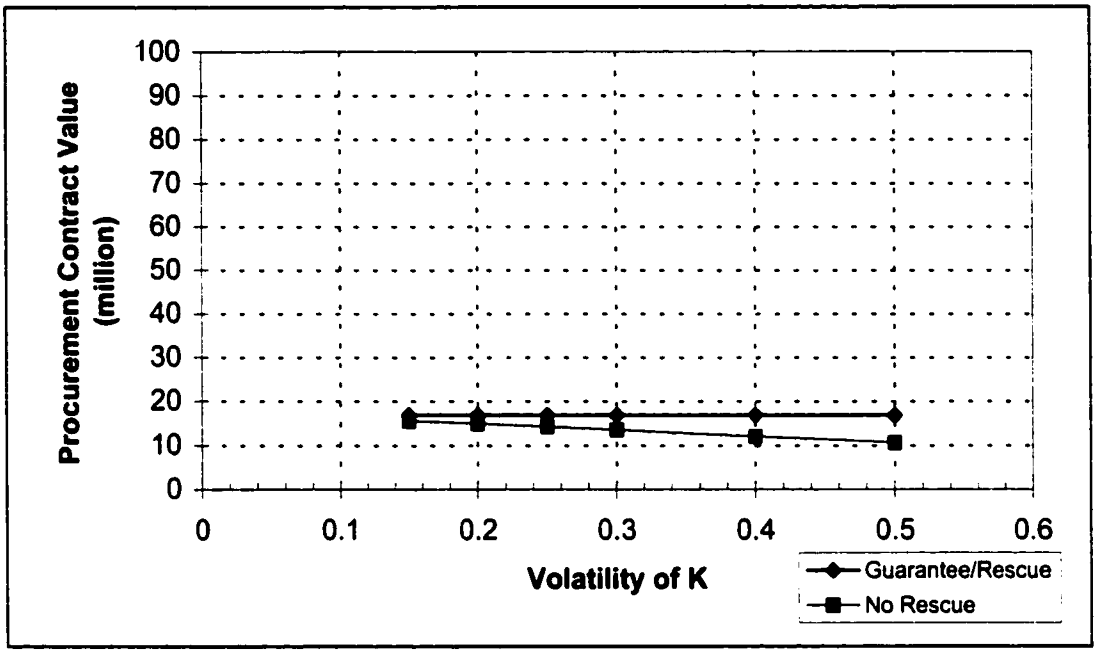

Fig. 8. 10 Procurement Contract Values w.r.t. Different Volatilities of K

8.4.4 Effects of Equity Ratios on the Overall Construction Contract Values

The Channel Tunnel project consists of the lump sum contract, the target cost contract and the procurement contract, and their values are £198 million, 203 million and 15 million, respectively, under the "no rescue" equilibrium. The overall value of the construction contract is £416 million, approximately 15% of the estimated cost, £2,788 million. When the project is under the "rescue" equilibrium or debt guarantee, the overall contract value is £444 million, approximately 16.1% of the estimated cost. In the financing decision making, it is important to know the overall value of the contract with respect to different equity ratios as shown in Fig. 8.11. Later the overall construction contract value will be combined with the equity value to examine the optimal financing decisions.

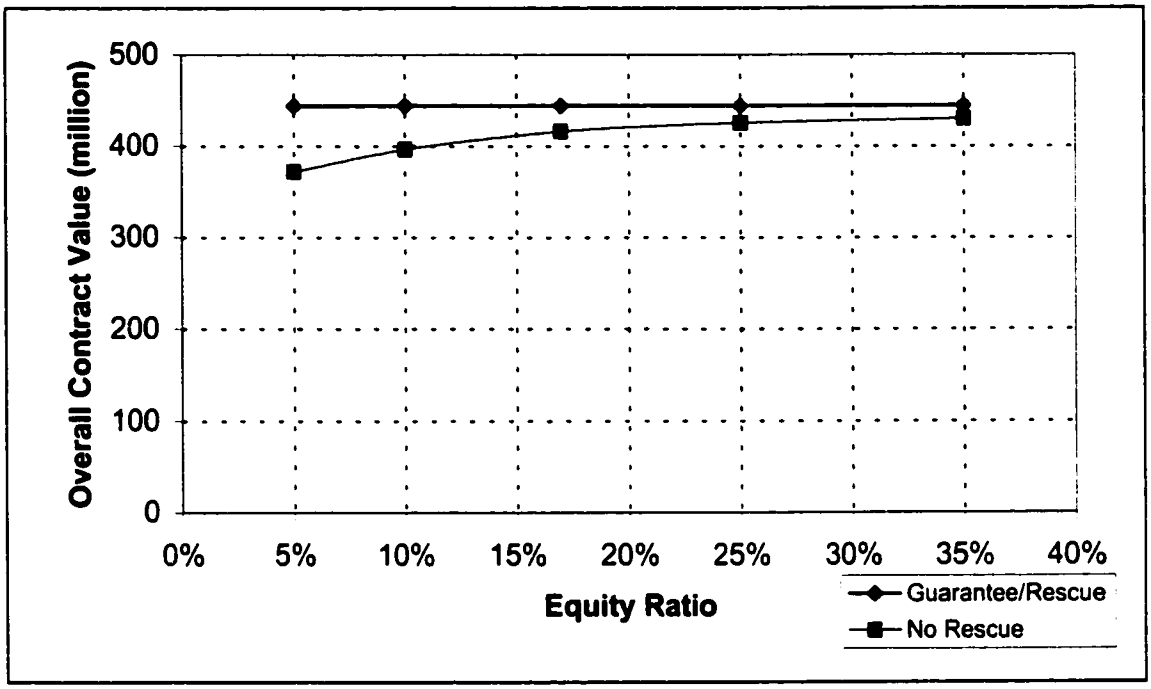

Fig. 8. 11 Overall Contract Values w.r.t. Different Equity Ratios

8.5 Project Financing Analysis

8.5.1 Channel Tunnel Project's Profit Structures

From sections 8.3 and 8.4, the equity value and overall construction contract value in the Channel Tunnel project have been obtained. It has been shown how these values vary with different equity ratios. As argued in Chapter 4, the developer should make decisions according to the project's profit structure instead of the equity value alone. The profit structure contains the equity value, the construction contract value, and the operating related contract value. However, in the Channel Tunnel project it is not clear in which operating related contracts that the developer has participated; thus, the operating related contracts will be ignored. As a result, the third component of the profit structure is equal to zero. Note that the discussion in Chapter 4 regarding the profit structure should not be restricted to "only" or "exact" three components in the profit structures. The profit structures may change due to different concession agreements or project characteristics.

8.5.2 Channel Tunnel Project's FNPV and the Optimal Equity Ratio

To analyze Eurotunnel's financing decision, some further assumptions will be made. These assumptions are critical to the analysis. However, the assumptions made here are subject to modification. One of the most important assumptions is that the developer includes the Eurotunnel and the contractor, TML, although they were separated by the requirement from the governments. The reason for this assumption was discussed in section 8.1. Under this assumption, the objectives of the Channel Tunnel developer is to maximize the project's full net present value, FNPV, given in equation (6.58). Since it is assumed that the developer did not participate any operating related contracts, the FNPV would become

$$FNPV = E + C - I \tag{8.17}$$

It is also assumed that the Channel Tunnel project is under the "no rescue" equilibrium, since the governments had clearly indicated that it would not provide any subsidies should things go wrong. Therefore, the developer's financing decision making shall be analyzed based on the "no rescue" equilibrium. However, for analyzing the government's BOT policies, the project's FNPVs under both the "rescue" equilibrium and debt guarantee shall be computed as well.

Since the developer invested only £253 million in the equity, from the developer's perspective, I should be equal to £253 million, instead of £1,023 million, given the project's total equity investment amount, £1,023. The developer's equity investment ratio is equal to £253/£1,023 = 24.73%. The amount of developer's equity investment amount will change with respect to different total equity investment or equity ratio. The question is how to determine the developer's equity investment amount given a certain equity ratio. This question will be answered by the end of this section.

In most cases, the developer's equity investment ratio is different from the *developer's equity owning ratio*. The developer's equity owning ratio, ψ_E, is defined as the percentage of the developer's equity ownership. For example, a developer may invest 20% of total equity investment, and be awarded by 50% of total shares of equity due to the developer's taking the initial investment and promoting risks. When the equity has higher value, the same amount of initial/developer's equity investment will become larger equity ownership or owning ratio. Equivalently, when the equity has higher value, to acquire the same percentage of ownership will require lower initial/developer's equity investment amount or ratio. In the Channel Tunnel project, according to the *Offer for Sale*, at the time of public placement, November 1987, the total shares on equity after dividing are 112,366,780 for EPLC (Eurotunnel, the U.K. side) and 112,366,780 for ESA (Eurotunnel, the French side). The total shares from public placement are 220,000,000. There are equal amount of shares of warrants combined with the stocks in the public placement as well. Ten warrants can be exercised in buying one share of stock at the price of £2.3 per share of stock. For simplicity, we will ignore the warrants' impacts on the equity value. According the figures above, we may obtain the developer's *equity owning ratio:*

$$(112{,}366{,}780)*2/[112{,}366{,}780*2+220{,}000{,}000]=50.53\%.$$

Here the developer's 24.73% of equity investment ratio was awarded by 50.53% equity ownership.

