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Fuzzy multi-criteria decision-making procedure for evaluating advanced manufacturing system investments

E. Ertugrul Karsak*, Ethem Tolga

Industrial Engineering Department, Galatasaray University, Ortaköy, Istanbul 80840, Turkey

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Abstract

In this paper, a fuzzy decision algorithm is proposed to select the most suitable advanced manufacturing system (AMS) alternative from a set of mutually exclusive alternatives. Both economic evaluation criterion and strategic criteria such as flexibility, quality improvement, which are not quantitative in nature, are considered for selection. The economic aspects of the AMS selection process are addressed using the fuzzy discounted cash flow analysis. The decision algorithm aggregates the experts' preference ratings for the economic and strategic criteria weights, and the suitability of AMS investment alternatives versus the selection criteria to calculate fuzzy suitability indices. The fuzzy indices are then used to rank the AMS investment alternatives. Triangular fuzzy numbers are used throughout the analysis to quantify the vagueness inherent in the financial estimates such as periodic cash flows, interest rate and inflation rates, experts' linguistic assessments for strategic justification criteria, and importance weight of each criterion. A comprehensive numerical example is provided to illustrate the results of the analysis.

Keywords: Fuzzy decision analysis; Multi-criteria decision-making; Investment analysis; Flexible manufacturing systems

1. Introduction

Investment evaluation methods play an important role in today's competitive manufacturing environment. Shrinking profit margins and diversification require careful analysis of investments, and the decisions regarding these investments are crucial to the survival of the manufacturing firm. Lately, the manufacturing firms have been investing in advanced manufacturing technologies such as group technology, flexible manufacturing systems, computer-integrated manufacturing systems, etc. to improve manufacturing performance in terms of cost, productivity, flexibility and quality, in an effort to compete with other industrialized firms in the global marketplace.

Flexibility in a manufacturing environment can be defined as the capability and ease of accommodating changes in the system. Flexibility ensures that manufacturing can be both cost effective and customized at the same time [1]. A single widely-accepted measure for flexibility does not exist, and thus, there is a continuing research on this subject [2]. Flexibility of advanced manufacturing systems provides faster throughput, reduces cost of retooling for design changes, allows for smoother scheduling, and provides an ability for production volume adjustments to handle unanticipated

* Corresponding author.

E-mail address: karsak@gsunv.gsu.edu.tr (E. Ertugrul Karsak).

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demand changes with low levels of inventory [3]. In summary, flexibility results in considerable enhancement in responding to changes in market demand, product design and product mix.

According to Meredith and Suresh [4], investment justification methods in advanced manufacturing technologies are classified into economic analysis techniques, analytical methods, and strategic approaches. These methods deviate from each other mainly due to the treatment of non-monetary factors. Economic justification methods of manufacturing investments have been discussed thoroughly in the past couple of decades [5]. Economic analysis methods are the basic discounted cash flow techniques such as present worth, annual worth, internal rate of return, etc., and other techniques such as payback period and return on investment which ignore time value of money. The application of these techniques to the evaluation of flexible manufacturing system (FMS) investments is analyzed in [6]. It is well known by engineering economy practitioners that accounting methods, which ignore time value of money, would produce inaccurate or at best approximate results.

Discounted cash flow (DCF) methods appear as the most popular economic justification methodology; however, determining cash flows (revenues, expenses) and discount rates as crisp values can lead to erroneous results in most of the real-life applications. The probabilistic cash flow analysis can be used if the probabilities of the possible outcomes are known. However, when the frequency distribution of the possible outcomes is not known as for the revenues and expenses of a new product line, most decision-makers employ experts' knowledge in modeling cash flows in the evaluation phase [7,8].

The conventional DCF methods do not appear to be suitable on their own for the evaluation of an advanced manufacturing system (AMS) investment due to the non-monetary impacts posed by the system. Sullivan [9] points out the inadequacy of traditional financial justification measures of project worth such as return on investment, payback, net present worth in considering the strategic merits of advanced manufacturing technologies. The results of the surveys conducted by Lefley [10] for justification of advanced manufacturing technology (AMT) in the UK, and by Lefley and Sarkis [11] for appraisal of AMT investments in the UK and US both indicate the support for the difficulty in assessing AMT investments due to their non-quantifiable benefits. Due to this difficulty, over 80% of the respondents in the US and UK point out that not all potential benefits of AMT investments are considered in the financial justification process. Furthermore, the results of the surveys state that subjective assessment of AMT investment with/without financial justification is observed in approximately 60% of the manufacturing firms responding to the questionnaire. Improvements in product quality, reliability, production efficiencies, competitiveness as a result of the versatility and flexibility of the system are the focal points in the justification stage of an AMS investment. Productivity, quality, flexibility and other intangibles should be examined in terms of potential returns through enhancement of long-term business competitiveness as well as in terms of a comprehensive evaluation of internal costs [5].

When flexibility, risk and non-monetary benefits are expected, and particularly if the probability distributions can be subjectively estimated, analytical procedures may be used. Strategic justification methods are qualitative in nature, and are concerned with issues such as technical importance, business objectives, competitive advantage, etc. [4]. When strategic approaches are employed, the justification is made by considering long-term intangible benefits. Hence, using these techniques with economic or analytical methods would be more appropriate. Fig. 1, which is an updated version of the classification initially proposed by Meredith and Suresh [4], resumes the justification methods for advanced manufacturing technologies.

Since certain criteria cannot be expressed in quantitative terms, a number of articles focus on integrating the qualitative and quantitative aspects to evaluate the benefits of AMS. Wabalickis [12] develops a justification procedure based on the analytic hierarchy process (AHP) to evaluate the numerous tangible and intangible benefits of an FMS investment. Naik and Chakravarty [13] point out the need for integrating the non-financial and strategic benefits of AMS with the financial benefits, and propose a hierarchical evaluation

	TECHNIQUES	ADVANTAGES	DISADVANTAGES
ECONOMIC	– Payback method – Return on investment – Discounted cash flow techniques (NPV, IRR)	– Ease in data collection – Intuitive appeal	– Do not take into account strategic and non-economic benefits – Consider a single objective of cash flows, and ignore other benefits such as quality and flexibility
STRATEGIC	– Technical importance – Business objectives – Competitive advantage – Research and development	– Require less technical data – Use the general objectives of the firm	– Necessity to use these techniques with economic or analytic ones since they consider only long-term intangible benefits

ANALYTIC	– Scoring models (AHP etc.)
	– Mathematical programming
	  – Integer programming
	  – Goal programming
	  – DEA
	– Stochastic methods
	– Fuzzy set theory
		– Uncertainty of the future and the multi-objectivity can be incorporated
		– Subjective criteria can be introduced in the modeling phase
			– Require more data
			– Usually more complex than the economic analysis

Fig. 1. Classification of justification methods for advanced manufacturing technologies.

procedure involving strategic evaluation, operational evaluation and financial evaluation. Shang and Sueyoshi [14] propose a selection procedure for an FMS employing the AHP, simulation, and data envelopment analysis (DEA). Small and Chen [15] discuss the results of a survey conducted in the US that investigates the use of justification approaches for AMS. According to their findings, manufacturing firms using hybrid strategies, which employ both economic and strategic justification techniques, attain significantly higher levels of success from advanced technology projects. Sambasivarao and Deshmukh [16] present a decision support system integrating multi-attribute analysis, economic analysis and risk evaluation analysis. They have suggested AHP, TOPSIS, and linear additive utility model as alternative multi-attribute analysis methods.

