Pricing Construction Risk: Fuzzy Set Application

By James H. Paek,[1] Associate Member, ASCE, Yong W. Lee,[2] Member, ASCE, and Jong H. Ock,[3] Student Member, ASCE

Abstract: Contractors may be faced with the problem of deciding the bidding price of a construction project, when the likelihood of the occurrence of risk events and the risk-associated consequences (monetary loss and gain) are uncertain. If the monetary loss resulting from risk events is not considered or is underestimated due to associated uncertainties, a construction enterprise may suffer a tremendous loss and eventually fail. In this paper, a risk-pricing method is proposed for analyzing and pricing construction project risk. The method consists of identifying risk elements and quantifying risk-associated consequences. The uncertainty in the values of the quantified consequences are represented by using a fuzzy set approach and incorporated directly into the bidding price decision process. The proposed risk-pricing method will assist contractors in the process of estimation under uncertainty. A real construction project is selected to illustrate how the proposed method can be put into practice.

INTRODUCTION

Contracting is an ideal business for those who enjoy taking chances. However, when construction projects (such as environmental remediation and mass transit system projects, and sometimes tunnel construction projects) are complex and risky, contractors are faced with a variety of situations involving many unknown, unexpected, frequently undesirable, and often unpredictable factors. In this case, contractors are subject to cost increases that are not seen in construction projects with a relatively low degree of risk (such as standard commercial building projects). These cost increases can be attributed to the uncertainties in every element of risk analysis such as limited engineering data, insufficient construction knowledge, and lack of documentation on operational history.

As construction industries become involved in the super projects of the private business sector, they are often required to furnish conceptual estimates to private owners. The industries then provide owners with a risk analysis estimate to determine a project's feasibility, or when owners are on a fast-track schedule and the design is still in the preliminary stages but the project must be completed within a set time frame. Such situations have served to renew interest in a more professional approach to new ways of defining the uncertainties and risk-cost analysis schemes required for successfully managing construction projects.

An important requirement for successful management in the contracting field is effective evaluation of the risks involved in the construction project, followed by sound decisions based on this evaluation, and appropriate action taken as a result of these decisions. To calculate risk-associated conse-

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[1]Asst. Prof., Dept. of Constr. Mgmt., Univ. of Nebraska, Lincoln, NE 68588-0500.

[2]Res. Associate, Dept. of Civ. Engrg., Univ. of Nebraska, Lincoln, NE.

[3]Grad. Res. Asst., Dept. of Civ. Engrg., Univ. of Nebraska, Lincoln, NE.

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quences (monetary losses and gains), most engineers and contractors generally use historical records and their know-how by direct experience in the construction field (Al-Bahar and Crandall 1990). In many cases, however, directly applicable historical data are inadequate or unavailable, and then the subjective judgment of experts with the relevant knowledge and experience is used. Even in this case, most of the data evaluated are uncertain because the historical records for a risk event are usually different for every project case, and the subjective judgment of experts varies from one expert to another (Al-Bahar and Crandall 1990).

Several methods can be used for uncertainty analysis. These include: (1) Probabilistic analysis; (2) interval analysis; and (3) fuzzy set analysis (Zimmermann 1987; Lee et al. 1991). In probabilistic analysis, statistical measures (mean and standard deviation) of input variables are used to estimate the mean and standard deviation of the results. This method is precluded if the input variables have a nonprobabilistic nature. Interval analysis uses ranges (intervals) for input variables to estimate plausible ranges of the results. However, it is difficult to define the intervals of input variables when the boundaries of the intervals are uncertain. A fuzzy set approach, pioneered by Zadeh (1965), is useful for uncertainty analysis where a probabilistic data base is not available and/or when (interval) values of input variables are uncertain. The fuzzy set approach has been widely applied to represent the uncertainties of real-life situations (Bogardi and Bardossy 1983; Anandalingam and Westfall 1988; Paek et al. 1992).

In this paper, the uncertainty in risk-associated consequences and its impact on results are characterized by an application of fuzzy set theory and incorporated into the bidding price decision process. The specific objectives of this paper are twofold. The first objective is to develop a risk-pricing method for quantifying risk-associated consequences under uncertainty and then incorporating the quantified consequences into the bidding price decision process. The second objective is to show how the risk-pricing method can be put into practice by applying the method to a real construction project.