Suppose that the developer needs to maintain at least 50.53% of total ownership in order to take control of the BOT firm. This section shall examine that, in order to keep 50.53% ownership, how much initial equity investment is required with respect to different total equity ratio. First, when the total equity investment is £1,023 million, the equity value is £2,325 million as shown in Fig. 8.1. Second, in the public placement, the £770 million equity investment from passive shareholders obtains (100-50.53)% = 49.47% of total ownership. The 49.47% of total equity has the value of £2,023*0.4947 = £1,150 million.

Therefore, in the Channel Tunnel project it is assumed that the public placement's market price is obtained by adjusting the equity value by a margin, η = £1,150/£770 = 1.5, and this margin will be assumed to prevail for all possible equity ratios. It is worth noting that the margin, η, will inevitably discourage the use of high equity ratio since higher equity ratio will have higher total equity value, and higher total equity value will cause greater loss of the developer due to the sale price margin. Third, once the adjusting margin, η, has been obtained given any equity ratio, *e*, one may obtain the sale price of the 49.47% of total equity: $0.4947\,E_A(e)/\eta$, where E_A is the total equity value. Fourth, the required initial equity investment will be equal to:

$$I = I_A(e) - (1-\psi_E)E_A(e)/\eta \tag{8.18}$$

where I_A is the project's total equity investment and is a function of equity ratio. Table 8.13 shows the required initial equity investment with respect to different equity ratios. Note that in the second row of Table 8.13, the required initial equity investment is -£158 million when the equity ratio is 5%. This means that when equity ratio is 5%, the sale price of the 49.47% equity will be greater than the total equity investment amount, £300 million by £158 million. In this case, if the developer sells the 49.47% of equity for £458 million, then the equity ratio would be greater than 5%. If the developer sells the equity for only £300 million, the developer will be giving up the £158 million. Therefore, the only reasonable adjustment is to find an appropriate equity ratio so that the required initial equity is equal to £0. This adjustment involves trial and error. Table 8.14 shows the adjusted initial equity investment when there is a *negative* initial equity investment amount.

Table 8. 13 Developer's Initial Equity Investment

Equity Ratio	Developer's Initial Equity (£ million)	Developer's Equity Investment Ratio
5%	-158	0%
10%	0.24	0%
17%	253	24.84%
25%	572	38.15%
35%	997	47.46%

Table 8. 14 Adjusted Initial Equity Investment

Equity Ratio	Developer's Initial Equity (£ million)	Developer's Equity Investment Ratio
<10%	N/A	N/A
10%	0	0%
17%	253	24.84%
25%	572	38.15%
35%	997	47.46%

According to these assumptions, the BOT project's evaluation criteria may be rewritten as

$$FNPV = \psi_E E_A + \psi_C C_A + \psi_O O_A - I \qquad (8.19)$$

Here ψ_C and ψ_O are the developer's participating ratios on the overall construction contract, C_A, and operating related contract, O_A, respectively, and I is given in equation (8.18). In the Channel Tunnel project, $\psi_E = 0.5053$, $\psi_C = 1$, $\psi_O = 0$. As a result, the Channel Tunnel's FNPV can be given as

$$FNPV = 0.5053 E_A - I + C_A \qquad (8.20)$$

The overall construction contract values, C_A, with respect to different equity ratios have been analyzed and are shown in Fig. 8.11. The computation of the equity value, E_A, has been shown in section 8.3. Combining the results from these analyses, the BOT investment's FNPVs with respect to different equity ratios can be obtained as shown in Fig. 8.12. Note that each curve's smallest equity ratio in Fig. 8.12 is obtained by adjusting the negative initial equity investment as shown in Table 8.14.

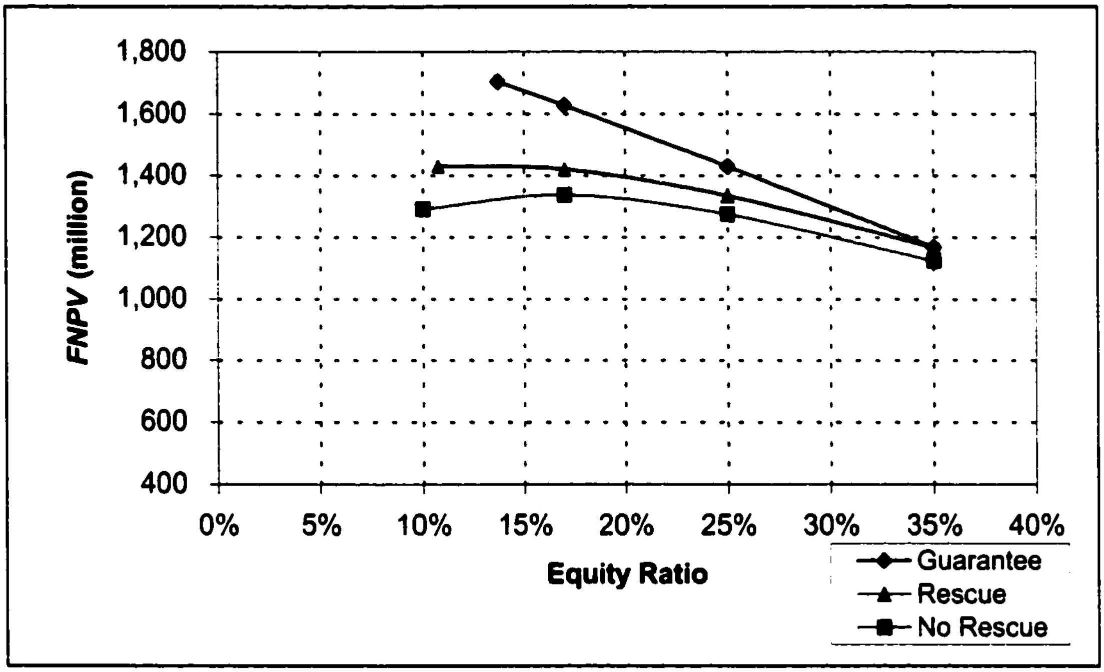

Fig. 8. 12 FNPVs w.r.t. Different Equity Ratios

According to Fig. 8.12, it is concluded that from the developer's perspective the project's theoretically optimal equity ratio is very close to 17%, the Channel Tunnel's actual equity ratio. Under the actual equity ratio, 17%, the Channel Tunnel's FNPV is £1,337 million, which includes the contract value, £416 million, and the profit from equity investment, £921 million. It is worth noting that if there is a government debt guarantee, the optimal financing arrangement is to raise the minimal 13.7% equity ratio entirely from by public placement while the developer can still have the 50.53% ownership. This arrangement would yield £1,705 million FNPV. Compared with the "no rescue" equilibrium's FNPV, the value of the debt guarantee is approximately £468 million. If the developer assumes that the project is under the "rescue" equilibrium, the FNPV at 17% equity ratio will increase by £84 million.

8.5.3 Financing Decision of a Project That is not Financially Viable – Under Symmetric Information

In the game theoretic model, it is critical to understand not only the financing decisions of a financially sound project, but also a project that is not financially viable. In the following analysis, it is assumed that the Channel Tunnel's revenue and cost figures are not as optimistic as those shown in *Offer for Sale.* Eurotunnel's projections are adjusted by reducing the revenue by 30% and increase the cost by 30%. The project after the adjustments is referred to as the "adjusted Channel Tunnel project" or "adjusted project." This section will examine the project financial viability from the shareholder's perspective and compute the value of total equity minus the total equity investment. Figure 8.13 shows the total equity investment profit of the adjusted project. In the "no rescue" equilibrium, one may find that the total equity investment profit is negative for all equity ratios. This means that the adjusted project is not financially viable. It will become viable to the shareholders only when the government provides the debt guarantee or the project is under the "rescue" equilibrium, and the equity ratio is low.

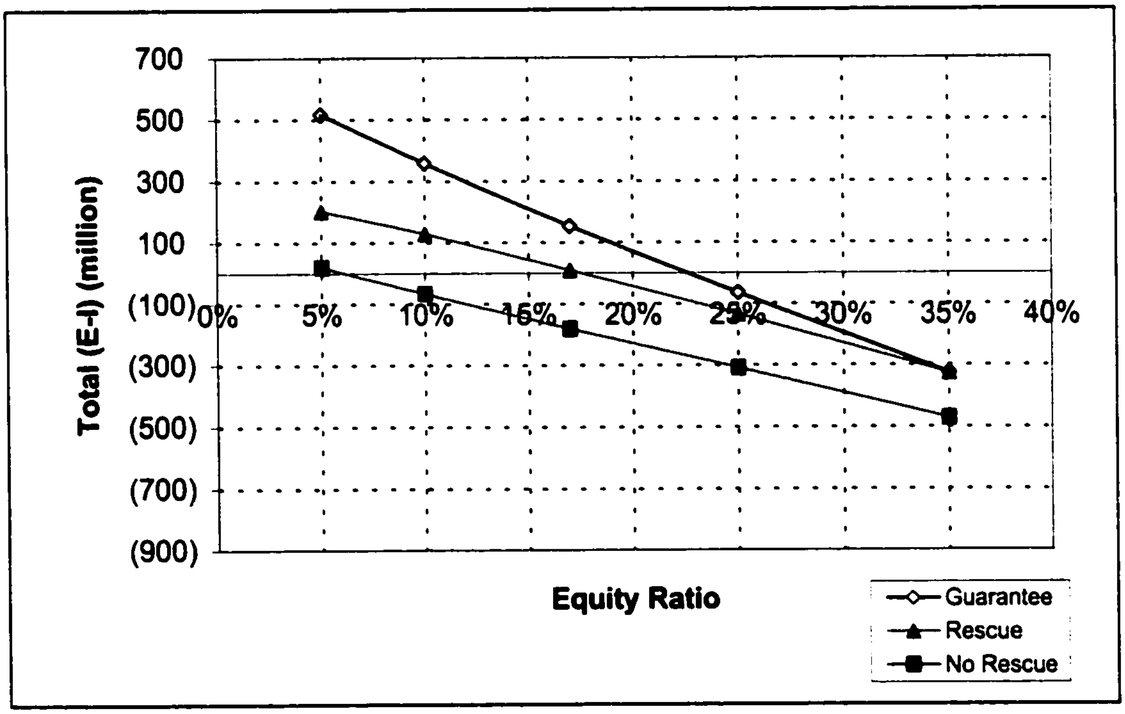

Fig. 8. 13 Total Equity Investment Profit w.r.t. Different Equity Ratios

As argued previously, the developer's project selection criteria are different from the shareholder's criteria. The developer will consider the project's FNPV. The Channel Tunnel project's FNPV is given by equation (8.20). Fig. 8.14 shows the adjusted project's FNPV's with respect to different equity ratios. According to Fig. 8.14, although the project is not viable from the perspective of shareholders, the BOT investment may be profitable to the developer by including the construction contract value and equity investment profit in the profit structures. Fig. 8.14 shows that for the adjusted Channel Tunnel project, lower equity ratio will yield higher FNPV.