An integrated multi-criteria procedure that takes into account both the economic criteria and the strategic justification criteria is required for proper evaluation of AMS investment alternatives. In general, scoring models, the analytic hierarchy process, outranking methods and goal programming can be listed among the deterministic methods for solving multiple criteria problems. Non-deterministic methods include game theoretical models, multi-attribute utility models, fuzzy linguistic methods and expert systems. In this paper, a fuzzy decision-making procedure is proposed as a computational-effective alternative to rectify some of the difficulties posed by the existing evaluation techniques.

2. The fuzzy approach

The fuzzy set theory was introduced by Zadeh [17] to deal with problems in which a source of vagueness is involved. The basic concepts of fuzzy sets, and algebraic operations of triangular fuzzy numbers are briefly introduced in Appendix A. This section presents a review of the literature on fuzzy decision-making techniques applied to the assessment of advanced manufacturing technologies, and fuzzy discounted cash flow analysis.

2.1. Fuzzy decision analysis for AMS selection

In general, the probability concept is related to the frequency of occurrence of events, captured by repeated experiments whose outcomes are recorded, while the fuzzy sets provide the appropriate framework to assess the possibility of events rather than their probability [18]. Bellman and Zadeh [19] point out the need for differentiation between randomness and fuzziness, with the latter being a major source of imprecision in many decision processes. They state that "By fuzziness, we mean a type of imprecision which is associated with fuzzy sets, that is, classes in which there is no sharp transition from membership to non-membership".

The arithmetic operations of fuzzy numbers generalize interval analysis [20]. The mathematical techniques for dealing with fuzziness are simpler in many ways than those of probability theory since in the fuzzy set theory the simpler notion of membership function corresponds to the notion of probability measure in probability theory. Addition of fuzzy numbers is shape-invariant in contrast to random variable convolution. That is, adding fuzzy numbers having triangular membership functions yields a fuzzy number with triangular membership function, adding the ones with trapezoidal membership functions gives a trapezoidal fuzzy number, etc. Nevertheless, one should note that since fuzzy models yield only best and worst case analysis and do not assume that errors compensate, there exists a trade-off between loss in precision and ease in computation [18].

The fuzzy set theory appears as an important tool to provide a multi-criteria decision framework that incorporates the vagueness and imprecision inherent in the justification and selection of advanced manufacturing systems. An effective way to express factors including flexibility, quality of the products, enhanced response to market demand, and reduction in inventory, which can neither be assessed by crisp values nor random processes, is using linguistic variables or fuzzy numbers.

Recently, fuzzy decision-making techniques have been applied to acquisition of advanced manufacturing technologies. Liang and Wang [21] propose a robot selection procedure employing the concepts of fuzzy set theory. Perego and Rangone [22] analyze and compare multi-attribute decision-making techniques that employ fuzzy set theory in the assessment and selection of advanced manufacturing technologies. Karsak [23] presents a two-phase decision framework, which employs data envelopment analysis in the initial phase to determine the technically efficient robot alternatives, considering cost and technical performance parameters. Vendor-related qualitative factors are treated in the second phase, and a fuzzy robot selection algorithm is used to rank the technically efficient robots.

2.2. Fuzzy discounted cash flow analysis

Fuzzy discounted cash flow analysis has been recently used by several authors as an alternative to the conventional cash flow models, where either deterministic cash flows and discount rates or cash flow estimates and/or discount rates accompanied by probability distributions are used. Fuzzy set theory enables us to employ fuzzy cash flows that take better account of the imprecision and vagueness in human judgments about the future in place of cash flows defined as crisp numbers or probability distributions [22].

Buckley [24] develops the elementary formulas for the fuzzy future worth and present worth, using both fuzzy cash flows and fuzzy interest rates over n periods, where n may be crisp or fuzzy. In his paper, the future worth formula for the continuous compounding of a discrete, single payment is also presented. Ward [8] presents fuzzy present worth and fuzzy internal rate of return models using flat fuzzy filter function (4F) numbers.

Chiu and Park [7] propose a discounted cash flow model in which the cash flows and discount rates are represented by triangular fuzzy numbers. They use present worth as the figure of merit, and first derive an exact present worth formulation, and later an approximate form with much less computational effort. They further state that when the future estimated discount rates are within an absolute range of 4%, the approximate present worth is a close estimate of the exact present worth. Wang and Liang [25] present two algorithms to perform benefit/cost ratio analysis in a fuzzy environment. Karsak [26] develops fuzzy analogues of the liquidity risk measures used in capital budgeting, which are required as a supplement especially when a superior alternative cannot be determined as a result of the discounted cash flow analysis.

It has been shown that lower present worths are obtained in the after-tax analyses incorporating inflation since depreciation calculations are not indexed to inflation [27]. Hence, inflation needs to be taken into account properly to broaden the applicability of fuzzy discounted cash flow analysis in inflation-prone economies. In this paper, fuzzy present worth analysis incorporating inflation is used as an economic evaluation criterion for the AMS investments. The results of the model are integrated in the fuzzy multi-criteria decision-making algorithm for evaluating the AMS investment alternatives.

3. Financial evaluation of AMS investments under conditions of inflation using fuzzy DCF analysis

A fuzzy present worth model, which is applicable to the economic justification of AMS investments in inflation-prone economies, is developed in this section. As in [3], which presents a synthesis of the capital budgeting problems related to the advanced automation projects, this paper uses the incremental (when compared to the existing manufacturing system) cash flows in the economic evaluation of the AMS alternatives. Notation that is used in the financial modeling phase is given below.