FUZZY SETS AND FUZZY NUMBERS

Fuzziness or uncertainty represents situations where membership in sets cannot be defined on a yes/no basis because the boundaries of the sets are vague. The central concept of fuzzy set theory is the membership function, which represents numerically the degree to which an element belongs to a set. In a classical set, a sharp or unambiguous distinction exists between the members and nonmembers of the set. In other words, the value of the membership function of each element in the classical set is either 1 for members (those that certainly belong to the set) or 0 for nonmembers (those that certainly do not). However, it is sometimes difficult to make a sharp or precise distinction between the members and nonmembers of a set. For example, the boundaries of the sets of very risky words, nice houses, or numbers much greater than 1.0 are fuzzy.

Since the transition from member to nonmember appears gradual rather than abrupt, the fuzzy set introduces vagueness (with the aim of reducing complexity) by eliminating the sharp boundary dividing members of the set from nonmembers (Klir and Folger 1988). Thus, if an element is a member of a fuzzy set to some degree, the value of its membership function can be between 0 and 1. When the membership function of an element can only have values 0 or 1, the fuzzy set theory reverts to the classical set theory.

A special class of fuzzy sets is described by fuzzy members, which are values that belong to a given set with a certain degree of membership only. As an example of fuzzy members, let Q be a fuzzy number and its membership function be denoted by (Fig. 1):

$$\mu(Q) = 1 - \frac{(q - Q)}{\delta}, \qquad q - \delta \le Q \le q \quad \text{(1a)}$$

$$\mu(Q) = 1 - \frac{(Q - q)}{\gamma}, \qquad q \le Q \le q + \gamma \quad \text{(1b)}$$

$$\mu(Q) = 0, \qquad \text{otherwise} \quad \text{(1c)}$$

where q = the center value of the fuzzy number Q, and δ ($\delta > 0$) and γ ($\gamma > 0$) represent the left and right fuzzinesses from the center value q. When the values of δ and γ are equal to zero, Q = a nonfuzzy number by convention. As the values of δ and γ increase, Q becomes fuzzier and fuzzier.

CONSTRUCTION RISK ANALYSIS

The risk analysis consists of three distinct stages: (1) The identification of risk elements (seeing all foreseeable risks); (2) the quantification of risk-associated consequences (measuring monetary losses and gains resulting from risk events); and (3) risk control (reducing or removing the exposure of a given risk by substituting proven technology for experimental technology) (Park 1979).

Construction projects vary widely in their inherent risks. Standard commercial building projects have a relatively low degree of risk. Projects that are very sensitive to subsurface information, i.e., tunnels, have a higher degree of risk. Also, environmental remediation projects that have an undefined work scope, or are subject to changing government regulation, have a very high degree of risk (Frano 1991). Some risks are very common, for example, the unit price for the cost of concrete is almost always wrong, if only by a small amount. On the other hand, some risks are significantly less common, such as the likelihood that a hurricane will damage a job site. Loss refers to the damages sustained if the risk occurs. The common risks

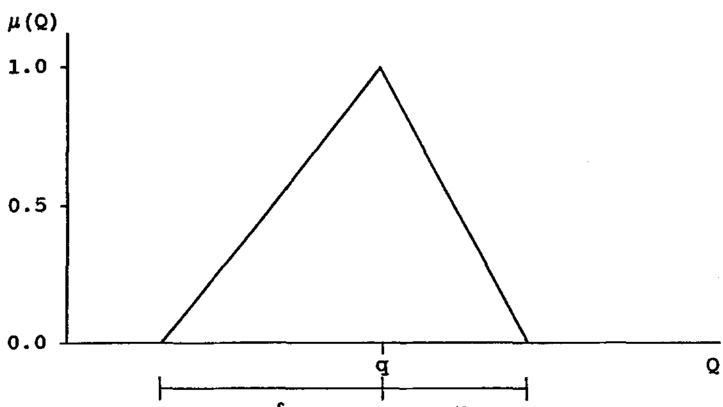

FIG. 1. Membership Function of Fuzzy Number Q

(i.e., an incorrect unit price for concrete) have relatively low losses, while the less common risks (i.e., a hurricane) have very large losses.

Some risks are well known enough that they can be insured against, with the contractor thereby assuming a direct cost and transferring the risk to an insurance company. Other risks, however, must be borne by the contractor.

The primary consideration in evaluating risks involves the potential returns or benefits that can be gained by taking the risks. In considering the risk of loss against the possibility of profit, the possibility of the occurrence of the risk events must first be estimated; next, the consequences of each risk on the business must be appraised; then, the cost of reducing risks by different degrees should be determined; and, finally, a judgment is necessary concerning whether the long-run profit expectation justifies the risk involved.

STRATEGIES OF CONSTRUCTION RISK MANAGEMENT

Risk management strategies are concerned with designing management actions to better understand the impact of the identified risk by modifying either the exposure or loss aspects of the risky situation. The strategies of risk management consist of three distinct stages: risk prevention and reduction, risk transfer, and risk retention (Paek 1991).