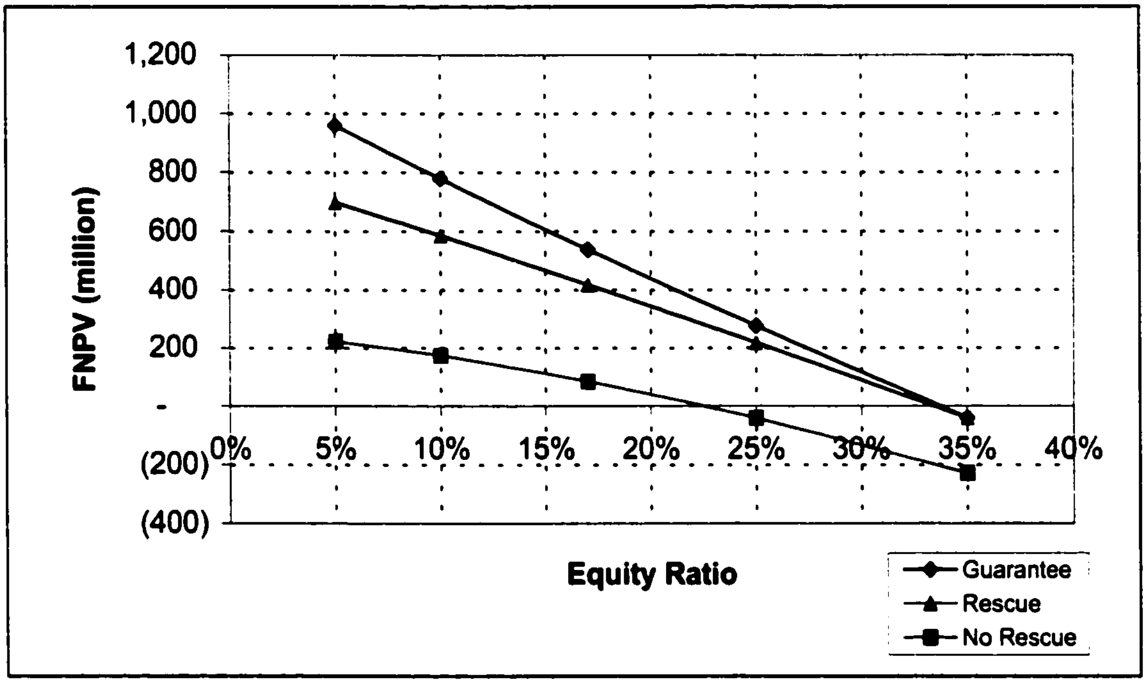

Fig. 8. 14 FNPV of the Unviable Project

8.6 Project Procurement Policies and Bidding Strategies—Under Asymmetric Information

This section assumes that the information is asymmetric; that is, the project developer has the information that the government or passive shareholders do not know. Particularly, the government or passive shareholders do not know whether the developer's released/proposed revenue and construction cost projections are the developer's "most likely" estimate or not. It is possible that the developer adopts the "optimistic" estimate in the proposal in order to earn higher evaluation points and increase the probability of being awarded the BOT project. This section examines whether effective government policies or developer signals exist.

8.6.1 FNPV and Government Policies When the Equity Owning Ratio Equals 50.53%

8.6.1.1 When the Project is Viable

Again, if the *Offer for Sale*'s projected revenue and costs are the most likely estimates, the project is considered financially viable. In this case, Fig. 8.12 shows that the FNPV will be in the range of £1,100 to £1,400 million. The highest equity ratio, 35%, will yield £1,124 million FNPV. An increase of 1% of equity ratio will reduce FNPV by approximately £11.8 million.

8.6.1.2 When the Project is Not Viable

If the *Offer for Sale*'s projections are the *most optimistic* estimates, it is considered that the project is not viable, as noted in section 8.5.3. For the purpose of obtaining FNPV, the project's equity value will be computed according the *adjusted* estimation. The adjustment is to reduce the projected revenue by 30% and increase costs by 30%. The adjusted projections are considered as the *most likely* estimations. The project's equity value and the project's construction contract value will depend on both the most likely and optimistic estimates. In computing the developer's equity investment amount, I, in equation (8.18), the computation will be based on the proposed or optimistic projections because the equity value in equation (8.18), E_A, is based on the capital market's perception if the market believes the proposed information. In computing contract value, the proposed cost amount should be used only in $K_{L,0}$ and $K_{G,0}$ since the proposed cost is relevant in the lump sum and target cost contract. The *most likely* revenue and cost estimations will be used in the dynamics of the state variables, since these values are the actual sources of uncertainty.

Fig. 8.15 shows the adjusted Channel Tunnel project's FNPV under asymmetric information. Note that the highest equity ratio, 35%, will yield a positive FNPV, £16 million. To increase 1% of equity ratio will reduce FNPV by approximately £13.8 million. As argued earlier, the developer should undertake the project when $FNPV \geq 0$.

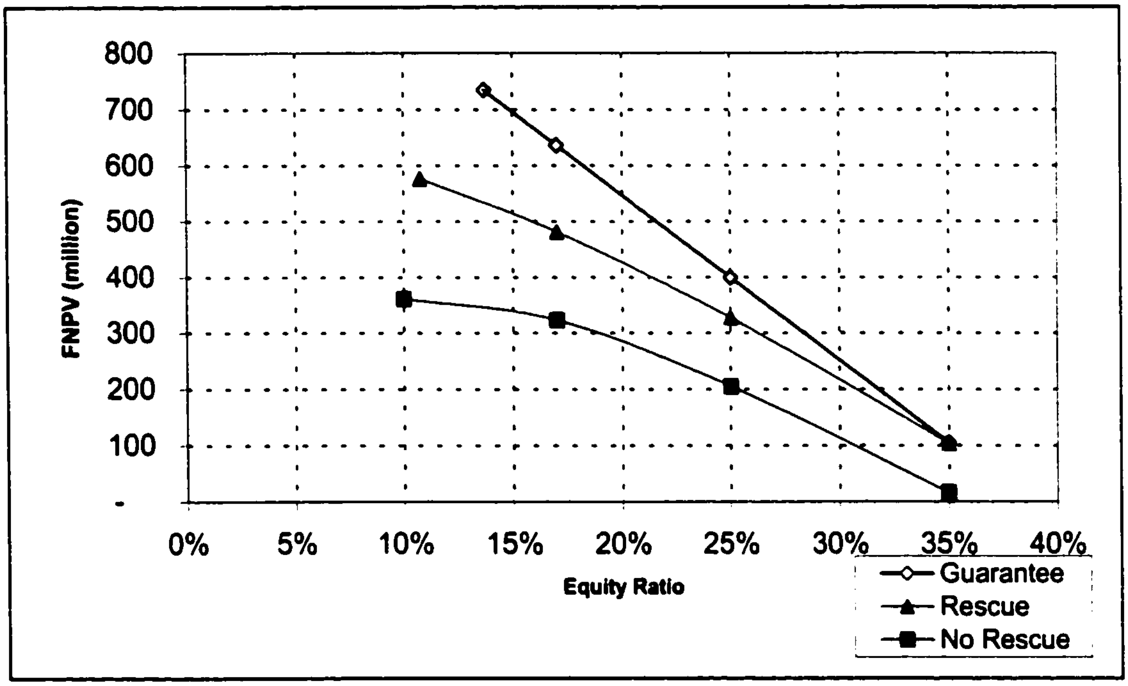

Fig. 8. 15 FNPV, When Developer's Equity Owning Ratio = 50.53%

8.6.1.3 Effectiveness of High Equity Ratio Criterion and No Construction Contract Criterion

Sections 8.6.1.1 and 8.6.1.2 show that when the developer's equity owning ratio is 50.53% and the information is asymmetric, the costs of increasing equity ratio in both cases are not significantly different. Plus, the 35% equity ratio will yield positive FNPV in both cases. Therefore, one may argue that when the developer's equity owning ratio is 50.53%, the high equity ratio criterion is not effective.

Another criterion or policy adopted by the British and French governments is the separation of the owning and the construction of the Channel Tunnel. Under this policy, the developer cannot participate in the construction contracts and therefore, the developer's FNPV will not include the construction contract value. Figure 8.16 shows the developer's possible FNPV that excludes the construction contract value. The curve of the FNPV of the adjusted project demonstrates that the developer will undertake the project as long as the equity ratio is not greater than 25%. Therefore, under the 50.53% equity owning ratio, the "no construction contract" policy will not be very effective unless the equity ratio requirement is high.

