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TECHNICAL NOTE --

A CLARIFICATION OF THE CONCEPTS OF "INCOME, WEALTH BASE, AND RATE OF RETURN IMPLICATIONS OF ALTERNATIVE PROJECT EVALUATION CRITERIA"

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ABSTRACT

A recent paper by Bernhard [*The Engineering Economist*, 38 (Spring 1993) 3, pp. 165-176)] asserts that the NPV *decision criterion*, and by implication the NPV *measure of worth*, which is part of the decision criterion, implies a different "investment" and "project income" (return) than the IRR decision criterion, except when the internal rate(s) of return are equal to the external rate(s) of return. In this paper, it is demonstrated that both the IRR *decision criterion* and the NPV *decision criterion* assume that the capital that remains invested (or borrowed) in an opportunity grows at the IRR of the opportunity and cash released by the decision about the opportunity would be invested to grow at the decision maker's marginal growth rate (or "external" rate).

INTRODUCTION

In the Spring 1993 issue of *The Engineering Economist*, Bernhard [3] provides an interesting discussion of the IRR and NPV decision criteria as measures of project acceptability when time-dependent IRRs and (reinvestment) interest rates are used. Bernhard builds on Bailey's [1] earlier work that demonstrates that the IRR *measure of worth* can be generalized to show that there are an infinite number of internal rates of return for a given project. Hence, there are an infinite number of unrecovered investments in the project and corresponding project returns that accrue at these internal rates of return, and that none of these project returns are dependent upon the "market interest rates" which are external to the project; whereas, the IRR *decision criterion*, through its decision rules, does imply that the project returns would be reinvested at a rate(s) external to the pro-

ject. However, Bernhard also asserts that the NPV *decision criterion*, and by implication the NPV *measure of worth*, which is part of the decision criterion, implies a different "investment" and "project income" (return) from that investment than the IRR decision criterion, except when the internal rate(s) of return are equal to the external rate(s) of return.

Bernhard's assertion rests principally on two propositions: (1) the "wealth" and "income" implied by the IRR decision criterion, $\{U_t, Z_t\}$, respectively, are generally not the same as the "wealth" and "income" implied by the NPV decision criterion, $\{B_t, Y_t\}$, respectively, and (2) the decision maker may liquidate an opportunity, summarized by the net cash flows A_t, at time t for an amount B_t. It is the viewpoint of this paper that these propositions do not demonstrate Bernhard's assertion, and instead, that the NPV *decision criterion* (and by implication, the NFV *decision criterion*) assume that the capital that remains invested (or borrowed) in an opportunity grows at the IRR of the opportunity and cash released by the decision about the opportunity would be invested to grow at the decision maker's marginal growth rate (or "external" rate). The viewpoint of this paper will be presented by comparing Bernhard [3] with an earlier and related paper by Lohmann [5].

A COMPARISON: VIEWPOINT AND ASSUMPTIONS

The following comparison will focus on the principal issues related to the reinvestment rate assumptions. For example, Bernhard employs the assumptions of certainty and a perfect capital market. [3, p. 165] These assumptions are not necessary to address the reinvestment rate assumptions, although they do facilitate the discussion. Further, although his discussion of the time-dependent IRRs and interest rates is interesting, this, too, is not necessary to address the reinvestment rate assumptions. To facilitate the following discussion, notation used by Bernhard will appear in boldface whereas notation used by Lohmann will appear in regular typeface. Additionally, it will be assumed that the interest rate i_t in Bernhard is equivalent to the marginal growth rate m in Lohmann, i.e., $i_t = m$ for all t. (Values of m_t could be used, but, again, they are not necessary to address the reinvestment rate assumptions.)

As was done in [5, p. 305], the viewpoint taken here is based on two central principles of economic decision making, namely,

1 • an economic decision between alternatives should be based on the prospective differences in the real (out-of-pocket) monetary consequences attributable (relevant) to the decision. All that is common between the alternatives is irrelevant to the decision, because commonalities cancel out in taking the differences; and

2 • the real (out-of-pocket) monetary consequences relevant to the decision should be summarized by the real (out-of-pocket) net cash flows that identify the amount of each net cash flow and its time of occurrence.