In addition to risk characteristics of the project, risk management strategy must address several subsidiary issues. One important question is the viewpoint adopted during the risk analysis. For example, contract language transfers risk from one party to another. Risk must be viewed from a specific point of view relative to contract language. Risk management strategy must also recognize the underlying goals of the analysis. Some analyses are conducted to help in go/no-go decisions for projects, and others are conducted to provide a basis for proactive projectwide risk management. Risk management strategy must also take into account such fundamental items as the amount of time available for the analysis, the data available, and the expertise available to the project. A final issue that must be considered is the fact that people need help in making accurate judgments about risk-associated consequences.

Risk management strategy must account for all of the preceding factors and then provide specific instructions on how to conduct risk assessment study. Specific information and procedures must be provided concerning identifying risks by measuring the uncertainties of the risk management process.

RISK-PRICING ALGORITHM

To determine the bidding price of a construction project, contractors traditionally use the following simple calculation:

$$BP = AC + PF \quad \text{(2)}$$

where BP = bidding price (stipulated price); AC = actual cost (direct, indirect, and overhead cost) to perform a construction project; and PF = contractor's profit.

Eq. (2) is clear and easy to implement. However, contractors are usually faced with the problem of deciding the bidding price (BP) when there are risk elements consisting of positive risks (potential monetary losses) and negative risks (potential monetary gains) in a project work, and their corresponding consequences are uncertain. The risk elements involved and their consequences may come both from historical records that show a construction firm's experience in past projects and from the opinion of experts. In many cases, however, directly applicable historical records concerning the risks and their consequences are not available in adequate amount, and the opinion of experts varies from person to person. In this case, it is difficult to precisely estimate risk-associated consequences.

Fig. 2 shows a framework for pricing construction risks while considering risk-associated consequences. In this paper, risk-associated consequences are estimated as fuzzy numbers (fuzzy consequences) to reflect their uncertainty. As mentioned earlier, the fuzzy numbers are values that belong to a given set (interval) with a certain degree of membership only. The uncertainty in the consequence of a risk event may be represented by two intervals (i.e., the most likely interval and the largest likely interval), which can be determined on the basis of the variability reflected in the historical records that contractors experienced in past projects and also the judgments of experts with relevant experience and knowledge. The two intervals can then be used to construct the membership function of the risk-associated consequence, as shown in Fig. 3.

To provide an example for showing how both "most likely" and "largest likely" intervals can be determined, let us assume five engineers participate in the process of evaluating subsurface conditions. Due to a lack of sub-