$\tilde{R}_j$ fuzzy pretax cash flow in constant dollars regarding incremental operating revenues such as labor savings, material savings, revenue from increased output, etc. obtained in period j ($j = 1, 2, \ldots, N$);
$\tilde{C}_j$ fuzzy pretax cash flow in constant dollars regarding incremental operating cost such as skilled labor cost, energy cost, etc. incurred in period j ($j = 1, 2, \ldots, N$);
$\tilde{A}_j$ fuzzy pretax cash flow in period j ($j = 0, 1, \ldots, N$), where $\tilde{A}_0$ represents initial costs including purchase of automated machinery and equipment, software costs, other installation costs less the trade-in value of the existing manufacturing system;
$\tilde{\tilde{A}}_j$ fuzzy after-tax cash flow in period j ($j = 0, 1, \ldots, N$);
D_j depreciation expense in period j ($j = 1, 2, \ldots, N$);
$\tilde{i}$ fuzzy after-tax interest rate;
$\tilde{f}_j$ fuzzy general rate of inflation in period j, which is the periodic rate of change of all prices in the economy;
$\tilde{h}_{Rj}$ fuzzy specific rate of inflation for the operating revenues obtained in period j;
$\tilde{h}_{Cj}$ fuzzy specific rate of inflation for the major operating cost items incurred in period j;
$\tilde{d}_{Rj}$ fuzzy differential rate of inflation of the revenues obtained in period j;
$\tilde{d}_{Cj}$ fuzzy differential rate of inflation of the major cost items incurred in period j.

The following assumptions are used throughout the analysis:

i. $\tilde{R}_j, \tilde{C}_j$ are end-of-period fuzzy predicates, and are represented by positive triangular fuzzy numbers;
ii. $\tilde{h}_{Rj}, \tilde{h}_{Cj}, \tilde{f}_j$ are time-dependent fuzzy predicates, whereas $\tilde{i}$ is a fuzzy predicate applicable to the entire planning horizon;
iii. $\tilde{h}_{Rj}, \tilde{h}_{Cj}$ and $\tilde{f}_j$ are positive triangular fuzzy numbers except $\tilde{h}_{R0} = \tilde{h}_{C0} = \tilde{f}_0 = (0, 0, 0)$;
iv. $\tilde{i}$ is a positive triangular fuzzy number;
v. planning horizon (N), tax rate (t), and depreciation rate are crisp values.

Triangular fuzzy numbers appear as useful means of quantifying the uncertainty due to vagueness regarding cash flows, interest rates and inflation rates. The reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation. For instance, assuming that an expert's estimate about the first year revenue from a manufacturing investment is "around 3 billion Turkish lira", it can be represented by a triangular fuzzy number as (2.8, 3, 3.15). In the light of the assumptions given above, we define the following triangular fuzzy numbers:

$$\tilde{R}_j = (R_{ja}, R_{jb}, R_{jc}),$$

$$\tilde{C}_j = (C_{ja}, C_{jb}, C_{jc}),$$

$$\tilde{h}_{Rj} = (h_{Rja}, h_{Rjb}, h_{Rjc}),$$

$$\tilde{h}_{Cj} = (h_{Cja}, h_{Cjb}, h_{Cjc}),$$

$$\tilde{f}_j = (f_{ja}, f_{jb}, f_{jc}),$$

$$\tilde{i} = (i_a, i_b, i_c).$$

Conducting extended algebraic operations on triangular fuzzy numbers, the following measures are obtained to determine the fuzzy after-tax cash flows. First of all, differential rates of inflation for the operating revenue and cost items are computed using Eqs. (1) and (2).

$$\tilde{1}\oplus\tilde{d}_{Rj}=\left(\frac{1+h_{Rja}}{1+f_{jc}},\frac{1+h_{Rjb}}{1+f_{jb}},\frac{1+h_{Rjc}}{1+f_{ja}}\right),\tag{1}$$

$$\tilde{1}\oplus\tilde{d}_{Cj}=\left(\frac{1+h_{Cja}}{1+f_{jc}},\frac{1+h_{Cjb}}{1+f_{jb}},\frac{1+h_{Cjc}}{1+f_{ja}}\right).\tag{2}$$

The inflation-adjusted operating revenue and cost items are denoted as

$$\tilde{R}_j\otimes\prod_{k=1}^{j}(\tilde{1}\oplus\tilde{d}_{Rk})=\left(R_{ja}\prod_{k=1}^{j}\frac{1+h_{Rka}}{1+f_{kc}},\; R_{jb}\prod_{k=1}^{j}\frac{1+h_{Rkb}}{1+f_{kb}},\; R_{jc}\prod_{k=1}^{j}\frac{1+h_{Rkc}}{1+f_{ka}}\right),\tag{3}$$

$$\tilde{C}_j\otimes\prod_{k=1}^{j}(\tilde{1}\oplus\tilde{d}_{Ck})=\left(C_{ja}\prod_{k=1}^{j}\frac{1+h_{Cka}}{1+f_{kc}},\; C_{jb}\prod_{k=1}^{j}\frac{1+h_{Ckb}}{1+f_{kb}},\; C_{jc}\prod_{k=1}^{j}\frac{1+h_{Ckc}}{1+f_{ka}}\right),\tag{4}$$

$$\tilde{A}_j=\left(\tilde{R}_j\otimes\prod_{k=1}^{j}(\tilde{1}\oplus\tilde{d}_{Rk})\right)\Theta\left(\tilde{C}_j\otimes\prod_{k=1}^{j}(\tilde{1}\oplus\tilde{d}_{Ck})\right).\tag{5}$$

Fuzzy pretax cash flow in period j, which is defined as $\tilde{A}_j=(A_{ja},A_{jb},A_{jc})$, is calculated using Eq. (5). Hence, the smallest possible, the most promising, and the largest possible pretax cash flow amounts in period j are determined using Eqs. (6), (7) and (8), respectively.

$$A_{ja}=\left(R_{ja}\prod_{k=1}^{j}\frac{1+h_{Rka}}{1+f_{kc}}-C_{jc}\prod_{k=1}^{j}\frac{1+h_{Ckc}}{1+f_{ka}}\right),\tag{6}$$

$$A_{jb}=\left(R_{jb}\prod_{k=1}^{j}\frac{1+h_{Rkb}}{1+f_{kb}}-C_{jb}\prod_{k=1}^{j}\frac{1+h_{Ckb}}{1+f_{kb}}\right),\tag{7}$$

$$A_{jc}=\left(R_{jc}\prod_{k=1}^{j}\frac{1+h_{Rkc}}{1+f_{ka}}-C_{ja}\prod_{k=1}^{j}\frac{1+h_{Cka}}{1+f_{kc}}\right).\tag{8}$$

Then the fuzzy after-tax cash flow in period j ($j=1,2,\ldots,N$) can be calculated using Eq. (9).

$$\tilde{\tilde{A}}_j=\left(A_{ja}(1-t)+\frac{tD_j}{\prod_{k=1}^{j}(1+f_{kc})},\; A_{jb}(1-t)+\frac{tD_j}{\prod_{k=1}^{j}(1+f_{kb})},\; A_{jc}(1-t)+\frac{tD_j}{\prod_{k=1}^{j}(1+f_{ka})}\right).\tag{9}$$