Bernhard defines the quantities U_t, Z_t, B_t, and Y_t as [3, pp. 166-167] as follows.

U_t ≡ Investor's 'unrecovered investment,' in dollars, in the project as of time t, immediately after the net cash flow then, as implied by the set, $(r_1, r_2,...,r_T)$, of internal rates of return.
= $A_{t+1}(1 + r_{t+1})^{-1} + A_{t+2}[(1 + r_{t+1})(1 + r_{t+2})]^{-1} + \cdots + A_T[(1 + r_{t+1})(1 + r_{t+2})...(1 + r_T)]^{-1}$,
where $t = 0,1,...,T$.

Z_t ≡ Time t income, in dollars, from the project to the investor, as implied by the criterion of a set, $(r_1, r_2,...,r_T)$, of internal rates of return.
= 0 for $t = 0$, and $r_t U_{t-1}$ for $t = 1,2,...,T$.

B_t ≡ Time t present value, in dollars, of project's post-t net cash flows.
= $A_{t+1}(1 + i_{t+1})^{-1} + A_{t+2}[(1 + i_{t+1})(1 + i_{t+2})]^{-1} + \cdots + A_T[(1 + i_{t+1})(1 + i_{t+2})...(1 + i_T)]^{-1}$, where $t = 0,1,...,T$. Note that $B_t = B_{t-1}(1 + i_t) - A_t$ for $t = 1,2,...,T$, and that $B_T = 0$.

Y_t ≡ Time t income, in dollars, from the project to the investor, as implied by the net present value criterion.
= NPV for $t = 0$, and $i_t B_{t-1}$ for $t = 1,2,...,T$.

Each proposition will be addressed separately, although they are interrelated.

PROPOSITION 1

In a decision about an opportunity j, whose cash flows are summarized by A_t, the investor's "unrecovered investment," U_t, and the "income" earned from the investment, Z_t, are related to a discussion of the reinvestment rate assumptions. Further, they are conceptually identical to the capital invested (or borrowed) function, $K[j,t]$, and the returns (or interest) on capital, $r[j,t]$, respectively, in Lohmann, where $K[j,t]$ and $r[j,t]$ are defined:

$K[j,t] = K[j,t-1](1 + r) - s[j,t]$ for $t = 0,1,...,H$, $K[j,t] = 0$ for $t < 0$ and $t \geq H$, and opportunity j is summarized by the net cash flow vector $\underline{s}_j$ whose elements are $s[j,t]$. The internal rate of return of opportunity j is represented as $r = IRR(\underline{s}_j) = \{r \mid K[j,H] = 0\}$, and H is the horizon time for the decision. All relevant net cash flows occur between $t = 0$ and H.

$r[j,t] = rK[j,t-1]$

The net cash flows in Bernhard, A_t, are equivalent to those in Lohmann, $s[j,t]$, i.e., $s[j,t] = A_t$. The only difference between U_t and $K[j,t]$, and Z_t and $r[j,t]$, is that Bernhard permits time-dependent IRRs whereas Lohmann assumes a time-constant rate, i.e, $r_t = r$ for all t.

However, a comparison of the "investment" B_t "implied (to be invested in the project) by the net present value criterion" [3, p. 168, parenthetical statement added] with U_t is a comparison of two fundamentally different quantities. Further, since Y_t is based on B_t, a comparison of Z_t and Y_t is also a comparison of fundamentally different quantities.

Assume a decision maker (owner) is faced with an independent decision about opportunity j at time $t = 0$ as summarized by the net cash flows A_t (or $\underline{s}_j$). It is an important distinction (discussed further in the next section) that the decision maker here is the "owner," as opposed to a "buyer" (market) interested in the purchase of all or part of opportunity j from the owner. A buyer would affect the owner's decision about opportunity j only through the owner's inclusion of the buyer's offer in the net cash flows that describe opportunity j. Thus, opportunity j's market (salvage) value is related to the discussion about the reinvestment rate assumptions only to the extent that it is a cash flow that needs to be included in the net cash flows A_t (or $\underline{s}_j$) describing opportunity j before the application of a specified decision criterion. Further, it is also important to the discussion to emphasize that the decision maker (owner) is faced with an independent accept/reject decision about an opportunity j, summarized by A_t (or $\underline{s}_j$), and, hence, a choice. Given such a choice, Lohmann illustrates [5, p. 306] that the cash released by a decision about opportunity j involves a choice between either receiving the benefit series, $b[j,t]$ (accept j), or the cost series, $c[j,t]$ (reject j), defined as:

$s[j,t] = b[j,t] - c[j,t]$, where

$b[j,t] = s[j,t]$ if $s[j,t] > 0$, else $b[j,t] = 0$, and

$c[j,t] = -s[j,t]$ if $s[j,t] < 0$, else $c[j,t] = 0$.

Thus, values of $A_t > 0$ for $t > 0$ in Bernhard correspond to the benefit series $b[j,t]$ defined in Lohmann and can be associated with the measure of worth, prospective future benefit, $B(\underline{b}_j)$,

$$B(\underline{b}_j) = \sum_{t=0}^{H} b[j,t](1+m)^{H-t}.$$

Further, values of $A_t < 0$ correspond to the cost series $c[j,t]$ and can be associated with the measure of worth, prospective future cost, $C(\underline{c}_j)$,

$$C(\underline{c}_j) = \sum_{t=0}^{H} c[j,t](1+m)^{H-t}.$$

Therefore, at $t = 0$, the decision time about opportunity j, it can be shown that $B(\underline{b}_j) - C(\underline{c}_j) = NFV(\underline{s}_j) = \{B_0 + A_0\}\{1 + m\}^H$. Further, to facilitate the discussion momentarily, assume opportunity j's net cash flows are such that $A_t > 0$ for $t > 0$ and $A_0 < 0$. In this special case, the quantity $B_0\{1 + m\}^H$ is equivalent to the measure of worth $B(\underline{b}_j)$. Thus, they both are estimates of the *incremental* increase (or decrease) in the decision maker's (owner's) future wealth if the decision maker invests the cash released (A_t for $t > 0$ in the net cash flow case posed) in marginal (external) investment opportunities, summarized by rate m, as a consequence of a decision to *accept* opportunity j. Similarly, $A_0\{1 + m\}^H$ is equivalent to the measure of worth $C(\underline{c}_j)$ and they are both estimates of the *incremental* increase (or decrease) in the decision maker's (owner's) future wealth if the decision maker invests the cash released (A_0 in the net cash flow case posed) in marginal (external) investment opportunities, summarized by rate m, as a consequence of a decision to *reject* opportunity j. Thus, in this case, B_0 is conceptually equivalent to the measure of worth value $B(\underline{b}_j)$, the only difference is the former is expressed in present value and the later in future value.

However, the values of B_t for $t > 0$ for the net cash flow case posed, and B_t for the more general case when opportunities have multiple values of $A_t > 0$ and $A_t < 0$ for various t, cannot be said to be equivalent to the measure of worth $B(\underline{b}_j)$ because: (a) in both cash flow cases, B_t includes cash flows only after time t for opportunity j whereas the measures of worth $B(\underline{b}_j)$ and $C(\underline{c}_j)$ always include *all* the net cash flows relevant to the decision about opportunity j, and (b), in the more general case, B_t includes some component cash flows associated with the decision to reject, namely $c[j,t]$, in addition to those associated with the decision to accept, namely $b[j,t]$. The point here is not to suggest that the quantity B_t is, in general, a measure of worth equivalent to the measures of worth $B(\underline{b}_j)$ and $C(\underline{c}_j)$, which it is not, but rather to illustrate some linkages between Bernhard and Lohmann with respect to B_t.