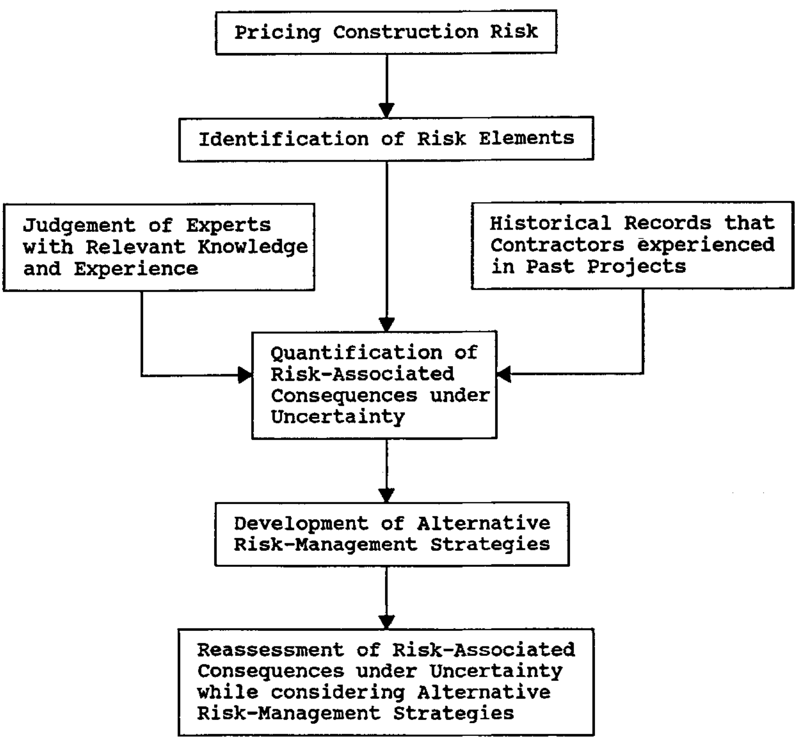

FIG. 2. Framework for Pricing Construction Risk

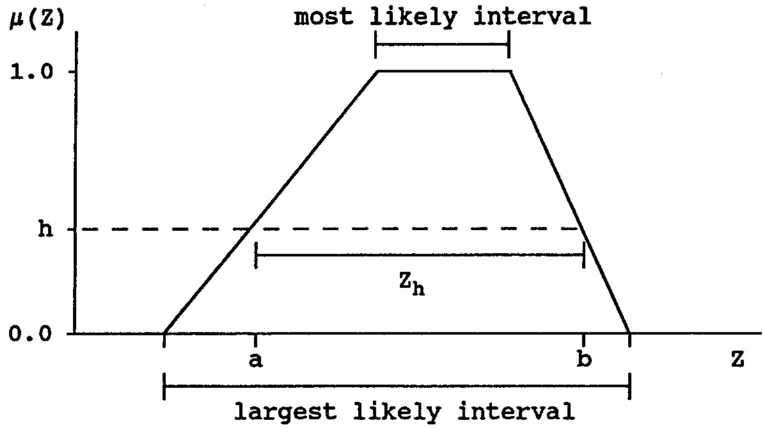

FIG. 3. Estimate of Imprecise Consequence ***Z*** **as Fuzzy Number**

surface information, each of them can differently interpret subsurface conditions and thus five different estimations can be made on the quantity of the potential extra excavation material when the unexpected case occurs. Further, if costs for transporting the five different amounts of material such as 1,040 m^3, 1,380 m^3, 1,530 m^3, 1,760 m^3, and 2,150 m^3 are \$3,120, \$4,140, \$4,590, \$5,280, and \$6,450, respectively, then the largest likely interval of the monetary loss resulting from the extra excavation material may be the range between the minimum and maximum values (i.e., \$3,120 and \$6,450) among the five values, and the most likely interval may be the range between the two values (i.e., \$4,290 and \$5,280) around the average of the five values.

For a mathematical formulation, let Z be a risk-associated consequence, $\mu(Z)$ be the membership function of Z, and Z_h be the domain (interval) of the consequence at the membership degree h (i.e., $a \leq Z_h \leq b$) (Fig. 3). If the trapezoidal shape is reduced to a vertical, line, it represents a crisp (nonfuzzy) number. However, it should be noted that the value of the consequence, Z, can be changed when contractors reduce or remove their liability for the consequence by adopting a proper risk-management strategy such as risk prevention, risk retention, and risk transfer.

With the views just described, the net loss, $L_{i,h}$, resulting from the ith positive risk event can be calculated as:

$$L_{i,h} = [Z_{i,h} - (D_{i,h}|S_i)] \quad (3)$$

where $Z_{i,h}$ = total monetary loss involved in the occurrence of the ith positive risk event; $D_{i,h}$ = the monetary amount that can be reduced or avoided by adopting the risk management strategy S_i for managing the ith positive risk event; and h = membership degree ($0 \leq h \leq 1$).

For example, assuming that the total monetary loss, $Z_{i,h}$, resulting from the ith positive risk event is from \$120,000 to \$180,000 and that a contractor retains all of the monetary loss [i.e., $(D_{i,h}|S_i) = 0$], then $L_{i,h}$ corresponds to $Z_{i,h}$. However, if the contractor can reduce the liability by one third of the monetary loss, $Z_{i,h}$, by adopting the risk transfer strategy, S_i, then $(D_{i,h}|S_i)$ = (\$80,000, \$120,000) and thus $L_{i,h}$ = (\$40,000, \$60,000).

In a formula similar to (3), the net gain, $G_{j,h}$, resulting from the jth negative risk event can be obtained by:

$$G_{j,h} = [Z_{j,h} - (D_{j,h}|S_j)] \quad (4)$$

where $Z_{j,h}$ = total monetary gain involved in the occurrence of the jth negative risk event; and $D_{j,h}$ = the monetary amount that can be reduced or avoided by adopting the risk management strategy S_j for managing the jth negative risk event.

The total net loss, T_h, involved in a construction project work can then be calculated by subtracting the sum of the net gains from the sum of the net losses, or

$$T_h = \sum_{i=1}^{m} L_{i,h} - \sum_{j=1}^{n} G_{j,h} \quad (5)$$

where m = the number of positive risk events; and n = the number of negative risk events.