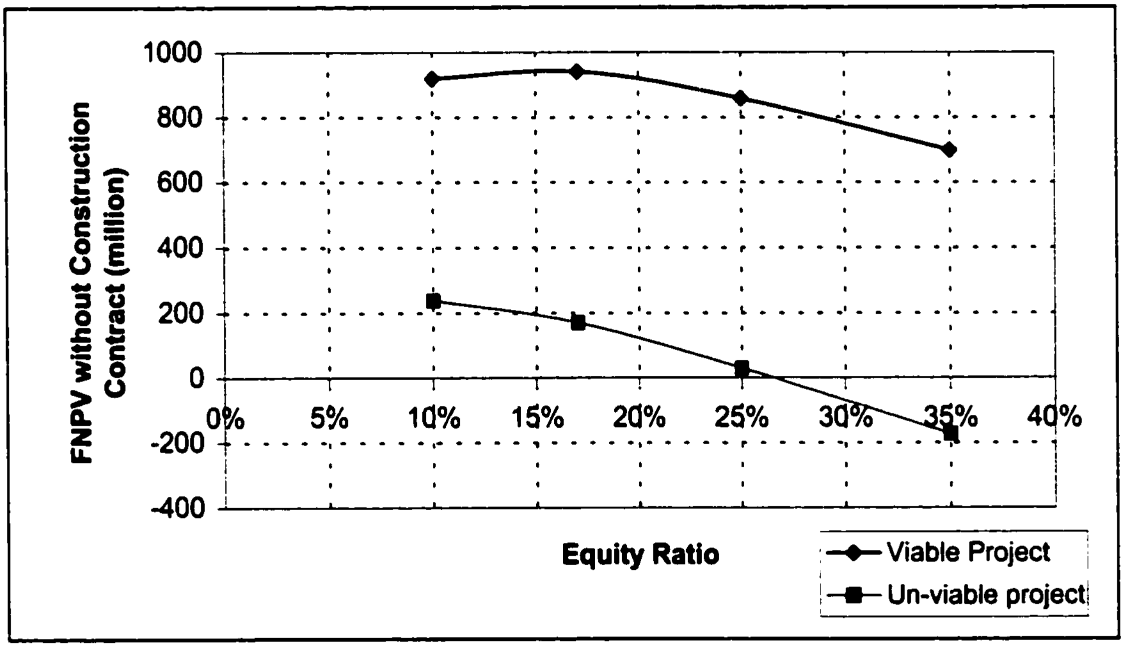

Fig. 8. 16 FNPV Without Construction Contract

8.6.2 The FNPV and Government Policies When the Equity Owning Ratio Equals 100%

When the developer has 100% equity ownership, the developer's equity investment profit, *E-I*, is equal to the project's total equity investment profit, $E_A - I_A$. Note that in the 100% equity ownership, the developer does not bear the loss from the equity discount margin, η. On the other hand, the developer cannot benefit from the asymmetric information. Under asymmetric information, the equity sold to the passive shareholders will be overpriced and the developer will benefit from the overpricing. These two effects will have different impacts on the projects with different viability.

8.6.2.1 When the Project is Viable

Figure 8.17 shows that the FNPV is in the range of £1,400 to £1,700 million, which is greater than the FNPV under 50.53% equity ownership. An increase of 1% of equity ratio will not reduce the FNPV; instead, it will increase the FNPV by approximately £7.1 million in average.

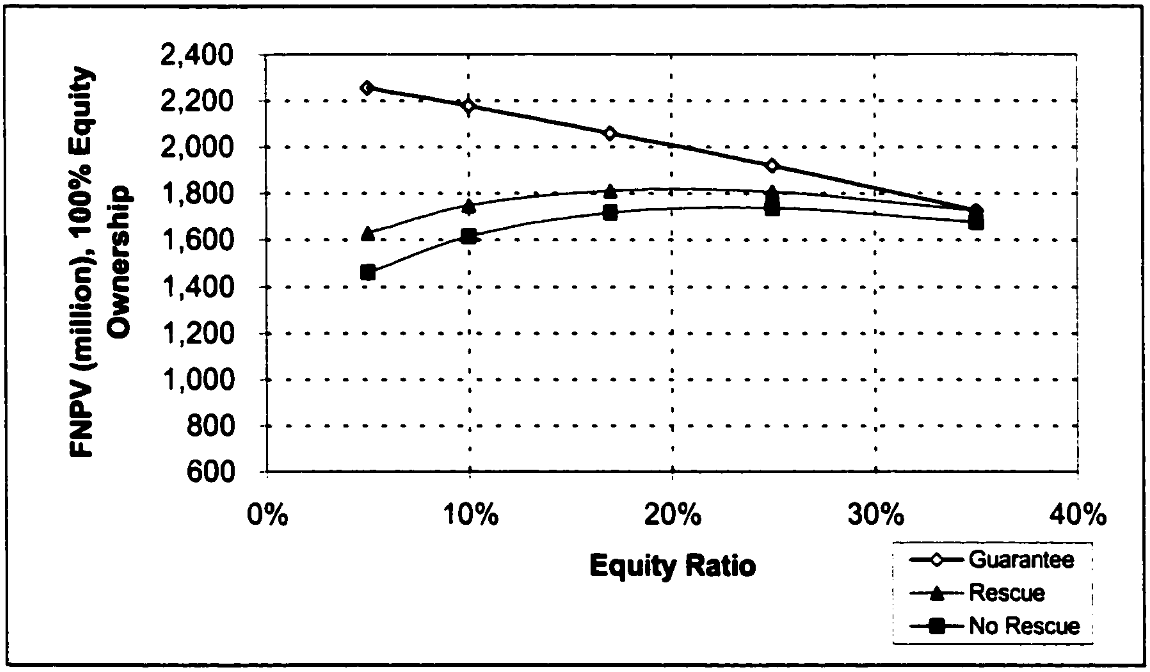

Fig. 8. 17 FNPV of a Viable Project With 100% Developer Equity Ownership

8.6.2.2 When the Project is Not Viable

The computation is similar to that in section 8.6.1.2. Figure 8.18 shows the *adjusted* Channel Tunnel project's FNPV under asymmetric information. Note that the highest equity ratio, 35%, will yield a negative FNPV, - £282 million. To increase 1% of equity ratio will reduce FNPV by approximately £13 million. According to the FNPV decision criterion, the developer should not undertake the project when the equity ratio is greater than 15%.

Figure 8.19 shows the comparison of the FNPV with different equity owning ratio. Due to the asymmetric information, the developer can overprice the equity of an unviable project, and gain profit from selling partial ownership to the passive shareholders. This is why the 50.53% equity owning curve in Fig. 8.19 is far above the 100% equity owning curve. However, when the developer has the 100% equity ownership, the developer cannot gain any profits from selling the ownership, and hence the asymmetric information is valueless.