Let $\tilde{\tilde{A}}_j$ be a triangular fuzzy number represented by $\tilde{\tilde{A}}_j=(\hat{A}_{ja},\hat{A}_{jb},\hat{A}_{jc}),\tilde{i}$ be a positive triangular fuzzy number denoted by $\tilde{i}=(i_a,i_b,i_c)$, and $\tilde{1}$ be the non-fuzzy number "1", i.e. $\tilde{1}=(1,1,1)$, then Eq. (10) can be used to derive the present worth (PW) of the after-tax fuzzy cash flows.

$$PW=\tilde{\tilde{A}}_0\oplus[\tilde{\tilde{A}}_1\otimes(\tilde{1}\oplus\tilde{i})^{-1}]\oplus\cdots\oplus[\tilde{\tilde{A}}_N\otimes(\tilde{1}\oplus\tilde{i})^{-N}].\tag{10}$$

If $\tilde{\tilde{A}}_j$ is positive, PW of $\tilde{\tilde{A}}_j$ can be obtained as

$$PW(\tilde{\tilde{A}}_j)=\left(\frac{\hat{A}_{ja}}{(1+i_c)^j},\frac{\hat{A}_{jb}}{(1+i_b)^j},\frac{\hat{A}_{jc}}{(1+i_a)^j}\right).\tag{11}$$

If $\tilde{\tilde{A}}_j$is negative, PW of $\tilde{\tilde{A}}_j$ is given as

$$PW(\tilde{\tilde{A}}_j)=\left(\frac{\hat{A}_{ja}}{(1+i_a)^j},\frac{\hat{A}_{jb}}{(1+i_b)^j},\frac{\hat{A}_{jc}}{(1+i_c)^j}\right).\tag{12}$$

The two representations given above can be combined using the maximum and minimum symbols, and the present worth of all fuzzy cash flows is obtained as

$$PW=\left(\sum_{j=0}^{N}\left[\frac{\max(\hat{A}_{ja},0)}{(1+i_c)^j}+\frac{\min(\hat{A}_{ja},0)}{(1+i_a)^j}\right],\; \sum_{j=0}^{N}\frac{\hat{A}_{jb}}{(1+i_b)^j},\; \sum_{j=0}^{N}\left[\frac{\max(\hat{A}_{jc},0)}{(1+i_a)^j}+\frac{\min(\hat{A}_{jc},0)}{(1+i_c)^j}\right]\right).\tag{13}$$

The cash flows and inflation rates are assumed to be independent throughout the present worth framework presented herein. If some of the periodic cash flows are dependent on the occurrence of other cash flows and/or periodic inflation rates are dependent, this can be incorporated in the analysis by employing the concept of conditioned fuzzy sets while defining the membership functions of fuzzy variables. A brief description of conditioned fuzzy sets [19] is provided in Appendix A.

4. Preference ranking system using linguistic variables

A linguistic variable can be defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill-defined to be described in conventional quantitative terms [28]. Linguistic variables are introduced to represent the value of natural or artificial languages such as 'around', 'very', 'little', etc. The value of a linguistic variable can be quantified and extended to mathematical operations using the fuzzy set theory. Throughout the analysis, we assume that there are a group of n decision-makers ($D_1, D_2, \dots, D_n$), who assess the importance weights of k criteria and suitability of m AMS alternatives under each of these k criteria. Feng and Xu [29] suggest that the importance weights can be determined going through the steps including determination of the initial value of the importance weight, consistency test, normalization, and adjustment. The process of assigning membership functions to fuzzy variables is either intuitive or based on some algorithmic or logical operations. Intuition is simply derived from the capacity of experts to develop membership functions through their own intelligence and judgment. Inference, rank ordering, angular fuzzy sets, neural networks, genetic algorithms, inductive reasoning, soft partitioning, and fuzzy statistics can be listed among the other methods stated in the literature to assign membership functions to fuzzy variables [30]. Ross [30] argues that the precise shapes of the membership functions are not that important in their utility, while the approximate placement of the membership functions on the universe of discourse, the number of partitions used, and the overlapping character are of vital importance for application purposes in fuzzy operations.

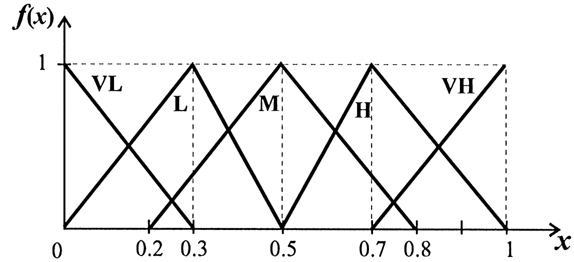

Fig. 2. Membership functions for importance weight of criteria (VL: (0, 0, 0.3), L: (0, 0.3, 0.5), M: (0.2, 0.5, 0.8), H: (0.5, 0.7, 1), VH: (0.7, 1, 1)).

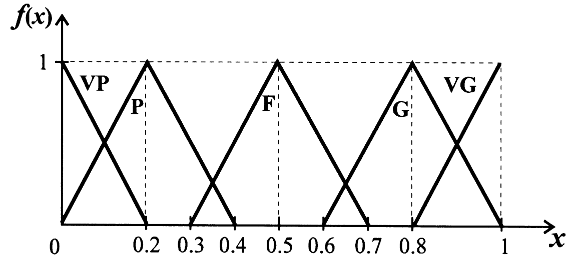

Fig. 3. Membership functions for linguistic values (VP: (0, 0, 0.2), P: (0, 0.2, 0.4), F: (0.3, 0.5, 0.7), G: (0.6, 0.8, 1), VG: (0.8, 1, 1)).

Throughout this paper, it is assumed that the decision-makers are using a set of weights $W = \{VL, L, M, H, VH\}$, where VL indicates 'very low', L 'low', M 'medium', H 'high', and VH 'very high', to show the importance of each criterion. The membership functions for importance weights are depicted in Fig. 2.

In order to determine the suitability of AMS alternatives versus the strategic criteria, the linguistic values, which are defined by the variable set $A = \{VP, P, F, G, VG\}$, are used. In here, VP, P, F, G, VG denote 'very poor', 'poor', 'fair', 'good', and 'very good', respectively. The membership functions of the linguistic values are shown in Fig. 3.

These membership functions have previously been used in research articles pertaining to application of fuzzy sets to robot selection and measuring manufacturing competence [21,31]. The triangular membership functions are chosen for application considering their intuitive representation and ease in computation. Nevertheless, the fuzzy decision-making algorithm presented herein is independent of the type of membership functions, and thus, trapezoidal or parabolic membership functions that are determined using the membership identification techniques discussed above could also be used.