In (5), the total net loss, T_h, varies with the membership degree h. With the help of two values $T_{h=0}$ and $T_{h=1}$, the total net loss, T, can be estimated as a fuzzy number with the following membership function (Lee et al. 1991) (Fig. 4):

$$\mu(T) = 1, \qquad \alpha < T < \beta \quad (6a)$$

$$\mu(T) = \frac{(T - A)}{(\alpha - A)}, \qquad A \leq T \leq \alpha \quad (6b)$$

$$\mu(T) = \frac{(T - B)}{(\beta - B)}, \qquad \beta \leq T \leq B \quad (6c)$$

$$\mu(t) = 0, \qquad \text{otherwise} \quad (6d)$$

where A and B = the lower- and upperbound values, respectively, of the $T_{h=0}$; and α and β = the lower- and upperbound values, respectively, of the $T_{h=1}$.

Obviously, the bidding price of a construction project should be decided as a point or crisp value (not interval value), but the total net loss, T, estimated as a fuzzy number, may be an interval value (i.e., $A \neq B$) restricted by the membership function of (6). In this case, the fuzzy number T should be transferred into a crisp value that represents the fuzzy number T. In this paper, a ranking method is used to transfer the fuzzy number T

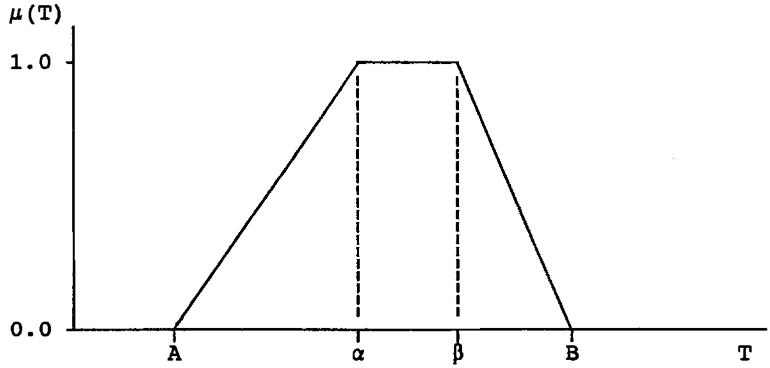

FIG. 4. Membership Function for Total Net Loss T

into the crisp value RC, making the ranking value of the fuzzy number T equal to that of the crisp value RC. Using the ranking method developed by Chen (1985) (Appendix I), the crisp value RC can be expressed as:

$$RC = \frac{V1 + V2}{2(W1 + W2)} \quad (7)$$

where

$$V1 = B^3(B + 3\alpha - 3A) - B^2(4\alpha A + \beta A + \alpha\beta) \quad (8a)$$

$$V2 = A^3(3B - 3\beta - A) + A^2(4\beta B + \alpha B + \alpha\beta) \quad (8b)$$

$$W1 = B^2(2B - 7A - \beta + 2\alpha) + 3(AB)(\beta - \alpha) \quad (8c)$$

$$W2 = A^2(7B - 2A - 2\beta + \alpha) - (\alpha\beta)(B - A) \quad (8d)$$

It should be noted that L_{min} and L_{max} in Appendix I correspond to A and B (Fig. 4), respectively. If the fuzzy number T has the membership function of a symmetric shape, that is $(\alpha - A) = (B - \beta)$ on Fig. 4, then the crisp value RC corresponds to the medium value between A and B, that is, $RC = (A + B)/2$. Furthermore, if $A = B$, then T in itself is a crisp value. In this case, RC is directly equal to T (i.e., $RC = T$).

To provide a numerical example for explaining what the calculation by (7) produces, that is, what the crisp value RC on the left side of (7) means, let the values of the variables A, B, α, and β (Fig. 4) be A = \$1,000,000, B = \$8,000,000, α = \$1,800,000, and β = \$2,200,000 (Fig. 5). Because the bidding price of a construction project is a crisp value (not interval value), the total net loss T should also become a crisp value. As shown in Fig. 5, the largest likely interval for T is from \$1,000,000 to \$8,000,000. When a contractor chooses a value among values constructing the interval, he or she might select the value A = \$1,000,000 with the most optimistic view of risk events, or the value B = \$8,000,000 with the most pessimistic view. However, the selection of the value A may result in a tremendous loss, while the selection of the value B may fail to win the bid. To overcome these problems, the value M = \$4,500,000 or k = \$2,000,000 (Fig. 5) could then be considered as the value that represents the total net loss ranging from \$1,000,000 to \$8,000,000, where M = the medium value of the largest likely interval for T, that is, $M = (A + B)/2$, and k = the medium value