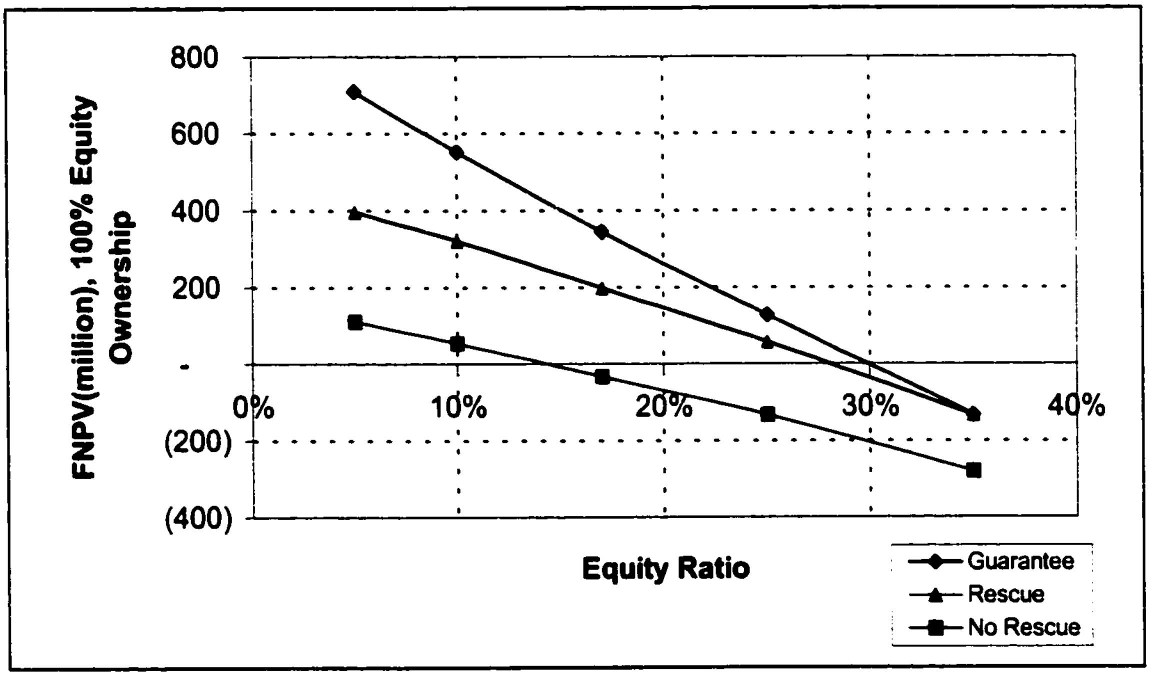

Fig. 8. 18 FNPV of an Unviable Project With 100% Developer Equity Ownership

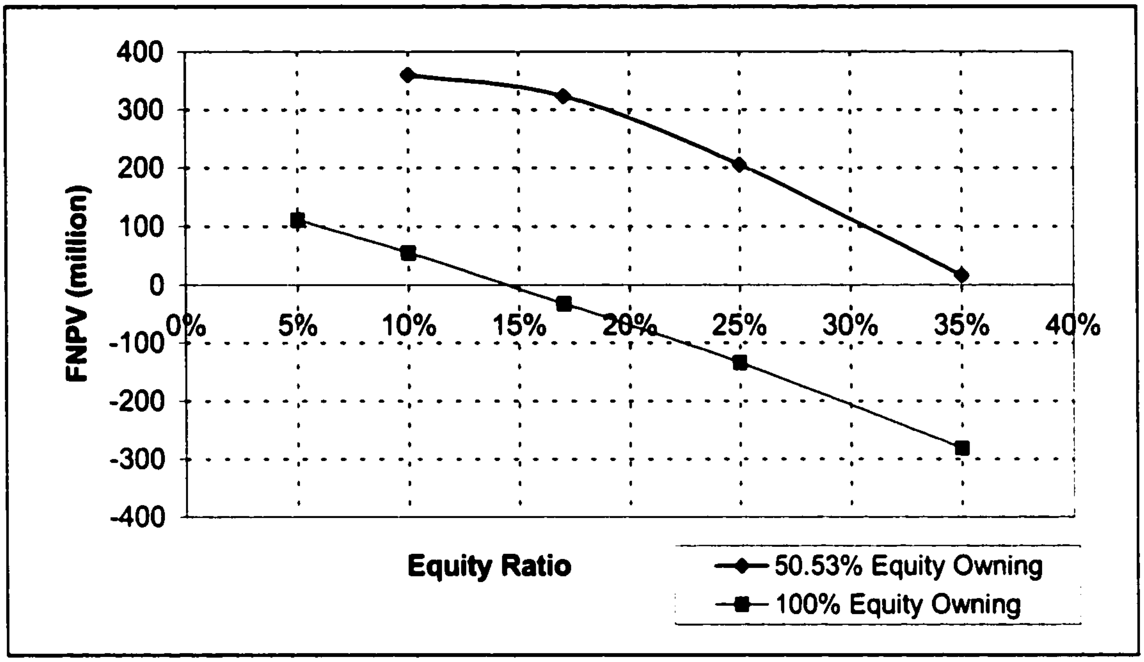

Fig. 8. 19 FNPV of Different Developer's Equity Owning Ratios

8.6.2.3 Effectiveness of the High Equity Ratio Criterion and the No Construction Contract Criterion

The cases in sections 8.6.2.1 and 8.6.2.2 show that when the developer's equity owning ratio is 100% and the information is asymmetric, the costs of increasing equity ratio in both cases are significantly different. For the viable project, increasing equity ratio is beneficial in average to the developer, whereas for the unviable project, the cost of increasing equity ratio is positive. Plus, the unviable project's FNPV will become negative when the equity ratio is greater than 15%. As a result, under the 100% developer's equity owning ratio, the high equity ratio criterion is effective. However, the weakness of this criterion is that it is more difficult for the developer to arrange a high equity ratio with 100% equity ownership, especially with a large-scale project. In fact, as Walker and Smith (1995) argued, developers are often very reluctant to devote their limited capital resources in the BOT's equity.

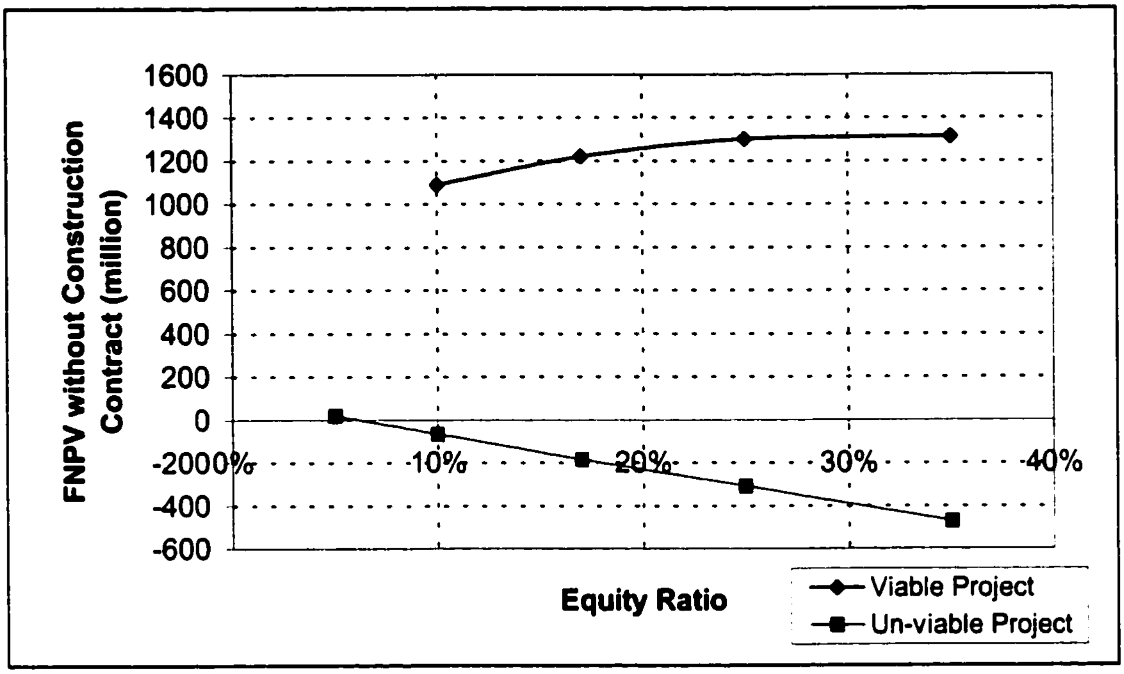

Fig. 8. 20 FNPV Without Construction Contract Under 100% Equity Owning Ratio

As shown in Fig. 8.20, the "no construction contract" criterion/policy under 100% equity owning ratio is more effective than the high equity ratio criterion in the Channel Tunnel's case. Also this criterion eliminates the difficulties of arranging 100% equity ownership of a "high" equity ratio when dealing with a large-scale project. According to Fig. 8.20, an equity ratio as low as 6% is sufficient to differentiate the viability of a project.

8.6.3 Developer's Equity Owning Ratio as a Criterion

Sections 8.6.1 and 8.6.2 demonstrate that the common criterion, "high equity ratio," and the British and French governments' "no construction contract" policy will be effective when the developer's equity owning ratio is 100%. The 100% equity ownership also precludes the developer from gaining benefit from overpriced passive equity under information asymmetry. As a result, it is reasonable to infer that the effectiveness of the "high equity ratio" and "no construction contract" depends on how high the developer's equity ownership is. In other words, one may conclude that the developer's equity owning ratio itself can become a criterion for proposal evaluation. The government can utilize this criterion to screen out unviable projects or their developers. This criterion is consistent with the shareholder and government's perspectives, since the developer will be evaluating the project from the shareholder and government's perspective when the equity owning ratio is high. However, since the developer has other stakes in profit structures, it is possible that the loss in the equity investment can be compensated by other profit components such as construction contracts. Therefore, sometimes it may be necessary to preclude the construction contracts from the project developer's participation so that the equity owning ratio criterion can be effective. However, this exclusion of construction contract participation may also discourage the developer's promoting efforts.

Although this section shows the levels of equity ratio or developer's equity ownership needed to make the criteria effective, in general, the conditions for effective criteria in each particular project will mainly depend on the characteristics of the project. This research contributes by providing a quantitative analytic framework for the analysis of the criteria's effectiveness.

8.6.4 Other Conventional Evaluation Criteria under Asymmetric Information

Further analysis will be made in this section in the government's project evaluation criteria according to the application of our framework on the Channel Tunnel project. Previously, it was concluded that the developer's equity owning ratio could be an effective evaluation criteria, together with the high equity ratio and no construction contract criteria. The point is that unless the project is viable, it is very costly to the developer to comply with these evaluation criteria. One project may be unviable to an inferior developer, yet may be viable to another developer with lower cost structures or particular technical or managerial specialties. However, the quality of the developer is unknown to the government and to the public. The purpose the evaluation criteria is to let the developers self-select into different levels of the criteria according to their characteristics so that only the appropriate developers will be selected.

By the same reasoning adopted in the research, one may use the real options and game theoretic model to analyze the effectiveness of other criteria that are often used by the BOT project evaluation. These conventional financial evaluation criteria may include:

- future operating cash flow
- tolls and other charges
- construction cost and duration
- concession period required
- possible profit-share scheme
- subsidies from the government, such as the debt guarantee or least revenue guarantee
- written commitments from the lending banks

8.6.4.1 High future operating cash flow or low tolls/other charges

The proposal's future operating cash flow projections, tolls estimation/plan, and construction costs are the project's most critical financial figures. The project's financial viability mainly depends on these cash flow and cost projections. However, they are also the most unreliable figures in the project proposal. One major reason, other than the difficulty of estimating those figures, is that the developer may have strong incentives to propose "optimistic" estimates as the "most likely" estimates in order to be awarded the project. As shown in Table 8.7, Eurotunnel's projections on the operating revenues were overestimated by approximately 51% or 47% of the projected revenue on average. That is, the actual revenues from 1994 to 1999 were only 49% or 53% of the projected amount in average. According to this research, it is easy to show that adopting the optimistic estimates of operating cash flow and tolls will not incur any actual cost upon the developer's actual FNPV, but "enhances" the appeared project viability and creates opportunity for overpricing the passive equity. The effectiveness of the criteria of future cash flows and tolls is low.

