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ERRORS IN PRESENT WORTH EVALUATIONS ATTRIBUTABLE TO THE END-OF-YEAR AND MID-YEAR CASH FLOW CONVENTIONS

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INTRODUCTION

Several technical notes in this journal [4], [5], [6], and [7] have addressed the problem of computational errors introduced in the use of the classic engineering economy end-of-year cash flow convention. They proposed modifications to the convention that were intended to improve the computational accuracy in performing discounting (or compounding) calculations. This note summarizes briefly the results of a rather detailed study of errors in the value of present worths attributable to the end-of-year cash flow convention and the mid-year cash flow convention. The results suggest that unless the analyst is dealing with high discount rates or has fewer than a half dozen cash flows within each year, the additional accuracy gained by the modifications discussed in these notes probably does not justify the additional calculations. The results also show that the mid-year convention tends to be more accurate than the end-of-year convention, and a formula to convert a present worth evaluated with the end-of-year convention to its counterpart evaluated with mid-year convention is presented.

CASH FLOW CONVENTIONS

An engineering economy study involves forecasts of the amounts and timing of future cash flows. To simplify both forecasting and the subsequent computations, certain cash flow conventions (models) about the timing and the pattern of the anticipated cash flow have evolved. Among the many cash flow conventions used, the end-of-year (EOY) cash flow convention is probabily the most widely known. It has been adopted in most engineering economy texts. In essence, it treats all cash flows occurring after time t and through time $t+1$ as though they actually occurred at time $t+1$. It should be remembered, however, that the *EOY* convention is really a special case of the more general end-of-period convention where an arbitrary period (unit) of time can be selected for the economy study. By making the time period sufficiently short, the errors attributable to the cash flow convention could be made arbitrarily small. The mid-year (*MY*) cash flow convention, which treats all cash flows occurring after time t and through time $t+1$ as occurring at time $t+1/2$, i.e., as a 'lump-sum' in the middle of the year, is thought to be the second most widely used convention.

Cash flow conventions are approximations to reality, of course, and are used to facilitate the solution of economy studies at the sacrifice of some accuracy in the results. Occasionally a cash flow convention will model the anticipated cash flows accurately. For example, some loans and leases can be modeled precisely with an *EDY* convention. However, the cash flows of many productive investment opportunities and borrowing opportunities of a non-contractural nature can only be approximated by either the *EOY* or the *MY* conventions.

Errors in representation lead to errors in evaluation, of course. One source of error is uncertainty about the amounts and timing of future cash flows, that is, errors in estimation. The effect of such errors is difficult, if not impossible, to evaluate with classical mathematical analysis, however recent research used computer simulation to study the effect of errors in estimation of the expected values of cash flows on the long run financial performance of a firm [1], [3]. Errors introduced in present worth computations attributable to the cash flow convention adopted can be evaluated more easily.

A STUDY OF COMPUTATION ERRORS

The authors made a detailed, general study of the errors attributable to the *EOY* and *MY* cash flow conventions in present worth computations. Mathematical fomulas were developed for determining the relative error in computing the present worth of six commonly encountered cash flow patterns: a single cash flow, uniform series, exponential series, arithmetic (gradient) series, step-exponential series and step-arithmetic (gradient) series. The formulas expressed the relative computational error in terms of the ratio of the actual present worth (of a hypothetical or actual cash flow series spanning n years with m cash flows occurring each year) to the computed present worth when the m actual cash flows each year were assumed to occur as 'lump-sums' either at the end or at the middle of each year, accordingly as the *EOY* or the *MY* convention was adopted. The present worth evaluation of the actual cash flow series assumed m compounding periods per year. The formulas were used to evaluate the effects of varying the following factors: (1) the number of cash flows each year, m $(1 \leq m < \infty)$, (2) the number of years, n, (3) the effective annual discount rate, r, and (4) the actual timing of the m intra-year cash flows either at the end, E, middle, M, or beginning, B, of the m periods within each year.

In this note, we define the error by formula

$$e = [PW \text{ using } EOY \text{ or } MY/PW \text{ of actual series}] - 1.0 \quad (1)$$

This ratio is the reciprocal of the one described above. Values of $e > 0$ represent overestimates and values of $e < 0$ represent underestimates of the actual *PW*.

A brief summary of the results of the general study follows.