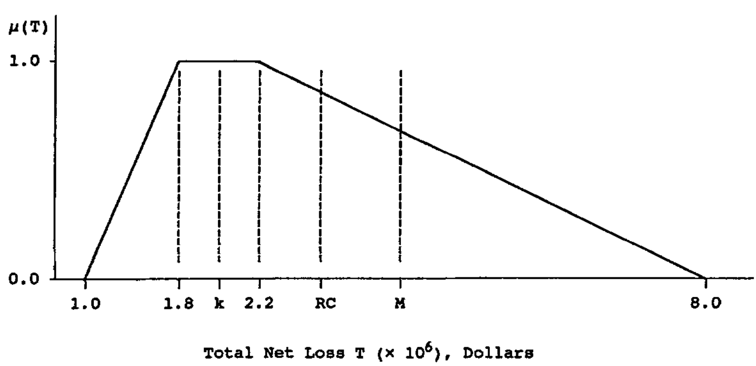

FIG. 5. Numerical Example for Explaining Meaning of *RC*

of the most likely interval for T, that is, $k = (\alpha + \beta)/2$. However, the selection of the value M is to take no account of the most likely values to occur (i.e., values between $1,800,000 and $2,200,000), while the selection of the value k is to pay no attention to the great difference between k = $2,000,000 and B = $8,000,000. Therefore, a compromised value between k and M should be determined. The compromised value (i.e., RC on Fig. 5) can then be calculated using (7), which was derived by applying the ranking method described in Appendix I. In the case of the present example, the value RC = $3,273,037. Always, the value RC is a crisp number and takes a value between $k = (\alpha + \beta)/2$ and $M = (A + B)/2$.

On the basis of the foregoing, the value RC may be added in the right side of (2) in order to remove or reduce the contractor's loss resulting from the risk elements associated with a construction project, or

$$BP = AC + PF + RC \quad \text{(9)}$$

If the value RC has a negative sign (i.e., $RC < 0$), it means that the sum of the gains is greater than the sum of the losses; otherwise, the sum of the losses is greater than the sum of the gains.

CASE STUDY

A real project for urban highway construction was chosen to show how computerized risk-pricing algorithm [(3)–(9)] can be put in practice. Since most of the input data used in the case study are proprietary information, we will call the UHCOC project.

The UHCOC project is for the construction of 32.18 km highway with 80 various types of bridge structures, which include approximately 21,870,000 m^3 of excavation material, 90,000 m^2 of retaining wall, and 45,000 m of various sizes of drainage pipes. The size of the proposed construction project was projected to be around $800 million, along with a completion schedule of 1,500 calendar days including the design phase. The UHCOC project was to be financed by local private owners raising the funds by issuing local bonds and charging road tolls.

One of the major construction incentives in the contract was the potential for sharing revenues generated from road tolls if the project could be completed early. Since the project's contract volume was very large and the job site was in a densely populated area, the private owners asked for a thorough and professional risk analysis. Major considerations in the analysis of contingencies to cover risks in the estimation process on this construction project were:

1. There had to be a 20% involvement by DBE (disadvantaged business enterprises (DBE).
2. The project had to pass inspection by the California Department of Transportation, even though financed totally by private owners.
3. 35% of the design was completed at the time of bid letting.
4. There was a liquidated damage clause in the contract, $200,000 per day extended up to maximum 500 days after original completion schedule.
5. All right-of-ways were not acquired at bid time.

The estimators, from the construction firm that bid the project, provided their own assessment of the major risk elements and the monetary consequences associated with each risk element. Also, many key joint venture

TABLE 1. Monetary Loss Estimates Related to Positive Risk Elements

Positive risk elements (1)	Most likely range[a] (2)	Largest likely range[a] (3)
(a) Estimation Risks		
Topsoil quantity overrun—150,000 cu yd	\$285–\$315	\$255–\$345
Additional retaining walls and pilings under retaining walls	\$4,500–\$5,250	\$3,500–\$5,500
Additional wickdrain pipe	\$142–\$150	\$120–\$150
Additional remedial excavation in lieu of wickdrain pipe	\$1,800–\$2,000	\$1,400–\$2,400
Rock quantity overrun—drill and shoot by 25%	\$3,230–\$3,570	\$2,550–\$4,250
Additional 1-mi hauling distance of drill and shoot rock	\$2,375–\$2,625	\$2,000–\$3,000
Disposal fee \$1.0/cu yd for drill and shoot rock	\$4,753–\$5,047	\$4,165–\$5,635
Increase in all storm drainage pipe by 6 in.	\$1,170–\$1,430	\$1,040–\$1,560
Increase in reinforced concrete pipe quantity by 15%	\$1,615–\$1,700	\$1,360–\$1,700
(b) Nonestimation Risks		
Schedule acceleration	\$6,750–\$7,500	\$5,250–\$8,625
DBE by 20%	\$900–\$1,000	\$800–\$1,150
Design growth (e.g., bigger cuts, more bridges)	\$5,100–\$6,600	\$3,000–\$7,500
Design/approval delays	\$3,600–\$4,400	\$2,800–\$5,200
Regulatory agencies	\$4,750–\$5,250	\$3,750–\$6,000
Disposal of excess materials	\$4,750–\$5,000	\$4,250–\$5,500