8.6.4.2 Low project cost or short duration

The effectiveness of the low construction cost or project cost criteria depends on the promoting scheme and contract types. For example, if the developer does not participate in the construction contract, adopting the optimistic cost estimation will not incur any costs to the developer, but makes the proposal look good. The effectiveness of the low cost criteria is then in doubt. However, if the developer is also the major constructor of the project, the effectiveness will depend on the type of the contract. If the contract is the lump sum contract and the contractor bears the major risks, to use "optimistic" estimates will increase the possibilities of future loss/liability due to cost overrun and therefore will incur cost to the contractor/developer. In this case, the low-cost criterion can be effective. The framework derived in this chapter to evaluate the value of various types of contracts can be used to analyze the effectiveness issue. If the contract is the target cost contract or the contract that the contractor bears minimal risks, it can be shown that to use "optimistic" estimation will have only limited impacts on the developer's profit structures. In this case, the low-cost criteria cannot be effective.

8.6.4.3 Concession period required and possible profit share scheme

The concession period or profit share scheme shown on the proposal is almost a binding contract although the concession contract has not been signed. Therefore, the concession period and profit share scheme will have material impact on the developer's profit structures. These two criteria may be effective under certain conditions.

8.6.4.4 Subsidies from the government

The subsidies from the government may include the government debt guarantee or future operating revenue guarantee. These guarantees typically are provided only when the project's future revenue is not high enough or too risky such that the project cannot be financially viable without the government guarantee. In this case, some proposals may require certain government guarantees. The government may prefer that the developer does not require any subsidies from the government in the developing process. However, it is difficult to use this criterion to judge a proposal. Especially when the government does not have clear policies regarding the post-awarding negotiation. For example, a developer who did not request for a subsidy in the proposal may request the subsidy during the post-awarding negotiation process. In this case, a proposal without requesting subsidies is not an effective judging criterion. It is argued that the government should thoroughly analyze the BOT project's financial viability from both the developer and shareholder's perspectives and decide whether subsidies are necessary before the tendering process. If it is decided that the subsidies may be necessary, the government should announce the subsidy policies upon the invitations for proposals. The government should allow request of subsidies in the proposal; however, the government should also prohibit any post-awarding request for the subsidies if the developer did not ask for the subsidies in the proposal. On the other hand, if the government determines that the subsidy is not necessary, the government should announce its policy of no subsidies toward the projects which includes the government's formal announcement that there will be no post-awarding request for subsidies.

8.6.4.5 Written commitments from the lending banks

Written commitments from the lending banks can be an effective evaluation criterion. The letter of commitments implies that the project will be able to use debt within reasonable cost/interest rate. As one may find in the Channel Tunnel project, the cost of debt plays a crucial role in the value of the project's future operating cash flows. In other words, the project's viability relies heavily on the cost of debt. A proposal without the lending banks' commitment may turn out to request the government for debt guarantee after the project awarding should the banks insist upon the debt guarantee. Furthermore, the lending banks' written commitments also show that the project's revenue and cost projections have been discussed with the banks and gain the trust from the banks. This will give the proposal's projections higher credibility and reduce some asymmetric information problems.

8.6.5 Developer's Developing Strategies and Effective Signals

From the BOT investment's asymmetric information analysis, one may find that the government's effective screening policy for searching "good" developers is in fact very close to the developer's effective signals to demonstrate that the developer is "good." For example, exclusion of the construction contracts from a developer's profit structures while the developer's equity owning ratio is high can be a very effective signal to the government. However, it is worth noting that if the government has very limited knowledge regarding the effectiveness of various evaluation criteria, the use of signals may be too costly to the developer. If the developer wants to adopt certain signal, the developer should make sure that the government understands why the signals are so effective that only a "good" developer can perform. The communications between the developer and government regarding the reasoning behind the signals adopted are crucial to the developer. Sometimes, the developer who adopts and sends out effective signals should take the responsibility to educate the public and government why the signals are effective. One important contribution of this research is to form a *theoretical foundation* for examining the effectiveness of the project evaluation criteria and signals.

Furthermore, it is well known that negotiations plays a crucial role in the BOT development process. The developer should be aware of what issues are open for negotiation and what are not. For example, if the government does not have very clear policies regarding the future subsidy, it is very possible that future subsidy can be negotiated after the project's awarding or even during the construction. Thus, it is not suggested that the developer evaluates the projects by assuming that there is no subsidy or re-negotiation possibility because this assumption will significantly reduce the value of the project's profit structures.

CHAPTER 9 CONCLUSIONS AND RECOMMENDATIONS

9.1 Conclusions

Since the 1970s, privatization has been recognized as an approach to solve the difficulty of funding public projects. Government is no longer the sole provider of public works. Instead, through the role of coordinating the utilization of resources, government has become a participant that provides infrastructure systems along with the private sector, which has brought vast funding and technology into the process. The major technique applied in infrastructure privatization is the non-recourse project financing, in which the BOT(Build-Operate-Transfer) approach is one of the major schemes in practice. Before a BOT project is undertaken, it needs to demonstrate its financial and technical viability. However, while it is relatively easy to demonstrate a project's technical viability, evaluating a BOT project's financial viability is complex and challenging.

Existing BOT research mainly relies on surveys and case studies that are prone to subjective and biased judgments. A systematic and theoretic framework for analyzing the financing, valuation, and tendering policies of a BOT investment does not exist. This research focuses on the characteristics of a BOT project and applies the modern financial theories and game theory to investigate the BOT's four major concerns: (1) the value of a BOT investment to the developers, (2) the project's financing decisions, (3) the developer's bidding strategies, and (4) the government's BOT policies. In this research, a quantitative model was developed that can dynamically evaluate the value of a BOT investment from the developer's perspective and determine a BOT project's capital structure. Furthermore, by incorporating with game theory, this research investigates the BOT projects' bidding strategies, the effective signals of a developer's commitments and the effectiveness of the government BOT policies. The BOT model developed in this research thesis provides potential major BOT developers, contractors, and governments with a method of evaluating BOT projects more realistically. The following conclusions can be drawn from this research:

Conclusion 1: **The profit structures of a BOT investment play a critical role in the decision-making process of the developer and government.** The profit structures include the equity value, construction contract value, and other operating related contract value. Profit structures are one of the most critical judging factors that the developer relies upon, since the profit structures are the developer's overall payoff of the BOT investment. The value of the profit structures can be solved with respect to many important decision variables such as the equity ratio. The decision criterion is the BOT's full net present value (FNPV), the value of the profit structures minus the equity investment amount. As a result, the developer may make decisions by maximizing the FNPV or expected FNPV under the competitive bidding.

Conclusion 2: **Modern option pricing theory, game theory, and the characteristics of a BOT project can be incorporated in a unified BOT model to evaluate a BOT investment.** The unified BOT model includes three analytical frameworks: (1) the BOT Investment Rationale and Decision Framework, (2) the BOT Profit Structures Valuation Framework, and (3) the BOT Tendering Framework. The BOT investment rationale and decision framework formulates the developer's profit structures maximization rationale. The five-step profit structures valuation model considers a BOT project's characteristics, and computes the value that is consistent with the capital market discipline. The model is based on the modern option pricing theory and game theory. In this model, the project's future operating cash flows and construction costs are the state variables of the value of profit structures. Crucial assumptions regarding the cost behaviors and the computation of the amount of total debt are made and formulated. A reverse binomial pyramid is derived to solve the value the profit structures. The payoff functions for each component of the profit structures are derived. Specifically, the payoff functions for different types of construction contracts are derived in Chapter 8, the Channel Tunnel case study. The BOT tendering model is derived to analyze the effectiveness of developer's developing strategies and government's tendering policies. The tendering model is based on the game theoretic signaling analysis.

Conclusion 3: **Because of the characteristics of BOT projects, there are two possible equilibria, the "rescue" and "no rescue" equilibria, and the two equilibria may have significant impacts on the profit structures.** As seen in practice, many negotiations and interactions take place between developers and governments. Therefore, it is critical to understand the negotiation and interaction processes, and thus impacts on the evaluation of a BOT investment. In this study we show that in many cases it is *not* rational for the developer to fully assume the completion/operating risk due to the developer's option to negotiate for subsidies should things go wrong. On the basis of the game theoretic analysis, two possible Nash equilibria are found in the BOT, the "rescue" and "no rescue" equilibria. These two possible equilibria play an essential role in the valuation of the developer's profit structures. Under the rescue equilibrium, the government will rescue the project when serious adverse events occur. The value of the profit structures will be significantly higher than that of the no rescue equilibrium.

Conclusion 4: **The real options and game theoretic model can help the developer and the government make developing strategies and effective policies, respectively, under asymmetric information.** The valuation of BOT investment gives the foundation of the developer's financing and bidding strategies. Knowing the developer's decision-making process further becomes the basis of making an effective government policy. Governments cannot make effective policies without understanding the developer's BOT investment valuation process. The developer's decisions and government policies will interact with each other. The BOT investment's game theoretic signaling analysis gives implications on developer's financing and tendering strategies and government's BOT policies.

Conclusion 5: **The Channel Tunnel case study shows (1) the BOT model can reasonably evaluate the developer's profit structures and the impacts of various**

project characteristics and developing decisions on each profit structures component, and (2) the BOT model can help developers and government make appropriate developing strategies and policies, respectively.