The most specific statement that can be made about Bernhard's "investment" B_t in relation to Lohmann's work is that it is either: (a) in the special net cash flow case, an estimate of the *incremental* increase (if $B_t > 0$) or decrease (if $B_t < 0$) of the decision maker's (owner's) future wealth (expressed in terms of present value) by investing the cash released after time t in (external) marginal investment opportunities at rate m as a consequence of the decision maker's decision to accept opportunity j at time 0, or (b) in the more general case, an estimate of the *incremental* increase or decrease of the decision maker's future wealth (expressed in terms of present value) by investing after time t some of the cash released by a decision to accept and some cash released by a decision to reject at rate m as a

consequence of the decision maker's accept or reject decision about opportunity j at time 0. In either case, however, it is an estimate of *incremental* wealth to be accrued from the reinvestment of the cash released at rate m and it is not an estimate of the capital that remains invested in opportunity j at time t. Only the unrecovered investment, U_t, represents the capital that remains invested (or borrowed) internally at time t in opportunity j if the decision maker accepts opportunity j at time 0. Hence, B_t and U_t are estimates of *different* quantities. What a decision maker might receive at time t from a buyer (market) for all or part of opportunity j is a matter of cash flow estimation for the decision maker (owner) and bidding strategy for the buyer (market). It is not related to the discussion of the reinvestment rate assumptions in the IRR, NPV, or NFV decision criteria used by the decision maker to judge the net cash flows describing opportunity j. If the decision maker accepts opportunity j and it so happens that the decision maker receives from a buyer at time t an amount equal to U_t, then it would result in no economic gain or loss to the decision maker during the period of time 0 to t than was otherwise expected by the decision maker at time 0 when opportunity j was accepted.

PROPOSITION 2

Bernhard states "with the assumed certainty and perfect capital market, B_t is the amount for which the project could be sold to another investor as of time t, immediately after the net cash flow, A_t, then, and thus is, indeed, the 'market value' of the project at that time." [3, p. 167] This statement co-mingles the concepts of identifying the relevant alternatives (Principle 1) and estimating their relevant real (out-of-pocket) monetary consequences (Principle 2) -- which apply regardless of the assumptions of either certainty or perfect capital markets -- with the concepts of cash flow estimation and techniques to estimate cash flows given one's assumptions about the future (e.g., certainty and perfect capital markets).

For example, assume a decision maker (investor) is considering opportunity j whose life is n_j, where n_j is the time beyond which there are no cash flow consequences relevant to the decision about opportunity j. The net cash flows A_t (or $\underline{A}_j$) describing opportunity j should, of course, include all the relevant monetary consequences related to the acquisition, ownership, and disposition (including salvage/market value) of the opportunity. If the decision maker also wishes to consider at time 0 an *alternative* in which opportunity j is liquidated at a time other than n_j, then the decision maker would need to construct the relevant alternative (Principle 1), call it opportunity j', and estimate the real net cash flows (Principle 2) associated with the alternative, including the estimated salvage/market value at time $n_{j'}$. Of course, the decision maker could construct multiple alternatives, each with its own different liquidation time. Having done so, the decision maker could proceed with an appropriate analysis using the desired

decision criterion. The point here is that the market/liquidation/salvage value, by whatever means chosen to estimate (e.g., B_t) and by under whatever assumptions (e.g., certainty and perfect capital markets), should simply be included in the net cash flow A_t (or $s[j,t]$) at the liquidation time. The decision criterion can then be applied.

CONCLUSION

The confusion about the underlying assumptions supporting the IRR and NPV, and other equivalent decision criteria, continues to be a source for the generation of alternative decision criteria to compensate for the IRR's and/or NPV's perceived shortcomings [2,4]. A resolution of this confusion is important because it is rooted in two fundamental principles of economic decision making. These principles are important and powerful concepts and their implications, although occasionally subtle, are central to engineering economy in general.

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BIOGRAPHICAL SKETCH

JACK R. LOHMANN is Professor of Industrial and Systems Engineering and Associate Dean for Academic Affairs, College of Engineering, at the Georgia Institute of Technology. His research and teaching issues include engineering economic decision analysis, capital budgeting under risk and uncertainty, and replacement analysis of evolving technologies. Dr. Lohmann has received several outstanding teaching citations and a number of awards for his professional contributions, including a Presidential Young Investigator Award from the White House Office of Science and Technology Policy, Outstanding Young Manufacturing Engineer from Society of Manufacturing Engineers, Director's Award for Excellence from the National Science Foundation, and Centenniel Certificate from the American Society for Engineering Education. He is also currently the Editor of THE ENGINEERING ECONOMIST.