Suppose that W_{tj} is the linguistic weight given to economic criteria $C_1, C_2, \dots, C_{k-s}$, and strategic justification criteria $C_{k-s+1}, \dots, C_k$ by decision maker D_j, and A_{itj} is the linguistic rating assigned to AMS alternative SA_i by decision maker D_j for the strategic justification criterion C_t. W_{tj} and A_{itj} are defined as follows, where s denotes the number of strategic justification criteria.

$$W_{tj} = (a_{tj}, b_{tj}, c_{tj}), \quad t = 1, 2, \dots, k; \; j = 1, 2, \dots, n,$$
$$A_{itj} = (o_{itj}, p_{itj}, q_{itj}), \quad i = 1, 2, \dots, m; \quad t = k-s+1, \dots, k; \; j = 1, 2, \dots, n.$$

Linguistic weights assigned to criteria and linguistic rating assessments made to AMS alternatives by the decision-makers are aggregated by the mean operator. We define W_t and A_{it} as follows:

$$W_t = (1/n) \otimes (W_{t1} \oplus W_{t2} \oplus \cdots \oplus W_{tn}), \quad t = 1, 2, \dots, k \tag{14}$$

$$A_{it} = \begin{cases} (o_{\mathrm{it}}, p_{\mathrm{it}}, q_{\mathrm{it}}) \\ \qquad i = 1, 2, \dots, m; \; t = 1, 2, \dots, k-s, \\ (1/n) \otimes (A_{it1} \oplus A_{it2} \oplus \cdots \oplus A_{itn}) \\ \qquad i = 1, 2, \dots, m; \; t = k-s+1, \dots, k. \end{cases} \tag{15}$$

Here, W_t is the weight of criterion C_t ($t = 1, 2, \dots, k$), whereas A_{it} is the rating of AMS alternative SA_i versus the economic criterion C_t ($t = 1, 2, \dots, k-s$), and the average linguistic ratings of AMS alternative SA_i for the strategic justification criterion C_t ($t = k-s+1, \dots, k$), respectively.

Let

$$a_t = \sum_{j=1}^{n} a_{tj}/n, \quad b_t = \sum_{j=1}^{n} b_{tj}/n, \quad c_t = \sum_{j=1}^{n} c_{tj}/n$$

for $t = 1, 2, \dots, k$, and

$$o_{it} = \sum_{j=1}^{n} o_{itj}/n, \quad p_{it} = \sum_{j=1}^{n} p_{itj}/n, \quad q_{it} = \sum_{j=1}^{n} q_{itj}/n$$

for $i = 1, 2, \dots, m$; $t = k-s+1, \dots, k$.

Then

$$W_t = (a_t, b_t, c_t), \quad t = 1, 2, \dots, k, \tag{16}$$

$$A_{it} = (o_{it}, p_{it}, q_{it}), \; i = 1, 2, \dots, m; \; t = 1, 2, \dots, k. \tag{17}$$

From here onwards a weighted mean operator is defined as follows:

$$F_i = (1/k) \otimes [(A_{i1} \otimes W_1) \oplus (A_{i2} \otimes W_2) \oplus \cdots \oplus (A_{ik} \otimes W_k)] \tag{18}$$

By Zadeh's extension principle, F_i is a fuzzy number defined by the following membership function:

$$f_{F_i}(x) = \begin{cases} -H_{i1} + [H_{i1}^2 + (x - Y_i)/T_{i1}]^{1/2}, \\ \qquad Y_i \leqslant x \leqslant Q_i, \\ H_{i2} - [H_{i2}^2 + (x - Z_i)/U_{i1}]^{1/2}, \\ \qquad Q_i \leqslant x \leqslant Z_i, \\ 0, \\ \qquad \text{otherwise}, \end{cases} \tag{19}$$

for $i = 1, 2, \dots, m$. F_i is not a triangular fuzzy number and would be defined as

$$F_i = (Y_i, Q_i, Z_i; H_{i1}, T_{i1}; H_{i2}, U_{i1}), \quad i = 1, 2, \dots, m.$$

The procedure for calculating Y_i, Q_i, Z_i, H_{i1}, T_{i1}, H_{i2} and U_{i1} are given in Appendix B. However, the following approximation, which will be employed throughout the paper, can be used to provide a triangular fuzzy number:

$$F_i \cong (Y_i, Q_i, Z_i), \quad i = 1, 2, \dots, m.$$

Finally, a ranking procedure is required to determine the ranking order of the AMS alternatives with respect to their fuzzy suitability index values. There exist a number of papers concentrating on the ranking of fuzzy sets, and in particular fuzzy numbers. Bortolan and Degani [32] present a comparative review of some of the methods for ranking fuzzy sets. They indicate that the methods generally provide consistent results for the simple examples while a number of problems are observed for the questionable cases. Jain [33] utilizes the concept of maximizing set, which takes into account both the maximum utility associated with various alternatives and the grade of membership of the utilities. A more sensitive rule, which also considers measuring the left trend by utilizing the minimizing set concept, is applied in Chen's method [34]. In this work, Chen's method for ranking fuzzy numbers is used, considering the consistency and ease of implementation.

Let $A_i = (a_i, b_i, c_i)$ be a triangular fuzzy number with the membership function given as follows:

$$f_{A_i}(x) = \begin{cases} 0, & x < a_i, \\ (x - a_i)/(b_i - a_i), & a_i \leqslant x \leqslant b_i, \\ (x - c_i)/(b_i - c_i), & b_i \leqslant x \leqslant c_i \\ 0, & x > c_i. \end{cases} \tag{20}$$

The membership functions of maximizing set M and minimizing set G are given in [34] as

$$f_M(x) = \begin{cases} [(x - x_{\min})/(x_{\max} - x_{\min})]^k, \\ \qquad x_{\min} \leqslant x \leqslant x_{\max}, \\ 0, \\ \qquad \text{otherwise}, \end{cases} \tag{21}$$

$$f_G(x) = \begin{cases} [(x - x_{\max})/(x_{\min} - x_{\max})]^k, \\ \qquad x_{\min} \leqslant x \leqslant x_{\max}, \\ 0, \\ \qquad \text{otherwise}, \end{cases} \tag{22}$$

where $x_{\max} = \sup S$, $x_{\min} = \inf S$, $S = \bigcup_{i=1}^{n} S_i$, and S_i is the support of A_i. The linear case is given by $k = 1$, while $k > 1$ represents risk-prone (convex) membership functions, and $0 < k < 1$ represents risk-averse (concave) membership functions. In here, the value of k is assigned to be 1. When $k = 1$, the ranking value of A_i is calculated using the following expression:

$$\begin{aligned} U_T(i) = 1/2[&(c_i - x_{\min})/((x_{\max} - x_{\min}) - (b_i - c_i)) \\ &+ 1 - (x_{\max} - a_i)/((x_{\max} - x_{\min}) \\ &+ (b_i - a_i))], \quad i = 1, 2, \ldots, n. \end{aligned} \tag{23}$$

Using Eq. (23), the ranking of the n triangular fuzzy numbers can be obtained on the basis of their respective $U_T(i)$.