(1) Except for the arithmetic series, the computational error is solely a function of the discount rate, r, and the number of cash flows per year, m. For the arithmetic series, the error is also a function of n; however, the incremental error attributable to n is small for $n < 7$ years and negligible for $n > 7$ years.

(2) For all cash flow patterns considered, the magnitude of the error increases as the discount rate, r, increases but the error approaches a limit as the number of cash flows per year, m, increases. Figure 1 illustrates this result graphically for a uniform series and two discount rates, 0.10 and 0.20, and for values of m ranging from one to twelve cash flows per year. In Figure 1, the left graph resulted when $r = 0.10$ and the right when $r = 0.20$. The three solid lines show the relative error from the use of the *EOY* convention, and the dashed from the use of the *MY* convention, when the actual cash flows occur either at the end (E), middle (M), or beginning (B) of the m periods within each year.

(3) As illustrated by Figure 1, the *EOY* convention never overestimates the actual *PW*, i.e., $e < 0$, but the *MY* convention can either underestimate or overestimate the actual *PW*. The *EOY* convention's maximum error results, regardless of the cash flow pattern, when the cash flows actually occur

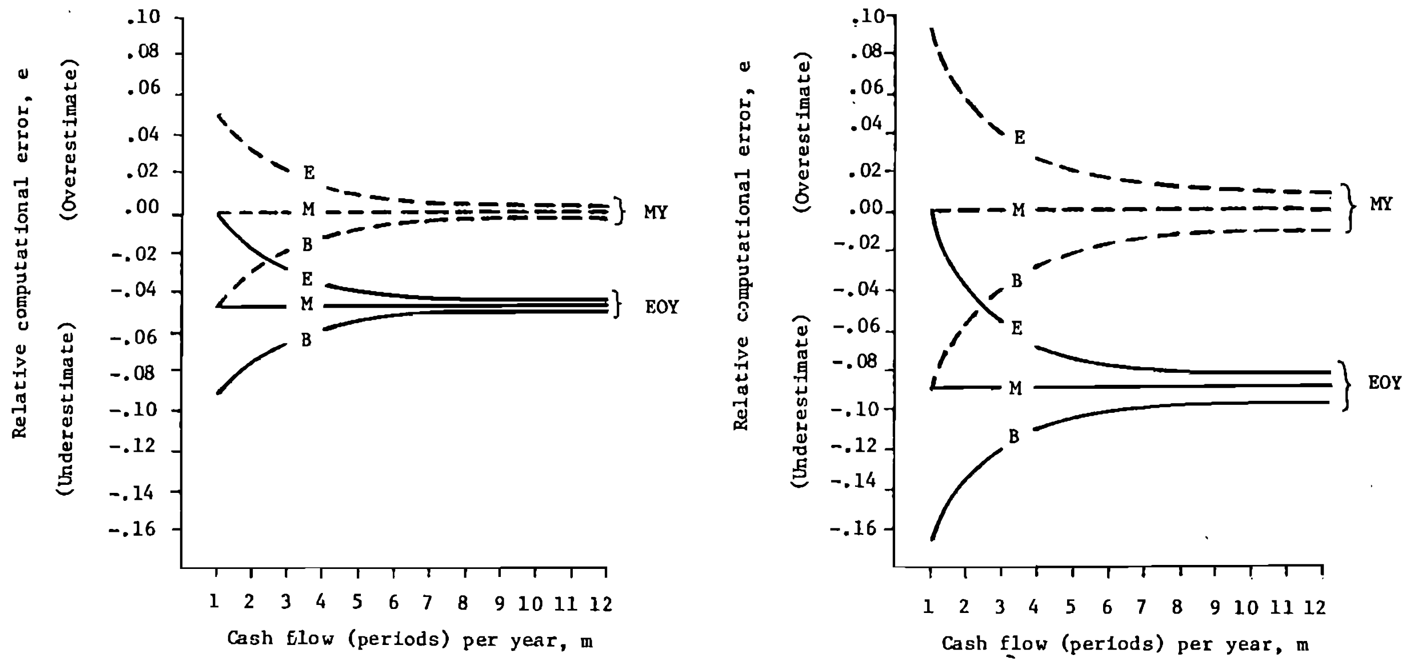

Figure 1. Example of relative computational error when the actual cash flows are a uniform series and the discount rate is r = 0.10 (left) and r = 0.20 (right). The sets of three curves for each convention (solid lines EOY and dashed lines MY) show the relative error when the actual cash flows occur either at the end (E), middle (M) or beginning (B) of the m periods per year.