[a]Each of monetary losses described should be multiplied by 1,000.

partners related to the UHCOC project gave their subjective judgment of the likely interval for each of risk-associated consequences. Tables 1 and 2 show the monetary losses and gains, respectively, associated with the risk elements in the UHCOC project. Usually, the "worst case scenario" approach is used for risk-associated loss estimates (i.e., the use of the upper-bound values of potential losses), and the "best case scenario" approach for risk-associated gain estimates (i.e., the use of the upper-bound values of potential gains). However, it may be difficult to precisely determine the upper-bound values of risk-associated consequences (monetary losses and gains) as point or crisp numbers when uncertainties exist in a construction project, as in the case of the UHCOC project.

To represent the uncertainty in the upper-bound value of the consequence resulting from each risk event related to the UHCOC project, both "most likely" and "largest likely" intervals for each consequence (Tables 1 and 2) were estimated by adopting the risk management strategy that the contractor retains all of each consequence, that is, $(D_{i,h}|S_i) = 0$ [(3)] and $(D_{j,h}|S_j) = 0$ [(4)]. These two intervals were then used to construct the membership function (Fig. 3) for each consequence. Tables 1 and 2 have two different kinds of risk elements: estimation risks and nonestimation risks. The estimation risks were defined as the quantifiable risks such as additional/less

TABLE 2. Monetary Gain Estimates Related to Negative Risk Elements

Negative risk elements (1)	Most likely range[a] (2)	Largest likely range[a] (3)
(a) Estimation Risks		
Less remedial excavation in lieu of wick-drain pipe	\$297–\$300	\$285–\$300
Less retaining walls and pilings under retaining walls	\$3,800–\$4,200	\$3,200–\$4,600
Fatten slopes on site waste from drill and shoot rock	\$2,700–\$3,000	\$2,400–\$3,000
Less tire/track/repair cost	\$1,067–\$1,133	\$935–\$1,265
Less equipment maintenance cost	\$1,140–\$1,260	\$996–\$1,404
Piling reduction by 6 ft per pile under bridge	\$873–\$900	\$720–\$900
Replace 78R-value rock with 50R-value rock	\$2,185–\$2,300	\$1,725–\$2,415
(b) Nonestimation Risks		
Schedule deceleration	\$4,750–\$5,000	\$3,750–\$5,750
Less design/approval delays	\$1,800–\$2,200	\$1,400–\$2,600

[a]Each of monetary gains described should be multiplied by 1,000.

remediation excavation in lieu of wickdrain pipes, additional/less retaining walls, and rock quantity overrun, etc. For example, retaining wall requirements increase/decrease in retaining wall quantities due to variations in advanced ground conditions. The nonestimation risks are nonquantifiable risks such as design growth, schedule acceleration/deceleration, design/approval delays. An example of nonestimation risks would be untimely design/approval delays that would slow the startup of work and result in overtime costs to stay on schedule. Conversely, early design/approval could save costs. As of September 1992, since the construction work has not been started, the exact numbers for take-off quantities and unit prices for each risk element have been held confidential at the firm's request.

In conjunction with this paper, a microcomputer-based software was developed to help decision makers incorporate the uncertainty in risk-associated consequences into the bidding price decision process. The software utilizes the risk-pricing algorithm described in the previous section and requires input data in the form shown in Tables 1 and 2. With the help of Tables 1 and 2, the software was used to calculate the "most likely" and "largest likely" intervals for total net loss and also the crisp value RC [(7)] that represents the total net loss estimated as a fuzzy number. Results of the calculation showed that the most likely interval is from \$25,427,000 to \$33,225,000 (i.e., α = \$25,427,000 and β = \$33,225,000 on Fig. 4), the largest likely interval from \$14,006,000 to \$43,104,000 (i.e., A = \$14,006,000 and B = \$43,104,000 on Fig. 4), and RC = \$28,968,350.