Through this research, the BOT project's valuation, financing, and tendering issues are thoroughly investigated. The major contribution of this research is to provide a theoretic and quantitative framework of evaluating the BOT investment's viability from different perspectives. Another important contribution is to utilize the valuation framework to derive the developer's optimal financing decisions and tendering strategies, and the government's effective tendering polices. This theoretic study contributes the BOT research in providing a more solid methodology for the research on BOT or project financing-related decision-making processes.

9.2 Future Research

The limitations regarding this research mainly come from the limitations on the real options methodology and the nature of the BOT investments. For example, it is not easy to precisely estimate the parameters required in the model. In particular, the rate of return shortfalls of the BOT project value and cost, the appreciation/growing rate of the future operating cash flow, the construction cost's growing rate, and the market rate of return are difficult to estimate; yet these parameters have significant impacts on the results. Also the dynamics of the BOT projects are subject to be verified empirically. Another limitation is that the model has to exogenously assume the loan interest rates under different conditions such as equity ratios. The assumptions regarding the loan interest rates may be subjective. Other limitations come from the application of game theory. In the analysis of the government's *rescue* or *not rescue* equilibria, it is assumed that the political cost of rescuing or not rescuing a failing project can be quantified. In fact, it is very difficult to quantify the political cost. Also, in the signaling analysis, since there is no analytical solution from the real options analysis, the signaling analysis cannot be complete. In other words, without the analytical form for the value of the profit structures, it is very difficult to work out the full set of solutions regarding the conditions of effectiveness of various criteria and signals.

This research prompts further investigations into the following areas.

1. *Theoretic and Empiric Studies on the Construction Real Options Framework*:
   Modern option pricing theory has been used in the project risk and value assessment and proved very powerful, especially in natural resources related industry, such as crude oil and pharmaceuticals. However, in construction, the methodology had not yet been explored until this research. Further theoretic and empirical investigations are needed to fully develop this new methodology. Directions may include:

- ☐ Studies on parameter calibrations in construction real options framework. (The accuracy of these parameters plays critical role in the success of the analysis.)
- ☐ Further theoretic studies to relax the assumptions adopted by this research thesis.
- ☐ Series of empirical studies on the validity of the construction real options applications and their parameter estimations.

2. *Game and Real Options Theoretic Infrastructure Management and Public Policy:*
   Conventional public policy research does not have solid quantitative research methodology. Future research may continue this study and further examine the game equilibria existing for the developers and government. More complete investigations on the developing strategies and public policies can be performed. This research may focus on:

- ☐ Thorough investigations on the conditions of the BOT signaling and screening equilibria.
- ☐ Empirical examinations of the effectiveness of various policies with respect to different project characteristics and other factors.
- ☐ The extension of the framework developed in this research to the infrastructure's life-cycle management and public policy.

REFERENCES

1. Aggarwal, R. (1993). *Capital Budgeting under Uncertainty: New and Advanced Perspectives.* Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
2. Aghion, P., and Bolton, P. (1992). "An incomplete contracts approach to financial contracting." *Review of Economic Studies*, 59, 473-494.
3. Amram, M., and Kulatilaka, N. (1999). *Real Options: Managing Strategic Investment in an Uncertain World.* Harvard Business School, Boston, Massachusetts.
4. Anderson, R. W., and Sundaresan, S. (1996). "Design and valuation of debt contracts." *The Review of Financial Studies*, 9, 37-68.
5. Augenblick, M., and Custer, B. S. (1990). "The build, operate and transfer ("BOT") approach to infrastructure projects in developing countries." *Working papers*, The World Bank, Washington, DC.
6. Barron, D., and Myerson, D. (1982). "Regulating a monopolist with unknown costs." *Econometrica*, 50, 911-930.
7. Baxter, N. (1967). "Leverage, risk of ruin and the cost of capital." *Journal of Finance*, 22, 395-403.
8. Beidleman, C. R., Fletcher, D., and Veshosdy, D. (1990). "On allocating risk: The essence of project finance." *Sloan Management Review*, 31, 47-55.
9. Bergman, Y. Z., and Callen, J. L. (1991). "Opportunistic underinvestment in debt renegotiation and capital structure." *Journal of Financial Economics*, 29, 137-171.
10. Binmore, K. (1992). *Fun and Games: A Text on Game Theory.* D. C. Heath and Company, Lexington, Massachusetts.
11. Black, F., and Cox, J. C. (1976). "Valuing corporate securities: some effects of bond indenture provisions." *Journal of Finance*, 31, 351-367.
12. Black, F., and Scholes, M. (1973). "The pricing of options and corporate liabilities." *Journal of Political Economy*, 81, 637-654.
13. Boyle, P. (1977). "Options: A Monte Carlo approach." *Journal of Financial Economics*, 4, 323-338.

14. Boyle, P. (1988). "A lattice framework for option pricing with two state variables." *Journal of Financial And Quantitative Analysis*, 23, 1-12.
15. Boyle, P., Evnine, J., and Gibbs, S. (1989). "Numerical evaluation of multivariate contingent claims." *Review of Financial Studies*, 2, 241-250.
16. Brealey, R. A., and Myers, S. C. (1991). *Principles of Corporate Finance*. 4th ed., McGraw-Hill, New York, NY.
17. Brennan, M. J., and Schwartz, E. S. (1978). "Finite difference methods and jump processes arising in the pricing of contingent claims: A synthesis." *Journal of Financial And Quantitative Analysis*, 13, 461-474.
18. Brennan, M. J., and Schwartz, E. S. (1984). "Optimal financial policy and firm valuation." *Journal of Finance*, 39, 593-607.
19. Breenan, M. J., and Schwartz, E. S. (1985). "Evaluating natural resources investments." *Journal of Business*, 58, 135-157.
20. Bulow, J. and Rogoff, K. (1989). "Sovereign debt: Is to forgive to forget ?" *American Economic Review*, 79, 43-50.
21. Carr, P. (1988). "The valuation of sequential exchange opportunities." *Journal of Finance*, 43(5), 1235-1256.
22. Cho, I.-K., and Kreps, D. (1987). "Signaling games and stable equilibria." *Quarterly Journal of Economics*, 102, 179-221.
23. Copeland, T. E., Koller, T., and Murrin, J. (1996). *Valuation: Measuring and Managing the Value of the Companies*. 2nd ed., John Wiley & Sons Inc., New York, NY.
24. Copeland, T. E., and Weston, J. F. (1988). *Financial Theory and Corporate Policy*. Addison-Wesley, New York, NY.
25. Cox, J. C., Ingersoll, J. E., and Ross, S. A. (1985). "An intertemporal general equilibrium model of asset prices." *Econometrica*, 53, 363-384.
26. Cox, J. C., and Ross, S. A. (1976). "The valuation of options for alternative stochastic processes." *Journal of Financial Economics*, 3, 145-166.
27. Cox, J. C., Ross, S. A., and Rubinstein, M. (1979). "Option pricing: A simplified approach." *Journal of Financial Economics*, 7, 229-263.

28. Datastream (2000). Financial Historical Database, Product of Thomson Financial Datastream.

29. Dias, A., Jr., and Ioannou, P. G. (1995a). "Debt capacity and optimal capital structure for privately financed infrastructure projects." *Journal of Construction Engineering & Management*, ASCE, 121, 404-414.

30. Dias, A., Jr., and Ioannou, P. G. (1995b). "Optimal capital structure for privately-financed infrastructure projects." *UMCEE Report* No. 95-10, University of Michigan-Ann Arbor, MI.

31. Dias, A., Jr., and Ioannou, P. G. (1995c). "A desirability model for the development of privately-promoted infrastructure projects." *UMCEE Report* No. 95-09, University of Michigan-Ann Arbor, MI.

32. Dixit, A. K., and Pindyck, R. S. (1994). *Investment under Uncertainty*. Princeton University Press, Princeton, New Jersey.

33. Dutta, P. K. (1999). *Strategies and Games: Theory and Practice*. The MIT Press, Cambridge, Massachusetts.

34. Dybvig, P., and Zender, J. (1991). "Capital structure and dividend irrelevance with asymmetric information." *Review of Financial Studies*, 4, 201-219.

35. Espinosa, M., and Rhee, C. (1989). "Efficient wage bargaining as a repeated game." *Quarterly Journal of Economics*, 91, 401-419.

36. Eurotunnel. (1987). *Prospectus: Offer for Sale.*

37. Eurotunnel. (2000). *Eurotunnel Interim Report of 2000.*

38. Evnine, J. J. (1983). *Three Essays in the use of Option Pricing Theory*. Ph. D. dissertation, University of California at Berkeley.

39. Fernandez, R., and Glazer, J. (1991). "Striking for a bargain between two completely informed agents." *American Economic Review*, 81, 240-252.

40. Ferson, W. E., and Harvey, C. R. (1993). "An exploratory investigation of the fundamental determinants of national equity market returns." *Working paper*, No. 4595, National Bureau of Economic Research, 1050 Massachusetts Ave., Cambridge, MA 02138.