5. Fuzzy multi-criteria AMS selection procedure

In this section, a fuzzy multi-criteria decision-making procedure integrating economic and strategic criteria for evaluating mutually exclusive AMS alternatives is proposed (Fig. 4). First, a decision-makers' committee is established, and incremental cash flows for the competing AMS alternatives, interest rate and inflation rates are estimated. Henceforth, strategic criteria such as reduction in floor space requirement, increase in process flexibility, increase in volume flexibility, and improvement in product quality, which pose difficulties in quantification, are integrated in the fuzzy decision-making procedure. The weights of the criteria, and the suitability of the AMS alternatives versus strategic criteria are determined. Then, a fuzzy present worth analysis is performed to evaluate the economic benefits of the AMS alternatives. Next, average linguistic ratings of the AMS alternatives for the strategic criteria are determined. Finally, the fuzzy suitability index values for the AMS alternatives are calculated, and the ranking of the AMS alternatives is obtained based on these index values.

The decision approach proposed in this section allows for use of crisp data when there is no subjectivity involved for certain criteria, and fuzzy data that may be expressed in linguistic terms or fuzzy numbers. The decision-making algorithm can be easily computerized and is capable of solving problems considering both a rich criteria set and a large number of alternatives.

6. An illustrative example

In this section, the fuzzy decision-making algorithm presented in the previous section is demonstrated via a numerical example. Suppose that a group of three decision-makers (D_1, D_2, D_3) is identified to evaluate the three AMS alternatives, namely three competing FMS proposals (SA_1, SA_2, SA_3) to replace the existing job shop consisting of an inventory area, a lathe, a milling machine, a drilling machine, a washing machine, an inspection machine, a load/unload area, a tool and fixture storage area, and a conveyor belt. Reduction in

Step 1. Identify the prospective AMS alternatives.
Step 2. Construct a decision-makers' committee and determine the incremental expected cash flows, interest rate, and inflation rates to be considered in the AMS selection problem.
Step 3. Determine the strategic attributes to be considered in addition to financial performance parameters.
Step 4. Determine the importance weights for the criteria.
Step 5. Calculate the aggregate weights for each criterion (W_t).
Step 6. Identify the suitability of the AMS alternatives versus strategic criteria.
Step 7. Calculate the economic criterion values (i.e. fuzzy after-tax present worth, fuzzy after-tax present worth of cost, etc.) of the AMS alternatives.
Step 8. Calculate the dimensionless indices for economic justification criterion.
Step 9. Compute the average linguistic ratings of the AMS alternatives for strategic justification criteria (A_{it}).
Step 10. Aggregate W_t and A_{it} with respect to each criterion, and then calculate the fuzzy suitability index values for the AMS alternatives.
Step 11. Rank the AMS alternatives with respect to their fuzzy suitability index values.

Fig. 4. The AMS selection algorithm.

floor space requirement, increase in process flexibility which indicates the ability to produce a given set of part types in several ways, increase in volume flexibility which indicates the ability to operate an FMS profitably at different production levels [35], and improvement in product quality are considered as strategic benefits along with the incremental cost and benefits reduced to monetary terms.

Since all the three FMS alternatives are expected to provide an approximately equal production level, the annual operating revenues will not be considered while calculating the economic figure of merit. The initial cost, and annual operating and maintenance cost estimates of the three FMS proposals for the five-year planning horizon are given as triangular fuzzy numbers in Table 1.

It is assumed that the FMS investment will be depreciated using the straight line method and a five-year life with zero salvage value at the end of the five years. The tax rate (t) is taken to be 45%. The after-tax interest rate is approximately 20%, and represented as $i = (17, 20, 22\%)$.

Inflation still appears to be the top ranking economic problem in certain countries. For instance, while investment in advanced manufacturing technologies is continuously in progress in Turkey, which can be noticed from the considerable increase in the percentage of manufactures as of total merchandise exports, the nation has been experiencing an average annual increase of 80.2% in consumer price index within the 1990–1996 period [36]. Hence, considering the persisting high inflation, a proper justification methodology for the FMS investment in such countries requires the differential rates of inflation to be taken into account. Consequently, expert estimates for the price level escalations related to the operating cost of the FMS alternatives, and general increase in the price levels for the five-year period are obtained. Triangular fuzzy numbers for annual specific rate of inflation estimates concerning cost items, and for annual general rate of inflation estimates are given in Table 2.

The decision-makers utilize the linguistic set of weights $W = \{VL, L, M, H, VH\}$ defined in Section 4 to identify the importance of the economic and strategic criteria. The weights assigned to the five criteria by the three decision-makers are given in Table 3.

The aggregate weights for each criterion are calculated by grouping the linguistic assessments of the three decision-makers. The aggregate weights calculated by employing Eq. (14) are given below:

$$W_1 = (0.567, 0.800, 1.000),$$

$$W_2 = (0.400, 0.633, 0.933),$$

$$W_3 = (0.633, 0.900, 1.000),$$

$$W_4 = (0.300, 0.567, 0.867),$$

$$W_5 = (0.467, 0.733, 0.933).$$

Table 1
Capital and operating cost estimates of the three FMS proposals (in millions of Turkish lira)

Year (j)	FMS alternative I (SA_1)	FMS alternative II (SA_2)	FMS alternative III (SA_3)
0	(25 000, 25 000, 25 000)	(30 000, 30 000, 30 000)	(27 500, 27 500, 27 500)
1	(9500, 10 500, 11 000)	(7500, 8000, 9000)	(10 500, 11 000, 11 500)
2	(10 500, 11 500, 12 000)	(8000, 9000, 9500)	(12 000, 12 500, 13 500)
3	(12 000, 13 000, 13 500)	(9000, 9500, 10 500)	(15 000, 15 500, 16 500)
4	(13 000, 14 000, 14 500)	(10 500, 11 000, 12 000)	(16 000, 17 000, 17 500)
5	(14 500, 15 000, 16 000)	(12 000, 12 500, 14 000)	(18 000, 18 500, 20 000)

Table 2
The fuzzy specific rates of inflation for cost items and fuzzy general inflation rates for the five-year period

$h_{C0} = (0, 0, 0)$	$f_0 = (0, 0, 0)$
$h_{C1} = (47, 50, 52\%)$	$f_1 = (63, 65, 68\%)$
$h_{C2} = (47, 50, 52\%)$	$f_2 = (60, 62, 65\%)$
$h_{C3} = (42, 45, 48\%)$	$f_3 = (57, 60, 62\%)$
$h_{C4} = (37, 40, 43\%)$	$f_4 = (47, 50, 52\%)$
$h_{C5} = (32, 35, 38\%)$	$f_5 = (37, 40, 42\%)$