at the beginning of each year, and the error is equal to

$$e_{EOY,max} = -r/(1+r) \qquad (2)$$

The minimum error is, of course, zero when the cash flows actually occur at the end of each year. The *MY* convention's maximum underestimate, regardless of the cash flow pattern, results when the cash flows actually occur at the beginning of the year, and is

$$e_{MY,max\ (under)} = (1+r)^{-1/2} - 1 \qquad (3)$$

Similarly, the *MY* convention's maximum overestimate results when the cash flows actually occur at the end of each year and is

$$e_{MY,max\ (over)} = (1+r)^{1/2} - 1 \qquad (4)$$

The minimum error for the *MY* convention occurs when the actual cash flows occur annually at the middle of the year and the error is, of course, zero.

(4) As m increases (approaching infinity), a discrete cash flow pattern approaches a corresponding continuous cash flow pattern as a limit, and the maximum and minimum relative errors from using either the *EOY* or the *MY* convention converge to either $e_{EOY,\infty}$ or $e_{MY,\infty}$ which represent the relative errors for the corresponding continuous cash flow patterns.

If the continuous cash flow pattern is exponential, of which the uniform is a special case, the relative error introduced by the *EOY* convention

$$e_{EOY,\infty} = [((z_0 - 1)/ln(z_0))/((z-1)/ln(z))] - 1 \qquad (5)$$

which exceeds (is more negative than) that introduced by the *MY* convention

$$e_{MY,\infty} = [((z_0 - 1)/ln(z_0))/(z-1)/ln(z))](1+r)^{-1/2} - 1 \qquad (6)$$

where $z = (1+r)/(1+\gamma)$, $z_0 = 1\ (1+\gamma)$, and γ is the rate of growth ($\gamma>0$) or decay ($\gamma<0$) of the cash flow in the exponential patterns. Note that a uniform cash flow pattern results when $\gamma = 0$.

If the continuous cash flow pattern is linear (the limit of an arithmetic series starting at time zero), the relative error by the *EOY* convention is

$$e_{EOY,\infty} = [((P/A,r,n)/2 + (P/G,r,n))/(P/\bar{G},r,n)] - 1 \qquad (7)$$

and the *MY* error is

$$e_{MY,\infty} = [((P/A,r,n)/2 + (P/G,r,n))/(P/\bar{G},r,n)](1+r)^{-1/2} - 1 \qquad (8)$$

where $(P/\bar{G},r,n)$ is the equivalence factor for a continuous linear flow function. See Oakford [2, pp. 62-64], the symbol used there is $LFF(g,n)$.

(5) In general, the *MY* convention introduces less computational error than the *EOY* convention. Notice in Figure 1 that the curves for the *MY* convention are centered around $e=0$ and their ranges are comparable to those for the *EOY* convention which lie below $e=0$. Similar results were observed for the other cash flow patterns considered in the general study cited above.

(6) Conversion of a *PW* evaluated with the *EOY* convention to its counterpart evaluated with the *MY* convention is a simple matter:

$$PW_{MY} = (1+r)^{1/2}\ PW_{EOY} \qquad (9)$$

CONCLUSION

For most cases, the impact of cash flow conventions is primarily a matter of the relative accuracy of the results. However, contradictory accept-reject decisions can result. For instance, consider a $3,750 investment which is expected to yield a uniform $100 end of month series for 5 years. The actual present worth of the uniform series with r = 0.2 is $3,906.88. However, using Figure 1 or computing directly, the *EOY* convention present worth of the uniform series is $3,588.73 and its *MY* present worth is $3,931.26. Consequently, the actual present worth is + $156.88 whereas the *EOY* present worth is - $161.27 and the *MY* present worth is $181.26. Other things being equal, the conventions suggest opposite decisions.

It is likely that errors in estimation of the amount and timing of the cash flow will have a greater influence on the results of an economy study than the computational error introduced by the cash flow convention adopted. As mentioned earlier, some research has already begun to investigate the influence of errors in estimation of the magnitudes (but not the timing) of cash flows on the long run financial performance of hypothetical firms [1], [3]. Further research is needed in this long neglected area.

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