41. Fetherston, D. (1997). *The Chunnel.* Times Books, Random House Inc., New York.

42. Finnerty, J. D. (1996). *Project Financing: Asset-Based Financial Engineering.* John Wiley & Sons Inc., New York, NY
43. Fischer, E. O., Heinkel, R., and Zechner, J. (1989). "Dynamic capital structure choice: theory and tests." *Journal of Finance*, 44, 19-40.
44. Franco-British Channel Link Financing Group. (1984). *Finance for a Fixed Channel Link*, 2 vols., London.
45. Fudenberg, D., and Tirole, J. (1991). *Game Theory.* The MIT Press, Cambridge, Massachusetts.
46. Gibbons, R. (1992). *Game Theory for Applied Economists.* Princeton University Press, Princeton, NJ.
47. Harrison, J. M. (1985). *Brownian Motion and Stochastic Flow Systems.* John Wiley & Sons, New York, NY.
48. Hart, O. (1995). *Firms Contracts and Financial Structure.* Oxford University Press, New York, NY.
49. Herbelot, O. (1992). *Option Valuation of Flexible Investments: the Case of Environmental Investments in the Electric Power Industry.* Ph. D. dissertation, Massachusetts Institution of Technology.
50. Hertz, D. B. (1964). "Risk analysis in capital investment." *Harvard Business Review*, 42, No. 1, 95-106.
51. Hingorani, A., Lehn, K., and Makhija, A. K. (1997). "Investor behavior in mass privatization: The case of the Czech voucher scheme." *Journal of Financial Economics* 44, 349-396.
52. Holliday, I., Marcou, G., and Vickerman, R. (1991). *The Channel Tunnel: Public Policy, Regional Development and European Integration.* Belhaven Press, New York.
53. Hull, J. C. (1997). *Options, Futures, and Other Derivatives.* 3rd ed., Prentice-Hall, NJ.
54. Hull, J. C., and White, A. (1994). "Numerical procedures for implementing term structure models II: Two-factor models." *Journal of Derivatives*, Winter 1994, 37-48.
55. Jensen, M.C., and Meckling, W. (1976). "Theory of the firm: Managerial behavior, agency costs and capital structure." *Journal of Financial Economics*, 3, 305-360.

56. Kim, E. H. (1978). "A mean-variance theory of optimal capital structure and corporate debt capacity." *Journal of Finance*, 33, 45-63.

57. Leland, H. E. (1994). "Corporate debt value, bond covenants, and optimal capital structure." *Journal of Finance*, 49, 1213-1252.

58. Leland, H., and Pyle, D. H. (1977). "Informational asymmetries, financial structure, and financial intermediation." *Journal of Finance*, 32, No. 2, 371-387.

59. Leland, H. E., and Toft, Klaus B. (1996). "Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads." *Journal of Finance*, 51, 987-1019.

60. Luenberger, D. (1998). *Investment Science.* Oxford University Press, Inc.

61. Majd, S., and Pindyck, R. S. (1987). "Time to build, option value, and investment decisions." *Journal of Financial Economics*, 18, 7-27.

62. Margrabe, W. (1978). "The value of an option to exchange one asset for another." *Journal of Finance*, 33, 177-186.

63. Mason, S. P. (1978). "The numerical analysis of certain free boundary problems arising in financial economics." *Harvard Business School Working Paper*, 78-52.

64. Mason, S. P., and Merton, R. C. (1985). "The role of contingent claims analysis in corporate finance." *Recent Advances in Corporate Finance*, edited by Altman, E. I. and Subrahmanyam, M. G., Richard D. Irwin, Homewood, Illinois.

65. McAfee, P., and McMillan, J. (1987). "Auctions and bidding." *Journal of Economic Literature*, 25, 699-738.

66. McDonald, R., and Siegel, D. (1984). "Option pricing when the underlying asset earns a below-equilibrium rate of return: A note." *Journal of Finance*, 39(1), 261-265.

67. McDonald, R., and Siegel, D. (1985). "Investment and the valuation of firms when there is an option to shut down." *International Economic Review*, 26(2), 331-349.

68. McDonald, R., and Siegel, D. (1986). "The value of waiting to invest." *Quarterly Journal of Economics*, 101(4), 707-727.

69. Mella-Barral, P., and Perraudin, W. (1997). "Strategic debt service." *Journal of Finance*, 52, 531-556.

70. Merton, R. C. (1973). "Theory of rational option pricing." *Bell Journal of Economics and Management Science*, 4, 141-183.

71. Merton, R. C. (1974). "On the pricing of corporate debt: the risk structure on interest rate." *Journal of Finance*, 29, 449-470.

72. Merton, R. C. (1990). *Continuous-time Finance.* Blackwell Publisher Ltd., Oxford, U.K.

73. Modigliani, F., and Miller, M. (1958). "The cost of capital, corporation finance and the theory of investment." *American Economic Review*, 48, 267-297.

74. Modigliani, F., and Miller, M. (1963). "Taxes and the cost of capital: a correction." *American Economic Review*, 53, 433-443.

75. Myers, S. C. (1976). "Using simulation for risk analysis." in *Modern Developments in Financial Management*, ed. S. C. Myers. Praeger.

76. Myers, S. C. (1984). "The capital structure puzzle." *Journal of Finance*, 39, 575-592.

77. Myers, S. C., and Majluf, N. (1984). "Corporate financing and investment decisions when firms have information that investors do not have." *Journal of Financial Economics*, 13, 187-221.

78. Myerson, R. B. (1991). *Game Theory: Analysis of Conflict.* Harvard University Press, Cambridge, Massachusetts.

79. Parkinson, M. (1977). "Option pricing: The American put." *Journal of Business*, 50, 21-36.

80. Pennacchi, G. G. (1997). *Theory of Financial Decision Making — Lecture Notes.* University of Illinois at Urbana-Champaign, IL.

81. Roger, N. (1999). "Recent trends in private participation in infrastructure." *Private Sector*, The World Bank Group, Washington, DC.

82. Rothchild, M., and Stiglitz, J. (1976). "Equilibrium in competitive insurance markets: an essay on the economics of imperfect information." *Quarterly Journal of Economics*, 90, 629-649.

83. Rubinstein, M. (1994). "Return to Oz." Risk, 7(11), 67-70.

84. Samuelson, P. A. (1958). *Economics.* 4th ed., McGraw-Hill Book Company, Inc. New York, NY.

85. Sappington, D. (1983). "Limited liability contracts between principal and agent." *Journal of Economic Theory*, 29, 1-21.
86. Shapiro, A. C. (1992). *Multinational Financial Management*. 4th ed., Allyn and Bacon, Needham Heights, Massachusetts.
87. Shen, L. L., Rowson K. H., and Zhang, Z. (1996). "Application of BOT system for infrastructure projects in China." *Journal of Construction Engineering & Management*, ASCE, 122, 319-323.
88. Smith, A. J. (1999). *Privatized Infrastructure: the Role of Government*. Thomas Telford Publishing, London E14 4JD.
89. Solt, M. E. (1993). "SWORD financing of innovation in the biotechnology industry." *Financial Management*, Summer 1993, 173-187.
90. Spence, A. M. (1973). "Job market signaling." *Quarterly Journal of Economics*, 87, 355-374.
91. Spence, A. M. (1974). *Market Signaling: Informational Transfer in Hiring and Related Screening Processes*. Harvard University Press, Cambridge, Massachusetts.
92. Tavis, R. L. (1995). *A Handbook of Building Investment: Using Spreadsheet Analysis-Architects' Edition*. Richard L Tavis, 2018 S. Cottage Grove, Urbana, IL 61801.
93. Tiong, R. L. K., Yeo, K.-T., and McCarthy, S. C. (1992). "Critical success factors in winning BOT contracts." *Journal of Construction Engineering & Management*, ASCE, 118, 217-228.
94. Tiong, R. L. K. (1995a). "Competitive advantage of equity in BOT tender." *Journal of Construction Engineering & Management*, ASCE, 121, 282-288.
95. Tiong, R. L. K. (1995b). "Risks and guarantees in BOT tender." *Journal of Construction Engineering & Management*, ASCE, 121, 183-188.
96. Tiong, R. L. K. (1995c). "Impact of financial package versus technical solution in a BOT tender." *Journal of Construction Engineering & Management*, ASCE, 121, 304-311.
97. Tiong, R. L. K. (1996). "CSFs in competitive tendering and negotiation model for BOT projects." *Journal of Construction Engineering & Management*, ASCE, 122(3), 205-211.

98. Tiong, R. L. K., and Alum, J. (1997). "Final negotiation in competitive BOT tender." *Journal of Construction Engineering & Management*, ASCE, 123, 6-10.

99. Trigeorgis, L. (1991). "A log-transformed binomial numerical analysis method for valuing complex multi-option investments." *Journal of Financial and Quantitative Analysis*, 26(3), 309-326.

100. Trigeorgis, L. (1993). "The nature of option interactions and the valuation of investments with multiple real options." *Journal of Financial Quantitative Analysis*, 28, 1-20.

101. Trigeorgis, L. (1996). *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. The MIT Press, Cambridge, Massachusetts.

102. Trigeorgis, L. (1995). *Real Options in Capital Investment: Models, Strategies, and Applications*. Praeger Publisher, Westport, CT.

103. United Nations industrial development organization (1996). *UNIDO BOT Guidelines*. United Nations, New York, NY.

104. Walker, C., and Smith, A. J. (1995). *Privatized Infrastructure- the BOT Approach*. Thomas Telford Inc., New York, NY.

105. Wearing, R. T. (1989). "Cash flow and the Eurotunnel." *Accounting and Business Research*, 20, No. 77, 13-24.

106. Zhang, W. R., Wang, S. Q., Tiong, R. L. K., Ting, S. K., and Ashley, D. (1998) "Risk management of Shanghai's private-financed Yan'an Donglu tunnels." *Journal of Engineering, Construction and Architecture Management*, 5(4), 399-409.

107. Ziegler, A. (1999). *A Game Theory Analysis of Options: Contributions to the Theory of Financial Intermediation in Continuous Time*. Springer-Verlag, New York.

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