Table 3
The importance of the decision criteria

	D_1	D_2	D_3
C_1	*VH*	*H*	*H*
C_2	*M*	*H*	*H*
C_3	*VH*	*H*	*VH*
C_4	*H*	*M*	*M*
C_5	*H*	*VH*	*M*

Table 4
The decision-makers' evaluation of the FMS alternatives with respect to reduction in floor space requirement (C_2)

	D_1	D_2	D_3
SA_1	*F*	*G*	*G*
SA_2	*G*	*VG*	*G*
SA_3	*P*	*F*	*P*

Table 5
The decision-makers' evaluation of the FMS alternatives with respect to increase in process flexibility (C_3)

	D_1	D_2	D_3
SA_1	*F*	*G*	*VG*
SA_2	*G*	*G*	*VG*
SA_3	*F*	*F*	*G*

The decision-makers use the linguistic variable set $A = \{VP, P, F, G, VG\}$, to assess the suitability of the three FMS alternatives under each of the strategic criteria. The linguistic ratings are presented in Tables 4–7, respectively.

The fuzzy after-tax present worth of cost is calculated for the three FMS proposals using the fuzzy DCF analysis presented in Section 3. The results are illustrated in Table 8.

In order to provide compatibility between economic justification criterion and linguistic ratings of strategic criteria, economic criterion values must be converted to dimensionless indices ($DISA_i$). The FMS alternative with the minimum after-tax present worth of cost should have the maximum rating value. The dimensionless indices for fuzzy after-tax present worth of cost are calculated by using Eq. (24), where P_i indicates the fuzzy after-tax present worth of cost of alternative i. The results are shown in Table 9.

$$DISA_i = \{P_i \otimes [P_1^{-1} \oplus P_2^{-1} \oplus \dots \oplus P_m^{-1}]\}^{-1}, \quad i = 1, 2, \dots, m. \tag{24}$$

Table 6
The decision-makers' evaluation of the FMS alternatives with respect to increase in volume flexibility (C_4)

	D_1	D_2	D_3
SA_1	*G*	*G*	*VG*
SA_2	*F*	*G*	*G*
SA_3	*F*	*P*	*F*

Table 7
The decision-makers' evaluation of the FMS alternatives with respect to improvement in product quality (C_5)

	D_1	D_2	D_3
SA_1	*VG*	*G*	*F*
SA_2	*G*	*G*	*VG*
SA_3	*G*	*F*	*G*

Table 8
Fuzzy after-tax present worth of cost of the FMS alternatives (in millions of Turkish lira)

SA_i	Present worth of cost (SA_i)
SA_1	(36 967, 38 854, 40 403)
SA_2	(38 351, 39 753, 41 717)
SA_3	(41 850, 43 359, 45 418)

Table 9
Dimensionless indices of economic justification criterion for the FMS alternatives ($DISA_i$)

FMS alternative (SA_i)	$DISA_i$
SA_1	(0.321, 0.348, 0.382)
SA_2	(0.311, 0.340, 0.369)
SA_3	(0.286, 0.312, 0.338)

Table 10
The average linguistic ratings of the FMS alternatives for strategic criteria

FMS alternative (SA_i)	Reduction in floor space ($t = 2$)	Increase in process flexibility ($t = 3$)	Increase in volume flexibility ($t = 4$)	Improvement in product quality ($t = 5$)
SA_1	(0.500, 0.700, 0.900)	(0.567, 0.767, 0.900)	(0.667, 0.867, 1.000)	(0.567, 0.767, 0.900)
SA_2	(0.667, 0.867, 1.000)	(0.667, 0.867, 1.000)	(0.500, 0.700, 0.900)	(0.667, 0.867, 1.000)
SA_3	(0.100, 0.300, 0.500)	(0.400, 0.600, 0.800)	(0.200, 0.400, 0.600)	(0.500, 0.700, 0.900)

The average fuzzy evaluation of the FMS alternatives concerning the strategic criteria (A_{it}, $i = 1, 2, 3$; $t = 2, 3, 4, 5$) is performed by using the linguistic assessments in Tables 4–7, and is demonstrated in Table 10.

Fuzzy suitability index values for the FMS alternatives are obtained by averaging the products of weights and linguistic ratings over all the criteria via a weighted mean operator. The results are presented in Table 11.

Eq. (23) is used to determine the ranking values of the three FMS alternatives. The ranking of the FMS alternatives are given in Table 12. It is observed that the ranking order is $SA_2 \succ SA_1 \succ SA_3$. SA_2 appears to be the most suitable FMS alternative as a result of the fuzzy multi-criteria decision procedure, and thus, is the first one to be considered for purchase. Here, we find out that an analysis based solely on economic measures would have resulted in the selection of the improper FMS alternative since SA_1, which yields the lowest after-tax present worth of cost of the three alternatives, should have been selected if the economic criterion was the only attribute considered in the analysis.

Table 11
Fuzzy suitability index values of the FMS alternatives

FMS alternative (SA_i)	Fuzzy suitability index
SA_1	(0.241, 0.493, 0.766)
SA_2	(0.265, 0.527, 0.803)
SA_3	(0.150, 0.344, 0.593)

Table 12
Ranking of the FMS alternatives using the fuzzy suitability index values

Ranking of SA_i	FMS alternative (SA_i)	Ranking value
1	SA_2	0.557
2	SA_1	0.522
3	SA_3	0.360

7. Conclusions

Diverse approaches have been proposed in the past decade to deal with evaluating AMS investment alternatives. The research articles pertaining to AMS evaluation focus primarily on the economic justification. Fuzzy discounted cash flow techniques are suggested for use in economic evaluation as an alternative to the traditional cash flow analysis. This paper presents a fuzzy present worth model for financial evaluation of AMS investments under conditions of inflation. The inaccuracy of the results obtained using the analysis ignoring inflation increases when after-tax cash flow analysis is performed. Consequently, differential rates of inflation are used in the fuzzy after-tax present worth model to properly account for the price-level changes.

Global competition in manufacturing environment has forced the firms to increase the quality and responsiveness to customization, while lowering the costs. An automated manufacturing system, when properly implemented, provides major strategic benefits to the manufacturing firm such as flexibility, improved product quality, and reduced lead time. In order to incorporate these notable benefits that cannot be reduced to monetary terms into the justification and selection process of the AMS, an integrated approach considering both economic and strategic criteria is required.