CONCLUSIONS

In this paper, a risk-pricing algorithm was provided to assist contractors in deciding the bidding price of a construction project where the consequences resulting from risk events related to the project are uncertainty. A microcomputer-based software, based on the risk-pricing algorithm, was developed. To use the software, it is not required that users (i.e., contractors) understand the algorithm. However, the algorithm would be valuable to some researchers and will be helpful, at least in understanding the results of calculation by the software, to the users of the software.

As shown in the case study, assessing risk-associated consequences contains elements of uncertainty. This paper shows that the uncertainty inherent in these elements and its impact on results can be characterized by applying fuzzy set theory and incorporated directly into the bidding price decision process.

Since the selection of risk elements tends to be case specific, realistic generalizations are hard to formulate. In other words, the kind of risk elements varies with the kind of construction projects, and the number of risk elements depends largely on the level of analysis desired (preliminary or detailed). However, care must be taken to identify all the risk elements where undesirable outcomes might fatally flaw a construction project, and it must also be taken to remove a potential of biased inputs.

For the case study, if the contractor takes the risk-management strategy that he retains all of the total net loss, the potential loss varies from $14,006,000 to $43,104,000, and then the value RC = $28,968,350, which is the value that represents the potential loss ranging from $14,006,000 to $43,104,000. Therefore, the value RC = $28,968,350 may be added in the final bidding price to remove the contractor's potential loss resulting from the risk elements associated with the UHCOC project. However, the final result (i.e., the value of RC) can be changed when the contractor reduces or removes his liability for the consequences by adopting other risk-management strategies such as risk prevention and risk transfer. In other words, the final result is sensitive to the adopted risk-management strategies.

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APPENDIX I. RANKING METHOD

When there are n fuzzy numbers $[L(x), x = 1, \ldots, n]$, the ranking method developed by Chen (1985) has been used to determine the ranking of the n fuzzy numbers. The method determines the ranking of n fuzzy numbers by using a maximizing set and a minimizing set (Fig. 6).

The maximizing set M is fuzzy subset with membership function $\mu_M(L)$ given as:

$$\mu_M(L) = \frac{L - L_{\min}}{L_{\max} - L_{\min}}, \qquad L_{\min} \le L \le L_{\max} \quad \text{(10a)}$$

$$\mu_M(L) = 0, \qquad \text{otherwise} \quad \text{(10b)}$$

where for $x = 1, \ldots, n$, $L_{\min} = \min(\min L_{h=0}[x])$ and $L_{\max} = \max(\max L_{h=0}[x])$. The right ranking value, $U_R(x)$, for the fuzzy number $L(x)$ is then defined as:

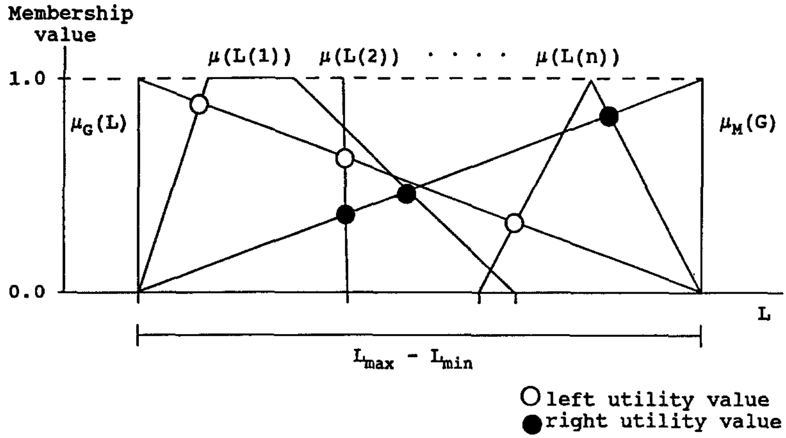

FIG. 6. Ranking Method of Fuzzy Numbers

$$U_R(x) = \max(\min\{\mu_M(L), \mu[L(x)]\}) \qquad (11)$$

The minimizing set G is a fuzzy subset with membership function $\mu_G(L)$ given as:

$$\mu_G(L) = \frac{L - L_{\max}}{L_{\min} - L_{\max}}, \qquad L_{\min} \le L \le L_{\max} \qquad (12a)$$

$$\mu_G(L) = 0, \qquad \text{otherwise} \qquad (12b)$$

The left ranking value, $U_L(x)$, for the fuzzy number $L(x)$ is then defined as:

$$U_L(x) = \max(\min\{\mu_G(L), \mu[L(x)]\}) \qquad (13)$$

Finally, the ranking or ordering value for the fuzzy number $L(x)$ then is written as:

$$U(x) = \frac{[U_R(x) + 1 - U_L(x)]}{2} \qquad (14)$$

APPENDIX II. REFERENCES

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