Since numerical estimates require more mental effort than linguistic descriptors, people are more likely to bias their evaluations if they are forced to provide numerical estimates of vague or imprecise items [37]. The strategic importance of the AMS investments can be effectively expressed using the fuzzy decision approach, as the fuzzy approach employs linguistic variables that are close to common language. In this paper, strategic criteria, which cannot be reduced to monetary terms, are integrated in the AMS selection procedure using a fuzzy multi-criteria decision-making algorithm. The fuzzy decision algorithm proposed in here helps to resolve the vagueness in AMS evaluation process by quantifying the non-monetary impacts. Hence, the decision-makers obtain a final ranking for the AMS alternatives by taking into account not only the economic criterion, but also the key strategic justification criteria by utilizing linguistic variables. The most suitable alternative among a set of mutually exclusive AMS investment alternatives is determined by applying a consistent and easily implemented method for ranking the fuzzy numbers.

The decision analysis presented in this paper can be easily computerized, and allows for assessment of AMS investments taking into account an excessive criteria set and a large number of alternatives. Considering its effectiveness in quantifying the vagueness and imprecision in human judgment, the fuzzy decision-making approach appears as a consistent and computational-efficient alternative to existing methods. Nevertheless, it is worth noting that the benefit obtained due to the ease in computation in applying fuzzy decision models may be balanced by a possible loss in precision since fuzzy models provide only best and worst case analysis and do not assume that errors compensate.

Appendix A

In this appendix, a brief introduction to the basic concepts of fuzzy sets and algebraic operations of triangular fuzzy numbers is presented.

A.1. Fuzzy sets

A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade corresponds to the degree to which individual is compatible with the concept represented by the fuzzy set. Hence, while a sharp distinction exists between the members and non-members of the class represented by the crisp set, the elements belonging in the fuzzy set do not possess that property.

The membership grades are frequently represented by real-number values ranging in the [0, 1] interval. A fuzzy subset A of a universal set X is defined by a membership function $f_A(x)$ which maps each element x in X to a real number in the [0, 1] interval. The function value $f_A(x)$ denotes the grade of membership of x in A, hence larger values imply higher degrees of set membership. A fuzzy set A is normal if and only if $\sup_x f_A(x) = 1$, otherwise it is subnormal. A fuzzy set is convex if and only if each of its α-cuts is a convex set. A fuzzy set A is defined as convex if and only if

$$f_A(\lambda x + (1-\lambda)y) \geqslant \min\,(f_A(x), f_A(y))$$

for $\forall x, y \in \Re^n$ and $\forall \lambda \in [0, 1]$. A fuzzy set $B(x)$ in $Y = \{y\}$ is conditioned on x if its membership function depends on x as a parameter which is expressed by $f_B(y \mid x)$. Using the mapping from X to the space of fuzzy sets in Y, any given fuzzy set A in X induces a fuzzy set B in Y that is denoted as

$$f_B(y) = \sup_x \min[f_A(x), f_B(y|x)],$$

where f_A and f_B indicate the membership functions of A and B, respectively.

A convex and normalized fuzzy set defined on $\Re$ whose membership function is piecewise continuous is called a fuzzy number. A fuzzy number is a triangular fuzzy number if its membership function can be represented as

$$f_A(x) = \begin{cases} 0, & x < a, \\ (x-a)/(b-a), & a \leqslant x \leqslant b, \\ (x-c)/(b-c), & b \leqslant x \leqslant c, \\ 0, & x > c. \end{cases}$$

A.2. The algebraic operations of triangular fuzzy numbers

Zadeh's extension principle is used to calculate membership function after mapping fuzzy sets through a function. The extension principle is defined as follows:

Let fuzzy sets $A_1, A_2, \ldots, A_n$ be defined on the universes $X_1, X_2, \ldots, X_n$. The mapping for these particular input sets can now be defined as $B = f(A_1, A_2, \ldots, A_n)$, where the membership function of the image B is expressed as

$$f_B(y) = \sup_{y = f(x_1, x_2, \ldots, x_n)} \{[\min f_{A_1}(x_1), f_{A_2}(x_2), \ldots, f_{A_n}(x_n)]\}.$$

Using the extension principle given above, the extended algebraic operations of the triangular fuzzy numbers $A_1 = (a_1, b_1, c_1)$, where $a_1 \leqslant b_1 \leqslant c_1$, and $A_2 = (a_2, b_2, c_2)$, where $a_2 \leqslant b_2 \leqslant c_2$, can be specified as follows:

Image of A_1:

$$-A_1 = (-c_1, -b_1, -a_1).$$

Inverse of A_1: If $a_1 > 0$,

$$A_1^{-1} \cong \left(\frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1}\right).$$

Addition: If $\oplus$ denotes extended addition,

$$A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2).$$

Subtraction: If extended subtraction is defined by Θ,

$$A_1 \,\Theta\, A_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2).$$

Multiplication: If k is a scalar constant and $\otimes$ denotes extended multiplication,

$$k \otimes A_1 = \begin{cases} (ka_1, kb_1, kc_1), & k > 0, \\ (kc_1, kb_1, ka_1), & k < 0, \end{cases}$$

$$A_1 \otimes A_2 \cong (a_1 a_2, b_1 b_2, c_1 c_2) \quad \text{if } a_1 \geqslant 0, a_2 \geqslant 0.$$

Division: If $a_1 \geqslant 0$, $a_2 > 0$ and $\varnothing$ denotes extended division,

$$A_1 \varnothing A_2 \cong \left(\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2}\right).$$

The approximations can be improved as more α-cuts are utilized; however, more α-cuts are likely to increase the complexity of computation in parameter estimations. Hence, only two values of α ($\alpha = 0$ and 1) are considered here. For a further treatment in theory of fuzzy numbers and its applications in decision-making, the reader is referred to texts devoted to this field [38,39].

Appendix B

In Section 4, a fuzzy number $F_i = (Y_i, Q_i, Z_i; H_{i1}, T_{i1}; H_{i2}, U_{i1})$ is obtained employing Zadeh's extension principle. The formulas used for calculating the elements of the fuzzy number F_i are given below.

$$T_{i1} = \sum_{t=1}^{k} (p_{it} - o_{it})(b_t - a_t)/k,$$

$$T_{i2} = \sum_{t=1}^{k} [o_{it}(b_t - a_t) + a_t(p_{it} - o_{it})]/k,$$

$$U_{i1} = \sum_{t=1}^{k} (q_{it} - p_{it})(c_t - b_t)/k,$$

$$U_{i2} = \sum_{t=1}^{k} [c_t(p_{it} - q_{it}) + q_{it}(b_i - c_i)]/k,$$

$$Y_i = \sum_{t=1}^{k} o_{it} a_t/k,$$

$$Q_i = \sum_{t=1}^{k} p_{it} b_t/k,$$

$$Z_i = \sum_{t=1}^{k} q_{it} c_t/k,$$

$$H_{i1} = T_{i2}/(2T_{i1}),$$

$$H_{i2} = -U_{i2}/(2U_{i1}).$$

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